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A-LEVEL MATHEMATICS • DECISION 1

Critical Path Analysis

Worked Solutions

Pearson Edexcel International A Level (WDM11) • Calculator allowed: show every step

Full worked solutions for the free Critical Path Analysis worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

B **Fluency** core skills page 2

C **Reasoning** explain it page 4

D **Problem** solving page 4

E **Exam** practice page 6

Tap a section to jump straight to it

Full worked solutions for every question on the Decision 1 Critical Path Analysis worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Decision 1 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 P ends at 5; Q at 7, R at 11; S starts at 11: **14**

A2 Smallest of $25-3$ and $25-8$: **17**

A3 $20-9-7$: **4**

STRETCH

A4 **B–E–K, 21**

A5 $\frac{75}{24} = 3.125$, round up: **4**

A6 E and F would join the same two events: **a uniqueness dummy** at the end of one of them.

A7 It must start by $9-4=5$ and cannot finish before $2+4=6$: $5 < t < 6$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** events 1 to 6: **0, 4, 7, 9, 15, 19** **(b)** **19 days** **(c)** **0, 4, 7, 13, 15, 19** **(d)** event 4 (9 and 13)

Forwards take the largest; backwards take the smallest.

B2 **(a)** A 0, B 1, C 0, D 4, E 0, F 4, G 4, H 0 **(b)** **A, C, E, H** **(c)** **A–C–E–H** **(d)** it has a float of 4

Float = late end − early start − duration.

B3 **(a)** **A** **(b)** **A and B** **(c)** D depends on A and B, but C depends on A only **(d)** **F and I**

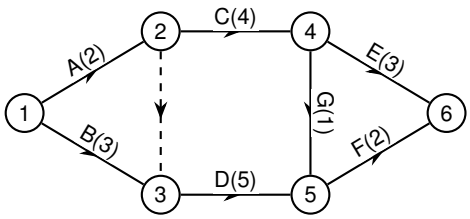
Follow dummies backwards to read precedences.

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B4 **(a)** **A and B** **(b)** D depends on A and B, but C depends on A only **(c)** see the network **(d)** **E and F**



Network for B4

One logic dummy, from the end of A to the start of D

B5 **(a)** **9** **(b)** $19 - 6 =$ **13** **(c)** G must start by 13 and cannot finish before 15, so it runs throughout $13 < t < 14$; E is critical from 7 to 15

Latest start and earliest finish fix the forced interval.

B6 **(a)** **38** **(b)** $\frac{38}{19}$: **2** **(c)** workers are whole numbers, and fewer would not have time **(d)** yes, the lower bound may not be achievable

A lower bound is not always enough.

STRETCH

B7 **(a)** **a logic dummy** **(b)** D needs both A and B; C needs A only **(c)** **1** **(d)** a dummy that stops two activities having the same start and end events

Two kinds of dummy, two purposes.

B8 **(a)** **4** **(b)** **19 days** (within the float) **(c)** **21 days** **(d)** **A, C, D, F, H**

Delays beyond the float extend the project.

B9 **(a)** **A and B** **(b)** **4** **(c)** worker 1: A 0–4, C 4–7, E 7–15, H 15–19; worker 2: B 0–6, D 6–11, F 11–13, G 13–19 **(d)** **19 days**

Two workers meet the lower bound here.

B10 **(a)** **9** **(b)** $15 - 9 - 5$: **1** **(c)** $15 - 9$: **6** **(d)** $9 + 9 + 4$: **22 days**

A new activity's float limits its duration.

C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 Late times take the *smallest* backward value: $\min(19-6, 15-2) = \boxed{13}$

C2 The critical path is the longest path through the network (A–C–E–H, 19 days), not the longest activity; E is critical because it lies on that path.

C3 Two activities may not join the same pair of events: add a uniqueness dummy so that P (or Q) ends at its own event, joined by the dummy.

C4 The floats must be shown (dashed) after each non-critical bar, so that the chart shows how far each activity can move.

C5 Day 14 is the interval $\boxed{13 < t < 14}$, not $14 < t < 15$.

C6 $\frac{50}{16} = 3.125$, so the lower bound is $\boxed{4}$: 3 workers can do only 48 days of work in 16 days.

C7 Using the float of 4 days exactly still finishes on time; only a delay of more than 4 days delays the project.

