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INTERNATIONAL A LEVEL MATHEMATICS • PURE 1

Pure Mathematics 1

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WMA11/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Algebraic Expressions Indices, surds, expanding and factorising	10	57	3	
2 Quadratics Completing the square, the discriminant, quadratic graphs	10	65	9	
3 Equations and Inequalities Simultaneous equations, inequalities, regions	10	69	15	
4 Graphs and Transformations Cubic and reciprocal graphs, intersections, transformations	10	68	21	
5 Straight-line Graphs Gradients, equations of lines, parallel and perpendicular	10	68	28	
6 Trigonometric Ratios Sine and cosine rules, area of a triangle, trig graphs	10	66	34	
7 Radians Radian measure, arc length, sector and segment areas	10	71	41	
8 Differentiation Gradients, second derivatives, tangents and normals	10	71	49	
9 Integration Integrating x^n , finding $y = f(x)$ from $f'(x)$	10	67	55	

Question booklet

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How to use this mark scheme

The same marking rules Pearson examiners use for WMA11.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

“Solutions relying on calculator technology are not acceptable.” When a question says this, a correct final answer with no algebraic working scores **no marks**. The notes under each scheme line say when this applies.



1

Chapter 1: Algebraic Expressions

Mark scheme. Marks are shown on the right of each line.

Question 1

4 marks

Part	Scheme	Marks
(a)	$27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}}$ or $27^{\frac{2}{3}} = 9$ or $(\sqrt[3]{27})^{-2} = 3^{-2}$ $\frac{1}{9}$ M1: deals correctly with either the negative sign (reciprocal) or the fractional power. A1: $\frac{1}{9}$ cao.	M1 A1 (2)
(b)	$(27x^6)^{\frac{2}{3}} = 9x^4$ or $(3x)^2 = 9x^2$ x^2 M1: numerator or denominator correctly simplified. A1: x^2 cao. Leaving $9x^4 \div 9x^2$ unsimplified scores A0.	M1 A1 (2)
Total for Question 1: 4 marks		

Question 2

6 marks

Part	Scheme	Marks
(a)	Takes out the common factor x : $x(2x^2 - 7x - 15)$ Attempts to factorise their quadratic: $(2x \pm 3)(x \pm 5)$ $x(2x + 3)(x - 5)$ A1: all three factors must be seen together, in any order. $(2x^2 + 3x)(x - 5)$ is not factorised completely: A0.	B1 M1 A1 (3)
(b)	Multiplies two brackets to get a three-term quadratic, e.g. $(x + 3)(2x - 1) = 2x^2 + 5x - 3$ Multiplies their quadratic by the third bracket: $(2x^2 + 5x - 3)(4 - x) = 8x^2 + 20x - 12 - 2x^3 - 5x^2 + 3x$ $-2x^3 + 3x^2 + 23x - 12$ dM1 depends on the first M1. A1: all four terms correct, in any order.	M1 dM1 A1 (3)
Total for Question 2: 6 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$\sqrt{50} = 5\sqrt{2}$ or $\sqrt{18} = 3\sqrt{2}$ $2\sqrt{2}$	M1 A1 (2)
(b)	$\frac{2\sqrt{2}}{3\sqrt{2}-4} \times \frac{3\sqrt{2}+4}{3\sqrt{2}+4}$ Numerator $2\sqrt{2}(3\sqrt{2}+4) = 12+8\sqrt{2}$ or denominator $18-16=2$ $6+4\sqrt{2}$ <i>First M1: multiplies top and bottom by $3\sqrt{2}+4$. Using the unsimplified numerator $\sqrt{50}-\sqrt{18}$ can score full marks. A decimal answer with no working scores 0.</i>	M1 M1 A1 (3)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	Attempts $(3-2\sqrt{x})(3-2\sqrt{x})$ giving 3 or 4 terms $9-12\sqrt{x}+4x$ <i>Accept $9-12x^{\frac{1}{2}}+4x$ with the terms in any order.</i>	M1 A1 (2)
(b)	Divides each term by $x^{\frac{1}{2}}$, with correct index work on at least one term, e.g. $\frac{4x}{x^{\frac{1}{2}}} = 4x^{\frac{1}{2}}$ Two of the three terms correct $9x^{-\frac{1}{2}}-12+4x^{\frac{1}{2}}$ <i>Accept $A=9, p=-\frac{1}{2}, B=-12, C=4, q=\frac{1}{2}$ stated separately. Follow through their expansion from (a) for M1 only.</i>	M1 A1 A1 (3)
Total for Question 4: 5 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	$BC = \frac{13+7\sqrt{3}}{2+\sqrt{3}}$ $\frac{(13+7\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})} = \frac{26-13\sqrt{3}+14\sqrt{3}-21}{4-3}$ $\boxed{5+\sqrt{3}}$ Alternative: $(2+\sqrt{3})(a+b\sqrt{3}) = 13+7\sqrt{3}$ gives $2a+3b=13$ and $a+2b=7$. M1 for both equations, M1 for solving them, A1 for $a=5, b=1$.	M1 M1 A1 (3)
(b)	Uses Pythagoras: $AC^2 = (2+\sqrt{3})^2 + (5+\sqrt{3})^2$ Squares one surd length correctly: $(2+\sqrt{3})^2 = 7+4\sqrt{3}$ or $(5+\sqrt{3})^2 = 28+10\sqrt{3}$ $AC^2 = 35+14\sqrt{3} = 7(5+2\sqrt{3})$ * A1* cso: both squares correct and $35+14\sqrt{3}$ seen before the given answer. Follow through their BC for the M marks only.	M1 M1 A1* (3)
Total for Question 5: 6 marks		

Question 6

6 marks

Part	Scheme	Marks
(a)	$2^{2x+1} = 2 \times (2^x)^2 = 2p^2$ $2^{x+4} + 2^x = 16 \times 2^x + 2^x = 17p$ $2p^2 + 8 = 17p \Rightarrow 2p^2 - 17p + 8 = 0$ * A1* cso: both B marks, with no errors in the rearrangement.	B1 B1 A1* (3)
(b)	Solves the quadratic: $(2p-1)(p-8) = 0$, so $p = \frac{1}{2}$ or $p = 8$ Uses $2^x =$ one of their values of p to find x, e.g. $2^x = 8 \Rightarrow x = 3$ $\boxed{x = -1, x = 3}$ Answers with no working score 0, because the question says solutions relying on calculator technology are not acceptable. dM1: accept $x = \log 8 \div \log 2$.	M1 dM1 A1 (3)
Total for Question 6: 6 marks		



Question 7

6 marks

Part	Scheme	Marks
	Writes the left-hand side of the first equation as a power of 2: $2^x \times 2^{2(y+1)} = 2^{x+2y+2}$	M1
	$32\sqrt{2} = 2^5 \times 2^{\frac{1}{2}} = 2^{\frac{11}{2}}$	B1
	Writes the left-hand side of the second equation as a power of 3: $\frac{3^{3x}}{3^{2y}} = 3^{3x-2y}$	M1
	$729\sqrt{3} = 3^6 \times 3^{\frac{1}{2}} = 3^{\frac{13}{2}}$	B1
	Equates indices and solves $x+2y = \frac{7}{2}$ and $3x-2y = \frac{13}{2}$	M1
	$x = \frac{5}{2}, y = \frac{1}{2}$	A1
	Final M1 needs two linear equations from equating indices, and an attempt to eliminate a variable. Answers with no working score 0. Accept $x = 2.5, y = 0.5$.	
		(6)
Total for Question 7: 6 marks		

Question 8

7 marks

Part	Scheme	Marks
(a)	Gradient of $AB = \frac{7-1}{2+2\sqrt{3}-\sqrt{3}} = \frac{6}{2+\sqrt{3}}$	M1
	$\frac{6}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{12-6\sqrt{3}}{4-3}$	M1
	$12-6\sqrt{3}$	A1
	Second M1: the rationalising step must be seen. A calculator answer with no rationalising scores M1 M0 A0.	
		(3)
(b)	Gradient of $l = \frac{-1}{12-6\sqrt{3}}$ (or $-\frac{2+\sqrt{3}}{6}$ after rationalising)	M1
	Equation of l : $y-1 = \frac{-1}{12-6\sqrt{3}}(x-\sqrt{3})$	M1
	Sets $y=0$ and solves: $x-\sqrt{3} = 12-6\sqrt{3}$	dM1
	$12-5\sqrt{3}$	A1
	First M1: uses the negative reciprocal of their gradient from (a). dM1 depends on the second M1. Seeing that $x-\sqrt{3}$ equals the gradient of AB avoids a second rationalisation, but either route scores full marks.	
		(4)
Total for Question 8: 7 marks		



