eClassroom

A-LEVEL MATHEMATICS • DECISION 1

Algorithms

Worked Solutions

Pearson Edexcel International A Level (WDM11) • Calculator allowed: show every step

Full worked solutions for the free Algorithms worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

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Tap a section to jump straight to it

Full worked solutions for every question on the Decision 1 Algorithms worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Decision 1 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 Swap 7,2; swap 7,6; keep 7,9; swap 9,3: **2 6 7 3 9**

A2 Pivot 40 (4th): **31 12 25 17 8 28 40**

A3 **[6, 4] [8, 3] [5, 7]**

STRETCH

A4 $\frac{39}{15} = 2.6$: **3**

A5 Middle $\left[\frac{1+8}{2}\right] = 5$: 17, reject 5–8; middle 3: 9: **found at position 3**

A6 Sorted 8, 7, 6, 5, 4, 3: **[8, 4] [7, 5] [6, 3]**

A7 p: 1, 2, 6, 24, 120 as k runs 1 to 5: **120** (this is 5!)

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** **4 7 2 8 5 9** **(b)** **4 2 7 5 8 9** **(c)** pass 3: **2 4 5 7 8 9**; pass 4: no swaps **(d)** **4 passes**

The sort ends with a pass that makes no swaps.

B2 **(a)** **8** (5th of 8) **(b)** **8 23 41 16 35 29 12 47** **(c)** pivot 35: **8 23 16 29 12 35 41 47**

(d) pivots 29, 47: 8 23 16 12 **29** 35 41 **47**; pivot 16: **8 12 16 23 29 35 41 47**

An even number of items uses the item right of the middle.

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B3 **(a)** **2.7** **(b)** **3.4 5.2 4.6 3.9 2.7 1.8 0.9** **(c)** **4.6 and 0.9** **(d)** **5.2 4.6 3.9 3.4 2.7 1.8 0.9**

Descending: larger items go to the left of the pivot.

B4 **(a)** $\frac{60}{20}$: **3** **(b)** **[7,12] [9,5,6] [13] [8]**: 4 bins **(c)** **[13,7] [12,8] [9,6,5]**: 3 bins **(d)** first-fit decreasing, as it meets the lower bound

Equal to the lower bound means optimal.

B5 **(a)** 27 (reject 1–6), 40 (reject 9–11), 36: **found** **(b)** 27, 13, 22, 18: **not found** **(c)** **4**

Each iteration halves what is left.

B6 **(a)** $\frac{40}{10}$: **4** **(b)** **7+3, 8+2, 6+4, 5+5**: 4 bins **(c)** **[7,3] [5,2] [8] [6,4] [5]**: 5 bins **(d)** the full-bin packing is optimal; first fit is not

Full bins can beat first fit.

STRETCH

B7 **(a)** **6, 3, 10, 5, 16, 8, 4, 2, 1** **(b)** **8** **(c)** **16** **(d)** the number of steps taken to reach 1

Follow each arrow to the box it points to.

B8 **(a)** **1 4 2 5 8**: 4 comparisons, 3 swaps **(b)** **1 2 4 5 8**: 3 comparisons, 1 swap **(c)** a swap was made, so a pass with no swaps is needed (2 comparisons) **(d)** $4+3+2+1$: **10**

Each pass compares one fewer pair.

B9 **(a)** $23+18 \leqslant n$: $n \geqslant 41$ **(b)** 12 fitted neither $41+12$ nor $31+12$: $n \leqslant 42$ **(c)** **41 or 42** **(d)** $37+8 = 45 > n$ and smaller sums also too big, which agrees

Every item that did not fit gives an upper limit.

B10 **(a)** **19**: everything left of it is larger and everything right is smaller **(b)** **6th** (right of the middle of ten)

(c) **43 35 21 40 50 57 39** and **5 17** **(d)** **40 and 17**

Only the pivot is guaranteed in its final place.

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C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 First fit places each item in the first bin it fits; the 2 fits Bin 1 ($5+3+2=10$): **[5, 3, 2] [7] [4, 6]**

C2 The order asked for is part of the question; an ascending list is a misread, which loses marks.

C3 He sorted each side instead of keeping the original order: **7 3 9 14 22 18 11**

C4 The algorithm stops only after a pass with no swaps; pass 4 makes **2 comparisons** and no swaps.