C8 **(a)** w workers can do at most $w \times T$ days of work in time T, so $w \geqslant \frac{\text{total}}{T}$; round up as w is a whole number

(b) on the longest path each event's early and late times are equal, so its activities have zero float; any longer path would give a later finish, so the completion time is the longest path

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

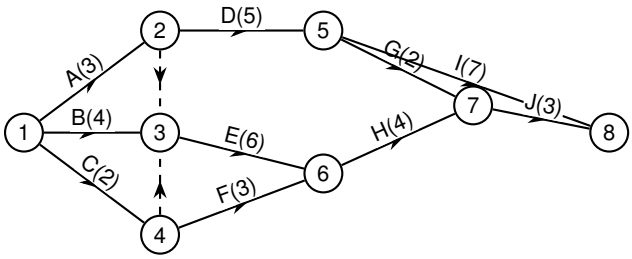
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CORE

D1 Dummies from the end of A and from the end of C to the start of E



Network for D1

D2 **(a)** early: **0, 5, 3, 6, 10, 12, 18, 22**; late: **0, 6, 3, 7, 10, 12, 18, 22** (events 1 to 8)

(b) **22 days**

D3 **(a)** $19 - 7 - 9$: **3 hours**

(b) total 67, $\frac{67}{23} = 2.91\ldots$: **3**

D4 G (critical, 9 to 12) and H (latest start 10, earliest finish 17) must both run: **G and H, at least 2 workers**

STRETCH

D5 **(a)** **19 days**; critical **B, E, H, I**

(b) D: **1**, J: **2**; D can start as late as **5**

D6 **(a)** D: A; E: A, B; G: C, D; H: E, F, I; I: C, D

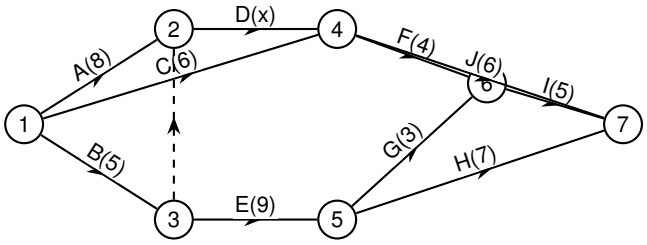
(b) the dummy 2–3 lets E depend on A without D depending on B; the dummy 4–5 lets G and I depend on C without D doing so

D7 **(a)** total 36, critical path 14: $\frac{36}{14} = 2.57\ldots$: **3**

(b) during $5 < t < 7$, C (3 to 7), D (5 to 7) and H (5 to 14) are critical and E (latest start 4, earliest finish 9) must also run: **4 workers are needed**

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D8 **(a)** one dummy, from the end of B to the start of D



Network for D8

(b) A–D–F–I is $17+x$; B–E–G–I is 22: only critical when $x > 5$

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** early times (events 1 to 10): 0, 4, 6, 3, 9, 10, 10, 15, 18, 24 *M1 A1*

late times: 0, 6, 6, 9, 13, 10, 16, 19, 18, 24 *M1 A1*

(b) $13-4-5=$ **4 days**: D can be delayed by up to 4 days without delaying the project *B1*

(c) critical B, E, I, L as solid bars; others with floats dashed (A 2, C 6, D 4, F 6, G 4, H 6, J 6, K 4) *M1 A1 A1*

(d) $\frac{59}{24} = 2.46$: lower bound 3; 3 workers can finish in 24 days (worker 1: A 0–4, D 4–9, G 9–15, K 15–20; worker 2: B 0–6, E 6–10, I 10–18; worker 3: C 0–3, F 3–10, J 10–12, H 12–15, L 18–24), so **3 workers** *B1 B1*

E2 **(a)** H: D, E; J: D, E, G; K: H, I; L: H, I *B1 B1*

(b) early (events 1 to 11): 0, 5, 4, 8, 11, 11, 12, 16, 18, 20, 25; late: 0, 5, 6, 12, 11, 13, 16, 16, 22, 20, 25 *M1 A1 A1*

(c) **A, D, H, L, N**; 25 weeks *B1*

(d) $22-12-6=$ **4 weeks** *B1*

(e) total 59; $\frac{59}{25} = 2.36$: **3 teams** *M1 A1*

(f) e.g. team 1: A 0–5, D 5–11, H 11–16, L 16–20, N 20–25; team 2: B 0–4, E 4–6, C 6–9, G 9–13, J 13–19, M 19–22; team 3: F 4–11, I 11–14, K 16–18 *M1 A1 A1*

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E3 **(a)** H is critical: event 5 has time 12 (B then E), so $12 + x + 6 = 28$ *M1*

$\boxed{x = 10}$ *A1*

late time of event 4 is $21 - 5 = 16$; float of C: $16 - 6 - y = 5$: $\boxed{y = 5}$ *A1*

(b) early (events 1 to 8): 0, 6, 4, 14, 12, 19, 22, 28; late: 0, 9, 4, 16, 12, 21, 22, 28 *M1 A1 A1*

(c) critical B, E, H, J solid; floats A 3, C 5, D 3, F 2, G 2, I 2, K 2 dashed *M1 A1 A1 A1*

(d) total 57; $\frac{57}{28} = 2.04$, so at least 3 workers *M1 A1*

with 3 workers the project finishes in 28 days: e.g. worker 1 B, E, H, J; worker 2 A, C, F, I; worker 3 D, G, K *M1*

$\boxed{\textbf{3 workers}}$ *A1*

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