Question 9

6 marks

Part	Scheme	Marks
(a)	Sets the equations equal and writes $\sqrt{2x} = \sqrt{2}\sqrt{x}$: $\sqrt{2}\sqrt{x} = \sqrt{x} + 3$ Collects the $\sqrt{x}$ terms: $\sqrt{x}(\sqrt{2} - 1) = 3 \Rightarrow \sqrt{x} = \frac{3}{\sqrt{2} - 1}$ $\sqrt{x} = \frac{3}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = \frac{3(\sqrt{2} + 1)}{2 - 1} = 3(\sqrt{2} + 1)$ * <i>A1* cso: the rationalising step must be shown. Squaring both sides at the start leads to a harder route; it scores if it reaches the given answer correctly.</i>	M1 M1 A1* (3)
(b)	Squares: $x = 9(\sqrt{2} + 1)^2 = 9(3 + 2\sqrt{2})$ $x = 27 + 18\sqrt{2}$ $y = 6 + 3\sqrt{2}$ <i>y from $y = \sqrt{x} + 3 = 3\sqrt{2} + 3 + 3$, or from $y = \sqrt{2}\sqrt{x} = \sqrt{2} \times 3(\sqrt{2} + 1)$. Decimal coordinates score M1 A0 A0.</i>	M1 A1 A1 (3)
Total for Question 9: 6 marks		



Question 10

6 marks

Part	Scheme	Marks
(a)	$\left(\sqrt{k+2}\right)^2 - \left(\sqrt{k}\right)^2 = (k+2) - k = 2 \quad *$ <i>Must show the difference of two squares, or all four terms of the expansion with the middle terms cancelling.</i>	B1* (1)
(b)	$\frac{1}{\sqrt{k} + \sqrt{k+2}} \times \frac{\sqrt{k+2} - \sqrt{k}}{\sqrt{k+2} - \sqrt{k}}$ $= \frac{\sqrt{k+2} - \sqrt{k}}{2}, \text{ using the result of part (a) } *$	M1 A1* (2)
(c)	Uses part (b) on at least two terms: $\frac{\sqrt{3} - \sqrt{1}}{2} + \frac{\sqrt{5} - \sqrt{3}}{2} + \dots$ The terms cancel in pairs, leaving $\frac{\sqrt{49} - \sqrt{1}}{2}$ $= \frac{7 - 1}{2} = 3 \quad *$ <i>A1* cso: the last term $\frac{1}{2}(\sqrt{49} - \sqrt{47})$ must be shown or clearly implied. A calculator sum of the terms scores 0.</i>	M1 M1 A1* (3)
Total for Question 10: 6 marks		



2

Chapter 2: Quadratics

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$(3x - 2)(x + 4) = 0$ (or the formula with $a = 3$, $b = 10$, $c = -8$) $x = \frac{2}{3}, x = -4$	M1 A1 (2)
(b)	$x = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 2 \times 1}}{2 \times 2} = \frac{6 \pm \sqrt{28}}{4}$ $\sqrt{28} = 2\sqrt{7}$ $x = \frac{3 \pm \sqrt{7}}{2}$ Accept $\frac{6 \pm 2\sqrt{7}}{4}$ for the second M1 only. Decimal answers with no working score 0. Completing the square is an equally valid method.	M1 M1 A1 (3)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	$a = 2$ $2(x^2 - 6x) + 23 = 2(x - 3)^2 - 18 + 23$ $2(x - 3)^2 + 5$	B1 M1 A1 (3)
(b)	5	B1ft (1)
(c)	$\frac{1}{5}$ (c): the reciprocal is largest when $2x^2 - 12x + 23$ is smallest. Follow through $\frac{1}{\text{their } c}$ for $c > 0$.	B1ft (1)
Total for Question 2: 5 marks		



Question 3

6 marks

Part	Scheme	Marks
(a)	$5 + 4x - x^2 = 5 - [(x - 2)^2 - 4]$ $9 - (x - 2)^2$, so $p = 9$, $q = -2$	M1 A1 (2)
(b)	A $\cap$ -shaped parabola y intercept (0, 5) x intercepts $(-1, 0)$ and $(5, 0)$, from $(x - 2)^2 = 9$ Maximum point (2, 9) <i>Coordinates may be written on the sketch or in the working. The maximum is follow through from their p and q.</i>	B1 B1 B1 B1ft (4)
Total for Question 3: 6 marks		

Question 4

4 marks

Part	Scheme	Marks
(a)	$b^2 - 4ac = 0$: $k^2 - 4 \times 3 \times 12 = 0$ $k = 12$ or $k = -12$	M1 A1 (2)
(b)	Substitutes one value and solves, e.g. $3x^2 + 12x + 12 = 3(x + 2)^2 = 0$ $k = 12$: $x = -2$ $k = -12$: $x = 2$ <i>Accept use of $x = -\frac{b}{2a}$ for the repeated root. The root must be matched to its value of k.</i>	M1 A1 (2)
Total for Question 4: 4 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	$(x^2 - 1)(x^2 - 9) = 0$ $x^2 = 1$ or $x^2 = 9$ $x = \pm 1, x = \pm 3$ <i>M1: treats the equation as a quadratic in x^2 (a substitution $u = x^2$ is fine). All four values are needed for the final A1.</i>	M1 A1 A1 (3)
(b)	$(2\sqrt{x} - 1)(\sqrt{x} - 3) = 0$ $\sqrt{x} = \frac{1}{2}$ or $\sqrt{x} = 3$ $x = \frac{1}{4}, x = 9$	M1 A1 A1 (3)
Total for Question 5: 6 marks		

Question 6

7 marks

Part	Scheme	Marks
(a)	$f(x) = a(x+2)(x-6)$ Uses (0, 9): $-12a = 9$, so $a = -\frac{3}{4}$ $f(x) = -\frac{3}{4}x^2 + 3x + 9$ <i>Omitting the constant a (using $(x+2)(x-6)$) scores M0. A method using $ax^2 + bx + c$ with three points is equally valid.</i>	M1 M1 A1 (3)
(b)	$-\frac{3}{4}(x^2 - 4x) + 9 = -\frac{3}{4}[(x-2)^2 - 4] + 9$ $12 - \frac{3}{4}(x-2)^2$	M1 A1 (2)
(c)	$(2, 12)$	B1ft (1)
(d)	$k < 12$ <i>The line $y = k$ must cross C twice, so it must be below the maximum. $k \leq 12$ scores B0.</i>	B1ft (1)
Total for Question 6: 7 marks		



Question 7

8 marks

Part	Scheme	Marks
(a)	$B = 5$ $-5\left(t^2 - \frac{12}{5}t\right) + 2 = -5(t - 1.2)^2 + 7.2 + 2$ $9.2 - 5(t - 1.2)^2$	B1 M1 A1 (3)
(b)	(i) 9.2 m (ii) 1.2 seconds	B1ft B1ft (2)
(c)	Solves $5t^2 - 12t - 2 = 0$: $t = \frac{12 \pm \sqrt{144 + 40}}{10}$, or $(t - 1.2)^2 = 1.84$ $t = 2.56$ seconds <i>The negative root must be rejected. Accept 2.56 with no units.</i>	M1 A1 (2)
(d)	A valid limitation, e.g. air resistance is ignored; the stone is modelled as a particle; the ground is assumed to be flat; the model does not apply after the stone hits the ground.	B1 (1)
Total for Question 7: 8 marks		