STRETCH

C5 The middle is $\left[\frac{1+8}{2}\right] = 5$ (17), not the 4th; and the middle item is rejected with its half, leaving 2, 5, 9, 13

C6 First-fit decreasing: [6, 5] [4, 3, 3] [3]: 3 bins; but **[6, 3, 3] [5, 4, 3]** uses 2 bins

C7 No two 6s fit in one bin ($12 > 10$), so each needs its own bin: **3 bins**

C8 **(a)** pass 1 makes $n-1$ comparisons, pass 2 makes $n-2$, …, down to 1: $\frac{n(n-1)}{2}$, as required

(b) **66**; for example **4 3 2 1** into ascending order ($3+2+1=6$)

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 **(a)** $\frac{2320}{700} = 3.31\ldots$: **4**

(b) **[220, 340] [180, 410] [260, 150, 270] [300, 190]**: 4 vans, which is optimal

D2 **(a)** **19**: all numbers before it are greater and all after it are smaller

(b) pivots 40, 17: 43 50 57 **40** 35 21 39 19 **17** 5; pivots 50, 21: 57 **50** 43 40 35 39 **21** 19 17 5; pivot 39: **57 50 43 40 39 35 21 19 17 5**

D3 **(a)** c increases at $k = 1, 2, 5, 10$: **output 4**

(b) the number of factors of N; an output of 2 means N is prime

D4 Bin 1: $23 + 18 = 41 \leqslant n$; the 12 did not fit Bin 1 (53) or Bin 2 ($31 + 12 = 43$), so $n \leqslant 42$: $n = 41$ **or** 42

STRETCH

D5 **(a)** pass 1: x stays before 18, so $x < 18$, and moves past 5, so $x > 5$; pass 2: x moves past 9 and 15, so $x < 9$: $5 < x < 9$

(b) **18 12 15 9 7 5 3**, **18 15 12 9 7 5 3**, then a pass with no swaps

D6 **(a)** middle 8: 30, reject 1–8; middle 12: 45, reject 12–15; middle 10: 38, reject 9–10; middle 11: 41, reject: **40 is not in the list**

(b) **4**

D7 **(a)** $\frac{118}{40} = 2.95$: **3**

(b) **[23, 16] [21, 19] [14, 11, 8, 6]**

(c) 3 trolleys equals the lower bound, so it is optimal

D8 **(a)** N: 29, 26, 21, 14, 5; k: 3, 5, 7, 9, 11; c: 1, 2, 3, 4, 5; $5 < 11$ so stop: **output 5**

(b) the integer part of $\sqrt{N}$ (the sum of the first c odd numbers is c^2)

(c) **7**

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $\frac{273}{60} = 4.55$ *M1*

5 discs *A1*

(b) **[24, 26] [18, 23, 12] [34, 11] [27, 28] [38] [32]** *M1 A1*

(c) pivot 27: 34 38 28 32 **27** 24 26 18 23 12 11 *M1*

pivots 28, 23: 34 38 32 **28** 27 24 26 **23** 18 12 11 *A1*

pivots 38, 26, 12: 38 34 32 28 27 26 24 23 18 12 11 *A1*

38 34 32 28 27 26 24 23 18 12 11 *A1*

(d) **[38, 18] [34, 26] [32, 28] [27, 24] [23, 12, 11]** *M1*

5 discs equals the lower bound, so optimal *A1*

E2 **(a)** pivot 5.4: 4.8 2.3 1.7 3.9 2.9 0.8 4.2 3.3 **5.4** 6.1 *M1*

pivot 2.9: 2.3 1.7 0.8 **2.9** 4.8 3.9 4.2 3.3 5.4 6.1 *A1*

pivots 1.7, 4.2: 0.8 **1.7** 2.3 2.9 3.9 3.3 **4.2** 4.8 5.4 6.1 *A1*

pivot 3.3: **0.8 1.7 2.3 2.9 3.3 3.9 4.2 4.8 5.4 6.1** *A1*

(b) middle 6: 3.9, reject 6–10 *M1*

middle 3: 2.3, reject 1–3; middle 5: 3.3, reject 4–5 *A1*

3.5 kg is not in the list *A1*

(c) **[6.1, 3.9] [5.4, 4.2] [4.8, 3.3, 1.7] [2.9, 2.3, 0.8]** *M1 A1 A1*

(d) $\frac{35.4}{10} = 3.54$, so at least 4 sacks; 4 were used, so optimal *M1 A1*

STRETCH

E3 **(a)** Bin 1: $33 + 34 + 10 = 77 \leqslant n$ *B1*

Bin 2: $39 + 17 + 21 = 77 \leqslant n$ *B1*

the 8 did not fit Bin 3: $30 + 20 + 25 + 8 = 83 > n$, so $n \leqslant 82$ *M1*

$77 \leqslant n \leqslant 82$ *A1*

(b) the 8 joins Bin 1: $39 + 35 + 8 = 82 \leqslant n$ *M1*

$n = 82$ *A1*

(c) $\frac{296}{82} = 3.6\ldots$, so at least 4 bins; 4 were used *M1 A1*

(d) **34 39 33 17 30 21 20 25 24 35 10 8** *M1 A1*

11 comparisons, 8 swaps *A1*

(e) at most $10 + 9 + \ldots + 1$ further comparisons: **55** *M1 A1 A1*

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