eClassroom

INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 2

Further Pure 2

Chapter 5: First-Order Differential Equations

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM02/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Inequalities** Rational and modulus inequalities, critical values, graphs	10	80	separate file	
2	**Series** Method of differences, partial fractions, telescoping sums	10	80	separate file	
3	**Complex Numbers** Exponential form, de Moivre's theorem, trigonometric identities, nth roots	10	80	separate file	
4	**Further Argand Diagrams** Loci, regions and transformations of the complex plane	10	80	separate file	
5	**First-Order Differential Equations** Integrating factors, substitutions, general and particular solutions	10	80	2	
6	**Second-Order Differential Equations** Auxiliary equations, complementary functions, particular integrals, substitutions	10	80	separate file	
7	**Maclaurin and Taylor Series** Higher derivatives, standard and compound series, Taylor series, series solutions	10	80	separate file	
8	**Polar Coordinates** Polar curves, areas, tangents parallel and perpendicular to the initial line	10	80	separate file	

Full mark scheme Free courses More worksheets

Name ..... Class ..... Date .....

★★★★★

Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

CHAPTER 5 • KEY SKILLS

5 First-Order Differential Equations

Specification FP2: First-order differential equations • 10 questions • 80 marks • about 96 minutes

1 Standard form

Write $\frac{dy}{dx} + P(x)y = Q(x)$: divide by the coefficient of $\frac{dy}{dx}$ first.

2 Integrating factor

$e^{\int P\,dx}$, simplified: $e^{2\ln x} = x^2$, $e^{-\int \tan x\,dx} = \cos x$

3 Multiply through

Every term is multiplied by the integrating factor; the left side becomes $\frac{d}{dx}(\text{IF} \times y)$

4 Integrate the right

Integrate IF $\times$ Q(x), often by parts; watch the signs and coefficients.

5 The constant

Add c when you integrate, then divide everything by the IF: $y = \ldots + ce^{-2x}$, not $\ldots + c$

6 Particular solutions

Substitute the condition into the general solution to find c, then write "$y = \ldots$".

7 Substitutions

From $z = y^{-1}$: $\frac{dz}{dx} = -y^{-2}\frac{dy}{dx}$. Replace every y and $\frac{dy}{dx}$, showing all steps.

8 Reverse the substitution

Put the original variable back: if $z = \frac{1}{y}$ then $y = \frac{1}{z}$, taking the reciprocal of the whole expression.

9 Interpreting

Use the solution to find greatest or least values, limits as $x \to \infty$ and asymptotes.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Standard form	•	•	•	•	•	•		•	•	
2 Integrating factor	•	•	•	•	•	•		•	•	
3 Multiply through	•	•	•	•	•	•		•	•	
4 Integrate the right			•	•	•	•				
5 The constant	•	•	•	•	•	•	•	•	•	•
6 Particular solutions			•			•	•	•	•	•
7 Substitutions					•	•	•			•
8 Reverse the substitution					•	•	•			•
9 Interpreting								•	•	•
Interleaved: integration by parts and trigonometric integrals (P3, P4)			•	•	•	•				
Interleaved: separating variables (P4)							•			•
Interleaved: differentiation, stationary points and asymptotes (P2, P3)								•	•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Standard form, integrating factor, multiply every term, integrate, add c, then divide everything by the integrating factor. In a substitution "show that", show each replacement.

★★★★★

Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

1. Find the general solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \mathrm{e}^{-x}$$

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 1 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 1 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

2. Using an integrating factor, solve

$$x\frac{dy}{dx} + 2y = x^2, \qquad x > 0$$

expressing y as a function of x.

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 2 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 2 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

3.

$$\frac{dy}{dx} - y\tan x = \cos x, \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(a) Find the general solution of the differential equation.

(4)

(b) Find the particular solution for which $y = 0$ when $x = 0$

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 3 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 3 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

4. Solve the differential equation

$$\frac{dy}{dx} + \frac{y}{x} = \ln x, \qquad x > 0$$

giving y in terms of x.

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 4 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 4 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

5.

$$\frac{dy}{dx} + y = xy^2 \qquad \text{(I)}$$

(a) Show that the substitution $z = \frac{1}{y}$ transforms equation (I) into $\frac{dz}{dx} - z = -x$

(3)

(b) Hence find the general solution of equation (I), giving y in terms of x.

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 5 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 5 is 6 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

6.

$$\frac{dy}{dx} + 2y = 2xy^3 \qquad \text{(I)}$$

(a) Show that the substitution $z = y^{-2}$ transforms equation (I) into $\frac{dz}{dx} - 4z = -4x$ (II)

(5)

(b) Solve equation (II), and hence find the general solution of equation (I) in the form $y^2 = f(x)$

(4)

(c) Find the particular solution of (I) for which $y = 2$ when $x = 0$

(1)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 6 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

7.

$$\frac{dy}{dx} = \frac{x^2 + y^2}{xy}, \qquad x > 0,\ y > 0$$

(a) By writing $y = vx$, with v depending on x, reduce this to the separable equation $x\frac{dv}{dx} = \frac{1}{v}$

(4)

(b) Solve for v, and deduce an expression for y^2 in terms of x and an arbitrary constant.

(4)

(c) Determine the solution curve through the point (1, 2)

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 7 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

8. The speed $v\,\text{m s}^{-1}$ of a model car at time t seconds satisfies

$$\frac{\mathrm{d}v}{\mathrm{d}t} + \frac{v}{t+1} = 2, \qquad t \geqslant 0$$

and $v = 5$ when $t = 0$

(a) Show that $v = t + 1 + \dfrac{4}{t+1}$

(6)

(b) Find the least speed of the car and the time at which it occurs.

(3)

(c) Describe how the speed changes for large values of t.

(1)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 8 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

9.

$$\left(x^2+1\right)\frac{\mathrm{d}y}{\mathrm{d}x}+2xy=4x$$

(a) Find the general solution of the differential equation.

(5)

(b) Find the particular solution whose graph passes through (0, 3)

(2)

(c) For this particular solution, show that $y=2+\frac{1}{x^2+1}$, and hence state the range of values of y.

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 9 is 10 marks)

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

10.

$$\frac{dy}{dx} = (y - x)^2 + 1$$

(a) Show that the substitution $u = y - x$ transforms the equation into $\frac{du}{dx} = u^2$

(3)

(b) Hence find the general solution of the original equation, giving y in terms of x.

(4)

(c) Find the particular solution for which $y = 1$ when $x = 0$, and state the equation of the vertical asymptote of its graph.

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 10 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk

Question 10 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 5 IS 80 MARKS

★★★★★
Join students already acing their exams.
Book your FREE lesson at www.eclassroom.org.uk