Question 8

6 marks

Part	Scheme	Marks
(a)	$(k+3)x^2 + 6x + (k-5) = 0$ and uses $b^2 - 4ac > 0$ $36 - 4(k+3)(k-5) > 0 \Rightarrow 36 - 4k^2 + 8k + 60 > 0$ $\Rightarrow 4k^2 - 8k - 96 < 0 \Rightarrow k^2 - 2k - 24 < 0$ * <i>A1* cso: the constant term $k - 5$ must be used, and the inequality sign must be correct throughout, including when dividing by -4.</i>	M1 M1 A1* (3)
(b)	$(k-6)(k+4) = 0$: critical values 6 and -4 $-4 < k < 6$ $-4 < k < 6, k \neq -3$ <i>B1: when $k = -3$ the equation is $6x - 8 = 0$, which is linear with only one root, so $k = -3$ must be excluded.</i>	M1 A1 B1 (3)
Total for Question 8: 6 marks		



Question 9

9 marks

Part	Scheme	Marks
(a)	$2x^2 - 3x + 5 = kx - x^2 + 2$ $3x^2 - 3x - kx + 3 = 0 \Rightarrow 3x^2 - (k+3)x + 3 = 0$ *	M1 A1* (2)
(b)	Exactly one point: $b^2 - 4ac = 0$, $(k+3)^2 - 4 \times 3 \times 3 = 0$ $(k+3)^2 = 36 \Rightarrow k+3 = \pm 6$ $k = 3, k = -9$	M1 M1 A1 (3)
(c)	$k = 3: 3x^2 - 6x + 3 = 0 \Rightarrow 3(x-1)^2 = 0$, so $x = 1$ $(1, 4)$	M1 A1 (2)
(d)	$(k+3)^2 - 36 < 0$, with critical values 3 and -9 $-9 < k < 3$	M1 A1 (2)
Total for Question 9: 9 marks		



Question 10

9 marks

Part	Scheme	Marks
(a)	$(x^2 - 3)^2 - 9$ attempted $(x^2 - 3)^2 - 9$, so $p = -3, q = -9$	M1 A1 (2)
(b)	$(x^2 - 3)^2 = 25 \Rightarrow x^2 - 3 = \pm 5$ $x^2 = 8$ (rejecting $x^2 = -2$) $x = \pm 2\sqrt{2}$	M1 M1 A1 (3)
(c)	$(x^2 - 3)^2 \geq 0$, so $f(x) \geq -9$ for all x , so $f(x)$ cannot equal -10	B1 (1)
(d)	$(x^2 - 3)^2 = A + 9 \Rightarrow x^2 = 3 \pm \sqrt{A + 9}$ Four solutions need two different positive values of x^2 : $A + 9 > 0$ and $3 - \sqrt{A + 9} > 0$ $-9 < A < 0$ <i>When $A = 0$ there are three solutions ($x = 0, \pm\sqrt{6}$) and when $A = -9$ there are two. A correct sketch of the W-shaped graph, with minimum points $(\pm\sqrt{3}, -9)$ and the local maximum at $(0, 0)$, used to justify both ends of the range, scores full marks.</i>	M1 M1 A1 (3)
Total for Question 10: 9 marks		



3

Chapter 3: Equations and Inequalities

Mark scheme. Marks are shown on the right of each line.

Question 1

6 marks

Part	Scheme	Marks
	Substitutes $y = 3x - 2$: $x^2 + (3x - 2)^2 = 20$	M1
	$10x^2 - 12x - 16 = 0$ (or $5x^2 - 6x - 8 = 0$)	A1
	Solves their quadratic: $(5x + 4)(x - 2) = 0$	M1
	$x = 2, x = -\frac{4}{5}$	A1
	Substitutes to find y	M1
	$x = 2, y = 4$ and $x = -\frac{4}{5}, y = -\frac{22}{5}$	A1
	<i>Substituting for x instead leads to $10y^2 + 8y - 176 = 0$ and scores in the same way. Answers with no algebra score 0. The final A1 needs the values correctly paired.</i>	
		(6)
Total for Question 1: 6 marks		

Question 2

7 marks

Part	Scheme	Marks
(a)	$5x - 3 > 2x + 12$, so $3x > 15$	M1
	$x > 5$	A1
		(2)
(b)	$(2x + 3)(x - 6) = 0$	M1
	Critical values $-\frac{3}{2}$ and 6	A1
	Chooses the inside region	M1
	$-\frac{3}{2} < x < 6$	A1
		(4)
(c)	$5 < x < 6$	B1ft
		(1)
Total for Question 2: 7 marks		



Question 3

4 marks

Part	Scheme	Marks
	Multiplies by x^2 : $5x^2 - 12x > 2x^2$	M1
	$3x^2 - 12x > 0$ (or $3x(x - 4) > 0$)	A1
	Critical values 0 and 4, and chooses the outside region	M1
	$x < 0$ or $x > 4$	A1
	<i>Multiplying by x gives $x > 4$ only and scores M0 A0 M0 A0, unless the cases $x > 0$ and $x < 0$ are considered separately and correctly.</i>	
		(4)
Total for Question 3: 4 marks		

Question 4

6 marks

Part	Scheme	Marks
(a)	$\cap$ -shaped parabola through $(-2, 0)$, $(2, 0)$ and $(0, 4)$	B1
	Straight line through $(-2, 0)$ and $(0, 2)$	B1
		(2)
(b)	$4 - x^2 = x + 2 \Rightarrow x^2 + x - 2 = 0 \Rightarrow (x + 2)(x - 1) = 0$	M1
	$(-2, 0)$ and $(1, 3)$	A1
		(2)
(c)	The region above the line and below the curve, between $x = -2$ and $x = 1$, shaded	B1
	The curve drawn dashed and the line drawn solid	B1
		(2)
Total for Question 4: 6 marks		

Question 5

5 marks

Part	Scheme	Marks
(a)	$2x^2 - x - 4 > x + 8 \Rightarrow 2x^2 - 2x - 12 > 0$	M1
	$(x - 3)(x + 2)$: critical values 3 and -2	A1
	Chooses the outside region	M1
	$x < -2$ or $x > 3$	A1
	<i>Final A1: accept $\{x : x < -2\} \cup \{x : x > 3\}$. $-2 > x > 3$ or "$x < -2$ and $x > 3$" is A0.</i>	
		(4)
(b)	$-2 \leq x \leq 3$	B1ft
		(1)
Total for Question 5: 5 marks		



Question 6

7 marks

Part	Scheme	Marks
(a)	$x^2 - 2x - 3 = x + 7$ $x^2 - 3x - 10 = 0$ $(x - 5)(x + 2) = 0$, and finds y for at least one value of x $P(-2, 5)$ and $Q(5, 12)$ <i>The factorisation or formula must be seen: the question does not allow a calculator solution.</i>	M1 A1 M1 A1 (4)
(b)	$y < x + 7$ $y > x^2 - 2x - 3$ $x > 0$ <i>Accept all strict or all non-strict inequalities, but not a mixture. Extra inequalities that cut off part of R score B0 for the last mark.</i>	B1 B1 B1 (3)
Total for Question 6: 7 marks		

Question 7

9 marks

Part	Scheme	Marks
(a)	$2x + 2y = 46$ (or $x + y = 23$) $x^2 + y^2 = 17^2 = 289$	B1 B1 (2)
(b)	Substitutes $y = 23 - x$: $x^2 + (23 - x)^2 = 289$ $2x^2 - 46x + 240 = 0 \Rightarrow x^2 - 23x + 120 = 0$ Solves: $(x - 8)(x - 15) = 0$ $x = 15, y = 8$ <i>Final A1: $x = 8, y = 15$ must be rejected because $x > y$.</i>	M1 A1 M1 A1 (4)
(c)	$x(23 - x) \geq 120 \Rightarrow x^2 - 23x + 120 \leq 0$ Critical values 8 and 15 and the inside region, $8 \leq x \leq 15$ Uses $x > y$, so $x > 11.5$: $11.5 < x \leq 15$ <i>The width is $23 - x$, so length greater than width means $x > 23 - x$. $8 \leq x \leq 15$ scores M1 M1 A0.</i>	M1 M1 A1 (3)
Total for Question 7: 9 marks		



Question 8

8 marks

Part	Scheme	Marks
(a)	$kx + 3 = \frac{1}{x}$ and multiplies by x : $kx^2 + 3x = 1$ $kx^2 + 3x - 1 = 0$ *	M1 A1* (2)
(b)	$b^2 - 4ac > 0$: $9 - 4 \times k \times (-1) > 0$, so $9 + 4k > 0$ $k > -\frac{9}{4}, k \neq 0$ <i>The sign of $c = -1$ is the common error. $k \neq 0$ is needed because k is non-zero (when $k = 0$ the line $y = 3$ meets C only once).</i>	M1 A1 (2)
(c)	$2x^2 + 3x - 1 = 0$: $x = \frac{-3 \pm \sqrt{9+8}}{4}$ $x = \frac{-3 \pm \sqrt{17}}{4}$ Finds y using $y = 2x + 3$ (or $y = \frac{1}{x}$ and rationalising) $\left(\frac{-3 + \sqrt{17}}{4}, \frac{3 + \sqrt{17}}{2} \right) \quad \left(\frac{-3 - \sqrt{17}}{4}, \frac{3 - \sqrt{17}}{2} \right)$	M1 A1 M1 A1 (4)
Total for Question 8: 8 marks		



Question 9

7 marks

Part	Scheme	Marks
(a)	$kx = 12 - 2x \Rightarrow x(k+2) = 12$ $x = \frac{12}{k+2}$ *	M1 A1* (2)
(b)	y coordinate $\frac{12k}{k+2}$; base from O to $(6, 0)$; area = $\frac{1}{2} \times 6 \times \frac{12k}{k+2}$ $\frac{36k}{k+2} = 24 \Rightarrow 36k = 24k + 48$ $k = 4$	M1 M1 A1 (3)
(c)	Two of $y \geq 0$, $y \leq 4x$, $y \leq 12 - 2x$ $y \geq 0$, $y \leq 4x$, $y \leq 12 - 2x$ Accept strict inequalities throughout. Follow through their k for the first B1.	B1ft B1 (2)
Total for Question 9: 7 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	$AP^2 = (p-3)^2 + (2p-1-0)^2$ $= p^2 - 6p + 9 + 4p^2 - 4p + 1 = 5p^2 - 10p + 10$ *	M1 A1* (2)
(b)	$5p^2 - 10p + 10 = 50 \Rightarrow p^2 - 2p - 8 = 0$ $(p-4)(p+2) = 0$, so $p = 4$ or $p = -2$ (4, 7) and (-2, -5)	M1 A1 A1 (3)
(c)	$5p^2 - 10p + 10 > 25 \Rightarrow p^2 - 2p - 3 > 0$ Critical values 3 and -1 and chooses the outside region $p < -1$ or $p > 3$ Using $AP > \sqrt{5}$ or $AP^2 > 5$ scores M0. The inequality must be in AP^2 with $5^2 = 25$.	M1 M1 A1 (3)
(d)	$AP^2 = 5(p-1)^2 + 5$, so the least value of AP^2 is 5 when $p = 1$ $\sqrt{5}$ at (1, 1) Alternative: the perpendicular from A to l (gradient $-\frac{1}{2}$) meets l at (1, 1), then Pythagoras gives $\sqrt{5}$.	M1 A1 (2)
Total for Question 10: 10 marks		



4

Chapter 4: Graphs and Transformations

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	Takes out x : $x(x^2 - 6x + 9)$ $x(x - 3)^2$	M1 A1 (2)
(b)	Positive cubic shape (bottom left to top right) Passes through (0, 0) Touches the x -axis at (3, 0), a minimum point <i>Third B1: follow through a repeated root from their factorisation. A curve crossing at (3, 0) scores B0.</i>	B1 B1 B1ft (3)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	Two branches, both above the x -axis, symmetrical about the y -axis Both branches asymptotic to both coordinate axes	B1 B1 (2)
(b)	Same shape translated down, with both branches crossing the x -axis $y = -1$ stated as the horizontal asymptote $(-\sqrt{3}, 0)$ and $(\sqrt{3}, 0)$ <i>Third B1: from $\frac{3}{x^2} = 1$. Decimals ± 1.73 score B0 because the question asks for exact coordinates.</i>	B1 B1 B1 (3)
Total for Question 2: 5 marks		



Question 3

6 marks

Part	Scheme	Marks
(a)	Maximum $(-3, 4)$	B1
	Minimum $(0, -3)$	B1
		(2)
(b)	Maximum $(-1, 12)$	B1
	Minimum $(2, -9)$	B1
		(2)
(c)	Maximum $(-\frac{1}{2}, 4)$	B1
	Minimum $(1, -3)$	B1
	<i>(a) moves the graph 2 left; (b) multiplies y coordinates by 3; (c) divides x coordinates by 2.</i>	(2)
Total for Question 3: 6 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	Positive cubic touching the x -axis at O (a maximum) and crossing at $(3, 0)$	B1
	Coordinates $(0, 0)$ and $(3, 0)$ shown	B1
	Reciprocal curve with branches in the first and third quadrants, asymptotic to both axes	B1
		(3)
(b)	Divides by x ($x \neq 0$): $x^4 - 3x^3 = 4 \Leftrightarrow x^2(x - 3) = \frac{4}{x}$	M1
	2 solutions, because the two graphs meet at two points	A1
	<i>The curves cross once in the third quadrant (at $x = -1$) and once for $x > 3$. A1 needs the reason, linked to their sketch.</i>	(2)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
(a)	(i) $y = \frac{4}{x+3}$	B1
	(ii) $x = -3$	B1 (2)
(b)	A translation 2 units in the negative y direction, i.e. by the vector $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$	M1 A1 (2)
(c)	$\frac{4}{x} - 2 = 0 \Rightarrow x = 2$: $(2, 0)$	B1 (1)
Total for Question 5: 5 marks		



Question 6

9 marks

Part	Scheme	Marks
(a)	Positive cubic shape Touches the x -axis at $(-\frac{1}{2}, 0)$ and crosses at $(3, 0)$ Crosses the y -axis at $(0, -3)$	B1 B1 B1 (3)
(b)	$x > 3$ $f(-\frac{1}{2}) = 0$, so this point is not included; $f(x) < 0$ elsewhere for $x < 3$.	B1ft (1)
(c)	Expands $(4x^2 + 4x + 1)(x - 3)$ $4x^3 - 8x^2 - 11x - 3$	M1 A1 (2)
(d)	$4x^3 - 8x^2 - 11x - 3 = -3 \Rightarrow x(4x^2 - 8x - 11) = 0$ $x = \frac{8 \pm \sqrt{64 + 176}}{8} = \frac{8 \pm 4\sqrt{15}}{8}$ $x = 1 \pm \frac{\sqrt{15}}{2}$ Accept $\frac{2 \pm \sqrt{15}}{2}$. The root $x = 0$ is the y intercept itself, so it is not one of the "other" points. Decimals only score M1 M0 A0.	M1 M1 A1 (3)
Total for Question 6: 9 marks		



Question 7

8 marks

Part	Scheme	Marks
(a)	$f(x) = kx(x - 6)$ and uses $(3, -9)$: $-9k = -9$ $f(x) = x(x - 6) = x^2 - 6x$	M1 A1 (2)
(b)	$g(x) = kx(x - 6)^2$ Uses $(2, 16)$: $k \times 2 \times 16 = 16$ $g(x) = \frac{1}{2}x(x - 6)^2$ <i>Omitting the constant k scores B0 M0 A0.</i>	B1 M1 A1 (3)
(c)	$x(x - 6) = \frac{1}{2}x(x - 6)^2 \Rightarrow x(x - 6) \left[1 - \frac{1}{2}(x - 6) \right] = 0$ $x = 8$ (as $x = 0$ and $x = 6$ are O and $(6, 0)$) $P(8, 16)$ <i>Expanding to a cubic and solving is acceptable. The y coordinate may come from either curve.</i>	M1 A1 A1 (3)
Total for Question 7: 8 marks		



Question 8

8 marks

Part	Scheme	Marks
(a)	Reciprocal shape with branches top right and bottom left of the asymptotes Asymptotes $x = 0$ and $y = -2$ Crosses the x -axis at $(\frac{3}{2}, 0)$	B1 B1 B1 (3)
(b)	At $x = -3$ on C : $y = \frac{3}{-3} - 2 = -3$, so $P(-3, -3)$ and $-3 = -3k + 1$ $k = \frac{4}{3}$	M1 A1 (2)
(c)	$\frac{4}{3}x + 1 = \frac{3}{x} - 2$ and multiplies by $3x$: $4x^2 + 9x - 9 = 0$ $(4x - 3)(x + 3) = 0$, so $x = \frac{3}{4}$ (as $x = -3$ is P) $Q(\frac{3}{4}, 2)$ <i>Knowing that $x = -3$ is one root is a check on the factorisation. The y coordinate may come from l or from C.</i>	M1 M1 A1 (3)
Total for Question 8: 8 marks		

Question 9

8 marks

Part	Scheme	Marks
(a)	Same shape as $y = f(x)$, translated down 2 Asymptotes $x = 1$ and $y = 0$ Crosses the y -axis at $(0, -3)$ and does not meet the x -axis	B1 B1 B1 (3)
(b)	Same shape, stretched horizontally by scale factor 2 Asymptotes $x = 2$ and $y = 2$ $(-1, 0)$ and $(0, -1)$	B1 B1 B1 (3)
(c)	$a = 2$ (from the horizontal asymptote) $b = 3$ (from $f(0) = 2 - b = -1$)	B1 B1 (2)
Total for Question 9: 8 marks		



Question 10

9 marks

Part	Scheme	Marks
(a)	Positive cubic shape Crosses the x -axis at $(-1, 0)$ and touches at $(2, 0)$ Maximum at $(0, 4)$ on the y -axis	B1 B1 B1 (3)
(b)	The line $y = k$ must lie between the minimum $y = 0$ and the maximum $y = 4$ $0 < k < 4$	M1 A1 (2)
(c)	$y = f(x + 3)$ is a horizontal translation, so it has the same number of solutions as $f(x) = 2$ 3 , because $0 < 2 < 4$	M1 A1 (2)
(d)	$c = 0$ (the minimum is already on the x -axis) $c = -4$ (the maximum moves down onto the x -axis)	B1 B1 (2)
Total for Question 10: 9 marks		



5

Chapter 5: Straight-line Graphs

Mark scheme. Marks are shown on the right of each line.

Question 1

4 marks

Part	Scheme	Marks
	$\text{Gradient} = \frac{-2-7}{4-(-2)} = -\frac{3}{2}$ $y-7 = -\frac{3}{2}(x+2) \text{ (or } y = -\frac{3}{2}x + c \text{ with } c \text{ found)}$ $3x + 2y - 8 = 0$ <i>Final A1: any integer multiple, e.g. $-6x - 4y + 16 = 0$, but it must include “= 0”.</i>	M1 A1 M1 A1 (4)
Total for Question 1: 4 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	$y = \frac{2}{5}x + 2$, so gradient $\frac{2}{5}$	B1 (1)
(b)	Gradient of $l_2 = -\frac{5}{2}$ $y + 1 = -\frac{5}{2}(x - 4)$ $y = -\frac{5}{2}x + 9$	B1ft M1 A1 (3)
(c)	$\left(\frac{18}{5}, 0\right)$	B1ft (1)
Total for Question 2: 5 marks		



Question 3

4 marks

Part	Scheme	Marks
	Midpoint of AB is $(1, 6)$	B1
	Gradient of $AB = \frac{4}{8} = \frac{1}{2}$, so the perpendicular gradient is -2	M1
	$y - 6 = -2(x - 1)$	M1
	$2x + y - 8 = 0$	A1
	Second M1: uses their midpoint with their perpendicular gradient. Using A or B instead of the midpoint scores M0.	
		(4)
Total for Question 3: 4 marks		

Question 4

7 marks

Part	Scheme	Marks
(a)	Attempts the gradients of PQ and QR $m_{PQ} = \frac{3-1}{2+2} = \frac{1}{2}$ and $m_{QR} = \frac{-3-3}{5-2} = -2$ $\frac{1}{2} \times (-2) = -1$, so PQ is perpendicular to QR and angle $PQR = 90^\circ$ Final A1*: needs the product -1 and a conclusion. A Pythagoras method ($PQ^2 + QR^2 = PR^2$: $20 + 45 = 65$) is equally valid.	M1 A1 A1*
(b)	Uses $\vec{QR} = \vec{PS}$, or $S = P + R - Q$ $S(1, -5)$	M1 A1 (2)
(c)	$PQ = \sqrt{20} = 2\sqrt{5}$ and $QR = \sqrt{45} = 3\sqrt{5}$, area $= 2\sqrt{5} \times 3\sqrt{5}$ 30 Decimal working such as 4.47×6.71 scores M1 A0, because the calculator warning requires the surd product to be shown.	M1 A1 (2)
Total for Question 4: 7 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	$205 = 3p + q$ and $405 = 7p + q$ Solves: $4p = 200$ $p = 50, q = 55$	M1 M1 A1 (3)
(b)	The cost increases by $\pounds 50$ for each extra day of hire	B1 (1)
(c)	$50d + 55 < 65d \Rightarrow d > \frac{11}{3}$ 4 days	M1 A1 (2)
Total for Question 5: 6 marks		

Question 6

8 marks

Part	Scheme	Marks
(a)	Gradient = $\frac{0 - 10}{5 - 0} = -2$ $y = -2x + 10$	M1 A1 (2)
(b)	Gradient of $l_2 = \frac{1}{2}$ $y = \frac{1}{2}x$	M1 A1 (2)
(c)	$\frac{1}{2}x = -2x + 10 \Rightarrow x = 4$ $P(4, 2)$	M1 A1 (2)
(d)	Area $OPA = \frac{1}{2} \times 10 \times 4 = 20$ and area $OPB = \frac{1}{2} \times 5 \times 2 = 5$ $20 = 4 \times 5$, so the area of OPA is four times the area of OPB * <i>Uses OA and OB on the axes as bases, with the coordinates of P as the heights. Alternatively $OP = 2\sqrt{5}$, $PA = 4\sqrt{5}$, $PB = \sqrt{5}$ with the right angle at P.</i>	M1 A1* (2)
Total for Question 6: 8 marks		



Question 7

8 marks

Part	Scheme	Marks
(a)	Midpoint $M(4, 6)$ Gradient of $AB = \frac{8}{6} = \frac{4}{3}$, so the gradient of l is $-\frac{3}{4}$ $y - 6 = -\frac{3}{4}(x - 4) \Rightarrow \boxed{3x + 4y - 36 = 0}$	B1 M1 A1 (3)
(b)	$AB = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$ *	B1* (1)
(c)	C is on the perpendicular bisector, so MC is the height: $\frac{1}{2} \times 10 \times MC = 25 \Rightarrow MC = 5$ $C\left(c, \frac{36-3c}{4}\right): (c-4)^2 + \left(\frac{12-3c}{4}\right)^2 = 25$ $\frac{25}{16}(c-4)^2 = 25 \Rightarrow (c-4)^2 = 16$, so $c = 8$ or $c = 0$ $\boxed{(8, 3) \text{ and } (0, 9)}$ <i>Both points are needed for the final A1. Using AC or BC as the height scores M0.</i>	M1 M1 A1 A1 (4)
Total for Question 7: 8 marks		

Question 8

7 marks

Part	Scheme	Marks
(a)	$kx = x(6-x) \Rightarrow x(6-x-k) = 0$, so $x = 6-k$ at P $y = k(6-k) = 6k - k^2$, so $P(6-k, 6k - k^2)$ *	M1 A1* (2)
(b)	Gradient of $PA = \frac{0 - (6k - k^2)}{6 - (6-k)} = \frac{-k(6-k)}{k}$ $= -(6-k) = k-6$ *	M1 A1* (2)
(c)	Perpendicular: $k(k-6) = -1 \Rightarrow k^2 - 6k + 1 = 0$ $k = \frac{6 \pm \sqrt{32}}{2} = 3 \pm 2\sqrt{2}$ $\boxed{k = 3 + 2\sqrt{2}}$ (since $k > 1$) <i>$3 - 2\sqrt{2} \approx 0.17$ is rejected. Decimal answers score M1 M0 A0 because of the calculator warning.</i>	M1 M1 A1 (3)
Total for Question 8: 7 marks		



Question 9

9 marks

Part	Scheme	Marks
(a)	Gradient of $AB = \frac{3}{6} = \frac{1}{2}$; line through C : $y - 4 = \frac{1}{2}(x - 10)$ $y = \frac{1}{2}x - 1$	M1 A1 (2)
(b)	Gradient -2 through A $y = -2x + 4$	M1 A1 (2)
(c)	$\frac{1}{2}x - 1 = -2x + 4 \Rightarrow x = 2$ $D(2, 0)$	M1 A1 (2)
(d)	$AB = \sqrt{45} = 3\sqrt{5}$, $DC = \sqrt{80} = 4\sqrt{5}$, height $AD = \sqrt{20} = 2\sqrt{5}$ $\text{Area} = \frac{1}{2} (3\sqrt{5} + 4\sqrt{5}) \times 2\sqrt{5}$ 35 <i>AD is the height because angle $DAB = 90^\circ$ and AB is parallel to DC. Splitting into two triangles is equally valid.</i>	M1 M1 A1 (3)
Total for Question 9: 9 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	Gradient of $l = -\frac{1}{2}$, so the perpendicular gradient is 2 $y = 2x - 4$	M1 A1 (2)
(b)	$x + 2(2x - 4) - 2 = 0 \Rightarrow 5x = 10$ $N(2, 0)$	M1 A1 (2)
(c)	N is the midpoint of PP' : $P' = (2 \times 2 - 5, 2 \times 0 - 6)$ $P'(-1, -6)$	M1 A1 (2)
(d)	$PP' = \sqrt{6^2 + 12^2} = \sqrt{180} = 6\sqrt{5}$	B1 (1)
(e)	$PP' = 6\sqrt{5}$ and NQ is the height: $\frac{1}{2} \times 6\sqrt{5} \times NQ = 15 \Rightarrow NQ = \sqrt{5}$ $Q(2 - 2t, t)$ on l : $(2t)^2 + t^2 = 5 \Rightarrow t = \pm 1$ $(0, 1)$ and $(4, -1)$	M1 M1 A1 (3)
Total for Question 10: 10 marks		



6

Chapter 6: Trigonometric Ratios

Mark scheme. Marks are shown on the right of each line.

Question 1

4 marks

Part	Scheme	Marks
(a)	$BC^2 = 8^2 + 11^2 - 2 \times 8 \times 11 \cos 50^\circ$ $BC = 8.48 \text{ cm}$	M1 A1 (2)
(b)	$\frac{1}{2} \times 8 \times 11 \times \sin 50^\circ$ 33.7 cm^2	M1 A1 (2)
Total for Question 1: 4 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	$\frac{\sin R}{9} = \frac{\sin 42^\circ}{7} \Rightarrow \sin R = 0.8603 \dots$ $R = 59.4^\circ$ $59.4^\circ \text{ and } 120.6^\circ$ <i>Second value from $180^\circ - 59.4^\circ$; it fits because $120.6^\circ + 42^\circ < 180^\circ$.</i>	M1 A1 A1 (3)
(b)	Angle $PQR = 180^\circ - 42^\circ - 120.6^\circ = 17.4^\circ$ and $PR = \frac{7 \sin 17.35 \dots^\circ}{\sin 42^\circ}$ 3.12 cm	M1 A1 (2)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	The largest angle is opposite the 9 cm side: $\cos \theta = \frac{5^2 + 7^2 - 9^2}{2 \times 5 \times 7}$ $= \frac{-7}{70} = -\frac{1}{10}$ *	M1 A1* (2)
(b)	Uses $\sin^2 \theta + \cos^2 \theta = 1$: $\sin \theta = \sqrt{1 - \frac{1}{100}}$ $\sin \theta = \sqrt{\frac{99}{100}} = \frac{3\sqrt{11}}{10}$ Area = $\frac{1}{2} \times 5 \times 7 \times \frac{3\sqrt{11}}{10} = \frac{21\sqrt{11}}{4}$ Uses $\sin^2 \theta + \cos^2 \theta = 1$ or a right-angled triangle with sides 1, $\sqrt{99}$, 10. Decimals only (17.4) score M1 A0 A0.	M1 A1 A1 (3)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	Tangent shape: two complete branches and parts of the branches at each end Asymptotes $x = 135^\circ$ and $x = 315^\circ$ $(0, -1)$, $(45^\circ, 0)$ and $(225^\circ, 0)$ The graph of $y = \tan x$ translated 45° to the right. y intercept: $\tan(-45^\circ) = -1$.	B1 B1 B1 (3)
(b)	2 One in each branch that covers all values: $x = 108.4^\circ$ and $x = 288.4^\circ$. The end sections take only values from -1 upwards (left) and -1 downwards (right).	B1 (1)
(c)	3 $\tan(x - 45^\circ) = -1$ at $x = 0^\circ, 180^\circ$ and 360°: both end points are included in the interval.	B1 (1)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
(a)	$a = 3$	B1
	$b = 2$ (two complete cycles in 360°)	B1
		(2)
(b)	$(90^\circ, -3)$	B1
	$(270^\circ, -3)$	B1
		(2)
(c)	4	B1
		(1)
Total for Question 5: 5 marks		

Question 6

8 marks

Part	Scheme	Marks
(a)	$AC^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \cos 110^\circ$	M1
	$AC = 12.4 \text{ m}$	A1
		(2)
(b)	$\frac{\sin(\angle ACD)}{5} = \frac{\sin 72^\circ}{12.407 \dots}$	M1
	22.5°	A1
	The obtuse value 157.5° is impossible, because $157.5^\circ + 72^\circ > 180^\circ$	B1
		(3)
(c)	$\text{Area } ABC = \frac{1}{2} \times 6 \times 9 \times \sin 110^\circ = 25.37 \dots$	M1
	Angle $CAD = 180^\circ - 72^\circ - 22.54^\circ = 85.46^\circ$, area	
	$ACD = \frac{1}{2} \times 5 \times 12.407 \dots \times \sin 85.46^\circ = 30.92 \dots$	M1
	56.3 m^2	A1
	Premature rounding (for example $AC = 12$) usually loses the final A mark.	(3)
Total for Question 6: 8 marks		



Question 7

9 marks

Part	Scheme	Marks
(a)	$Q(45^\circ, 0)$ P : maximum where $\cos 2x = -1$, i.e. $2x = 180^\circ$ $P(90^\circ, 4)$	B1 M1 A1 (3)
(b)	$5 = k - 4 \cos 60^\circ = k - 2$ $k = 7$	M1 A1 (2)
(c)	Minimum value $7 - 4 = 3$ where $\cos 2x = 1$ $(0, 3), (180^\circ, 3)$ and $(360^\circ, 3)$ <i>All three points are needed for A1: $x = 0^\circ$ and $x = 360^\circ$ are inside the interval.</i>	M1 A1ft (2)
(d)	Rearranges to $-4 \cos 2x = 2 - \frac{x}{45}$ and draws the line $y = 2 - \frac{x}{45}$ through $(0, 2)$ and $(90^\circ, 0)$, reaching $y = -6$ at $x = 360^\circ$ 3 solutions, because the line crosses C three times <i>The crossings are near $x = 51^\circ, 144^\circ$ and 205°. After $x = 270^\circ$ the line is below C all the way to $x = 360^\circ$ (it ends at $-6 < -4$).</i>	M1 A1 (2)
Total for Question 7: 9 marks		



Question 8

8 marks

Part	Scheme	Marks
(a)	$x = 0: y = 2 \sin 60^\circ = (0, \sqrt{3})$ $x + 60^\circ = 180^\circ \text{ or } 360^\circ$ $(120^\circ, 0) \text{ and } (300^\circ, 0)$	B1 M1 A1 (3)
(b)	Maximum $(30^\circ, 2)$ Minimum $(210^\circ, -2)$	B1 B1 (2)
(c)	$-2 < k < 2$ $-2 < k < 2, k \neq \sqrt{3}$ <i>When $k = \sqrt{3}$ the line meets C three times: at $x = 0^\circ, 60^\circ$ and 360°. When $k = \pm 2$ it meets C once.</i>	M1 A1 (2)
(d)	4 $y = 2 \sin(2x + 60^\circ)$ has period 180° , so two complete cycles: $2x + 60^\circ = 150^\circ, 390^\circ, 510^\circ, 750^\circ$.	B1 (1)
Total for Question 8: 8 marks		



Question 9

8 marks

Part	Scheme	Marks
(a)	180°	B1 (1)
(b)	$\tan 45^\circ = 1$ and $\frac{45}{45} = 1$, so $x = 45^\circ$ is a solution *	B1* (1)
(c)	(i) 3 ($x = -45^\circ, 0^\circ, 45^\circ$) (ii) 5 (iii) Each interval $180n^\circ < x < 180n^\circ + 90^\circ$ for $n = 1, 2, \dots, 49$ adds one solution, and the same for negative x 101 <i>(ii): the new solutions are in $180^\circ < x < 270^\circ$ and $-270^\circ < x < -180^\circ$, where $\tan x$ is positive and increases without limit. There are none where $\tan x$ and the line have opposite signs. (iii): 50 positive, 50 negative and $x = 0^\circ$. Common wrong answers are 100 (missing the origin) and 200.</i>	B1 B1 M1 A1 (4)
(d)	The line $y = -\frac{x}{45}$ passes through O with negative gradient; it meets the branch in $90^\circ < x < 180^\circ$ once and the branch in $-180^\circ < x < -90^\circ$ once 3, since the graphs also meet at the origin <i>For $0 < x < 90^\circ$, $\tan x > 0$ while the line is negative; for $-90^\circ < x < 0$ the signs are reversed, so there are no other solutions.</i>	M1 A1 (2)
Total for Question 9: 8 marks		



Question 10

9 marks

Part	Scheme	Marks
(a)	$\frac{\sin C}{10} = \frac{\sin 30^\circ}{6} \Rightarrow \sin C = \frac{5}{6}$ 56.4° Obtuse: 123.6° <i>Giving 56.4° as the final answer scores M1 A1 A0: the question states that angle ACB is obtuse.</i>	M1 A1 A1 (3)
(b)	$6^2 = 10^2 + x^2 - 2 \times 10 \times x \cos 30^\circ$ $\cos 30^\circ = \frac{\sqrt{3}}{2}, \text{ so } 36 = 100 + x^2 - 10\sqrt{3}x \Rightarrow x^2 - 10\sqrt{3}x + 64 = 0 \quad *$ $x = \frac{10\sqrt{3} \pm \sqrt{300 - 256}}{2} = 5\sqrt{3} \pm \sqrt{11}$ C is obtuse, so C is the nearer of the two points to A: $AC = 5\sqrt{3} - \sqrt{11} \quad *$	M1 A1* M1 A1* (4)
(c)	D gives the other root: $AD = 5\sqrt{3} + \sqrt{11}$, so $CD = AD - AC$ $2\sqrt{11}$ cm <i>Alternative: triangle BCD is isosceles with base angles 56.4°, so $CD = 2 \times 6 \cos(\angle BDC) = 12 \times \frac{\sqrt{11}}{6}$.</i>	M1 A1 (2)
Total for Question 10: 9 marks		



7

Chapter 7: Radians

Mark scheme. Marks are shown on the right of each line.

Question 1

4 marks

Part	Scheme	Marks
(a)	$\frac{5\pi}{6}$	B1 (1)
(b)	(i) $s = 6 \times \frac{5\pi}{6} = 5\pi \text{ cm}$ (ii) $A = \frac{1}{2} \times 6^2 \times \frac{5\pi}{6}$ $15\pi \text{ cm}^2$	B1 M1 A1 (3)
Total for Question 1: 4 marks		

Question 2

6 marks

Part	Scheme	Marks
(a)	$\frac{1}{2}r^2 \times 1.25 = 40 \Rightarrow r^2 = 64$ $r = 8$ *	M1 A1* (2)
(b)	$8 + 8 + 8 \times 1.25$ 26 cm	M1 A1 (2)
(c)	$AB^2 = 8^2 + 8^2 - 2 \times 8 \times 8 \cos 1.25$ (or $AB = 2 \times 8 \sin 0.625$) 9.36 cm <i>The angle must be used in radians: $\cos 1.25^\circ$ gives 0.17 cm and scores M1 A0.</i>	M1 A1 (2)
Total for Question 2: 6 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	Cosine curve translated $\frac{\pi}{3}$ to the right	B1
	$(0, \frac{1}{2})$	B1
	$(\frac{5\pi}{6}, 0)$ and $(\frac{11\pi}{6}, 0)$	B1
		(3)
(b)	Maximum $(\frac{\pi}{3}, 1)$	B1
	Minimum $(\frac{4\pi}{3}, -1)$	B1
		(2)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
(a)	π	B1 (1)
(b)	$3 \sin \frac{\pi}{3} = \frac{3\sqrt{3}}{2}$	B1 (1)
(c)	$2x = \frac{\pi}{2}$ or $\frac{5\pi}{2}$	M1
	$(\frac{\pi}{4}, 3)$ and $(\frac{5\pi}{4}, 3)$	A1
		(2)
(d)	4	B1
	Two complete cycles, and $0 < 2 < 3$, so the line $y = 2$ crosses each cycle twice.	(1)
Total for Question 4: 5 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	$\cos(\angle AOB) = \frac{10^2 + 10^2 - 12^2}{2 \times 10 \times 10} = 0.28$ $\angle AOB = 1.2870 \dots = 1.287 \text{ radians}^*$ Alternative: $\sin \frac{\theta}{2} = \frac{6}{10}$, so $\theta = 2 \arcsin 0.6$. The value must be seen to more than 4 s.f. (e.g. 1.28700) for A1*.	M1 A1* (2)
(b)	$\frac{1}{2} \times 10^2 (1.287 \dots - \sin 1.287 \dots)$ 16.4 cm²	M1 A1 (2)
(c)	$10 \times 1.287 \dots + 12$ 24.9 cm	M1 A1 (2)
Total for Question 5: 6 marks		



Question 6

9 marks

Part	Scheme	Marks
(a)	$\frac{1}{2}r^2 \times 2.1 = 58 \Rightarrow r^2 = \frac{116}{2.1}$ $r = 7.432\dots = 7.43 \text{ m}^*$	M1 A1* (2)
(b)	$\frac{\pi - 2.1}{2} = 0.521$ <i>The common error is $\frac{2\pi - 2.1}{2}$: AOD is a straight line, so the angles at O add to π.</i>	B1 (1)
(c)	Area of one triangle = $\frac{1}{2} \times 7 \times 7.432\dots \times \sin 0.5208\dots$ Total = $58 + 2 \times 12.94\dots$ 83.9 m²	M1 M1 A1 (3)
(d)	$AB^2 = 7^2 + 7.432\dots^2 - 2 \times 7 \times 7.432\dots \cos 0.5208\dots$, so $AB = 3.739\dots$ Arc $BC = 7.432\dots \times 2.1 = 15.61\dots$ Perimeter = $15.61\dots + 2 \times 3.739\dots + 14 = 37.1 \text{ m}$ <i>The radii OB and OC are internal and are not part of the perimeter. The base AD = 14 m is.</i>	M1 B1 A1 (3)
Total for Question 6: 9 marks		



Question 7

9 marks

Part	Scheme	Marks
(a)	$6 + 6 + 6 \times 1.1$ 18.6 km	M1 A1 (2)
(b)	Area of sector = $\frac{1}{2} \times 6^2 \times 1.1 = 19.8$ $R_1 + R_2 = 3R_1$, so $R_1 = \frac{19.8}{3}$ $= 6.6 \text{ km}^2$ * <i>A circular argument (using 6.6 to find OP and then the area again) scores 0.</i>	M1 M1 A1* (3)
(c)	$\frac{1}{2} \times 6 \times OP \times \sin 1.1 = 6.6$ $OP = 2.47 \text{ km}$	M1 A1 (2)
(d)	$AP^2 = 6^2 + 2.468\dots^2 - 2 \times 6 \times 2.468\dots \cos 1.1$ $AP = 5.35\dots$, so 5.4 km <i>Final A1 needs the answer to the nearest 100 m, with units. AP is not perpendicular to OB.</i>	M1 A1 (2)
Total for Question 7: 9 marks		



Question 8

9 marks

Part	Scheme	Marks
(a)	$\frac{\sin(\angle OBA)}{10} = \frac{\sin \frac{\pi}{6}}{7} \Rightarrow \sin(\angle OBA) = \frac{5}{7}, \text{ acute value } 0.7956 \dots$ Obtuse: $\pi - 0.7956 \dots = 2.3459 \dots = 2.346^*$	M1 A1* (2)
(b)	Angle $OAB = \pi - \frac{\pi}{6} - 2.3459 \dots = 0.2720 \dots$ and $OB = \frac{7 \sin 0.2720 \dots}{\sin \frac{\pi}{6}}$ 3.76 cm	M1 A1 (2)
(c)	Triangle = $\frac{1}{2} \times 10 \times 3.761 \dots \times \sin \frac{\pi}{6} = 9.40 \dots$ Sector = $\frac{1}{2} \times 3.761 \dots^2 \times 1.2 = 8.49 \dots$ 17.9 cm²	M1 M1 A1 (3)
(d)	$10 + 7 + 3.761 \dots \times 1.2 + 3.761 \dots$ 25.3 cm <i>OB is an internal line and must not be added. The boundary is OA, AB, the arc BC and CO.</i>	M1 A1 (2)
Total for Question 8: 9 marks		



Question 9

8 marks

Part	Scheme	Marks
(a)	$a = 3$	B1
	$\frac{\pi}{6} + b = \frac{\pi}{2}$, so $b = \frac{\pi}{3}$	B1
		(2)
(b)	$3 \sin \frac{\pi}{3}$: $\left(0, \frac{3\sqrt{3}}{2}\right)$	B1
		(1)
(c)	$x + \frac{\pi}{3} = \pi$ or 2π	M1
	$x = \frac{2\pi}{3}$ and $x = \frac{5\pi}{3}$	A1
		(2)
(d)	$y = 3 \cos(x + c)$ has its maximum where $x + c = 0$, so $\frac{\pi}{6} + c = 0$	M1
	$c = -\frac{\pi}{6}$	A1
	Equivalently, $\sin\left(x + \frac{\pi}{3}\right) = \cos\left(x + \frac{\pi}{3} - \frac{\pi}{2}\right)$. $c = \frac{11\pi}{6}$ is outside the given range.	(2)
(e)	$\left(\frac{7\pi}{6}, -3\right)$	B1
		(1)
Total for Question 9: 8 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	$2r + r\theta = 20$ and $\frac{1}{2}r^2\theta = 16$ Eliminates θ : $r\theta = 20 - 2r$, so $\frac{1}{2}r(20 - 2r) = 16$ $10r - r^2 = 16 \Rightarrow r^2 - 10r + 16 = 0$ *	B1 M1 A1* (3)
(b)	$(r - 2)(r - 8) = 0$, so $r = 2$ or $r = 8$ $r = 2$ gives $\theta = \frac{20 - 4}{2} = 8$ Rejects: $8 > 2\pi$, which is more than a full turn, so this is not a sector $r = 8, \theta = \frac{1}{2}$	M1 M1 A1 A1 (4)
(c)	$r\theta = 20 - 2r$, so $A = \frac{1}{2}r(20 - 2r) = 10r - r^2$ * $A = 25 - (r - 5)^2$, so the maximum area is 25 cm^2 when $r = 5$ $5\theta = 20 - 10$, so $\theta = 2$	B1* M1 A1 (3)
Total for Question 10: 10 marks		



8

Chapter 8: Differentiation

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	Reduces a power by one on any term: $x^n \rightarrow x^{n-1}$ Two of $12x^2$, $+12x^{-3}$, $+\frac{5}{2}x^{-\frac{1}{2}}$ $12x^2 + 12x^{-3} + \frac{5}{2}x^{-\frac{1}{2}}$ Writing $-\frac{6}{x^2}$ as $-6x^{-2}$ is essential. Adding $+c$ loses the final A1.	M1 A1 A1 (3)
(b)	Differentiates their $\frac{dy}{dx}$ again $24x - 36x^{-4} - \frac{5}{4}x^{-\frac{3}{2}}$	M1 A1 (2)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	Splits the fraction: $\frac{3x^2}{2x^{\frac{1}{2}}} - \frac{4}{2x^{\frac{1}{2}}}$ One term correct: $\frac{3}{2}x^{\frac{3}{2}}$ or $-2x^{-\frac{1}{2}}$ $\frac{3}{2}x^{\frac{3}{2}} - 2x^{-\frac{1}{2}}$ $(A = \frac{3}{2}, p = \frac{3}{2}, B = -2, q = -\frac{1}{2})$	M1 A1 A1 (3)
(b)	Differentiates their expression $\frac{9}{4}x^{\frac{1}{2}} + x^{-\frac{3}{2}}$	M1 A1 (2)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	y at Q : $(4+h)^2 - 3(4+h) = 16 + 8h + h^2 - 12 - 3h$ $= 4 + 5h + h^2$ Gradient = $\frac{4 + 5h + h^2 - 4}{(4+h) - 4} = \frac{5h + h^2}{h} = 5 + h$ *	M1 A1 A1* (3)
(b)	As $h \rightarrow 0$, Q moves towards P and the chord gradient tends to the gradient of the tangent at P 5 Check: $\frac{dy}{dx} = 2x - 3 = 5$ at $x = 4$.	B1 B1 (2)
Total for Question 3: 5 marks		

Question 4

6 marks

Part	Scheme	Marks
(a)	Reduces a power by one on either term $3t^{-\frac{1}{2}} - \frac{3}{2}t^{\frac{1}{2}}$	M1 A1 (2)
(b)	Substitutes $t = 3$: $\frac{3}{\sqrt{3}} - \frac{3}{2}\sqrt{3}$ $\frac{3}{\sqrt{3}} = \sqrt{3}$, so $\sqrt{3} - \frac{3}{2}\sqrt{3}$ $-\frac{\sqrt{3}}{2}$ A decimal answer (-0.866) with no surd working scores M1 M0 A0.	M1 M1 A1 (3)
(c)	At 3 minutes the volume of water is <i>decreasing</i> at a rate of $\frac{\sqrt{3}}{2}$ (about 0.87) litres per minute Needs "decreasing" (or equivalent) and the rate with units.	B1ft (1)
Total for Question 4: 6 marks		



Question 5

6 marks

Part	Scheme	Marks
(a)	$2^3 - 4 \times 2^2 + 7 = 8 - 16 + 7 = -1$, so P lies on C	B1 (1)
(b)	$\frac{dy}{dx} = 3x^2 - 8x$ At $x = 2$: gradient = -4 $y + 1 = -4(x - 2) \Rightarrow y = -4x + 7$	M1 A1 A1 (3)
(c)	Normal gradient $\frac{1}{4}$: $y + 1 = \frac{1}{4}(x - 2)$ $x - 4y - 6 = 0$	M1 A1 (2)
Total for Question 5: 6 marks		

Question 6

8 marks

Part	Scheme	Marks
(a)	Multiplies out two brackets, then the third $x^3 - 3x^2 - 6x + 8$	M1 A1 (2)
(b)	$f'(x) = 3x^2 - 6x - 6$ $f'(0) = -6$ and $f(0) = 8$ $y = 8 - 6x$ <i>Using $f(0)$ or $f''(0)$ as the gradient scores M0.</i>	M1 A1 A1 (3)
(c)	$x^3 - 3x^2 - 6x + 8 = 8 - 6x \Rightarrow x^3 - 3x^2 = 0$ $x^2(x - 3) = 0$, so $x = 3$ (as $x = 0$ is the point of contact) $P(3, -10)$	M1 M1 A1 (3)
Total for Question 6: 8 marks		



Question 7

9 marks

Part	Scheme	Marks
(a)	Reduces a power by one on any term $3x^2 - 12x + 5$	M1 A1 (2)
(b)	$R(-1, -10)$ Gradient $3 + 12 + 5 = 20$	B1 B1 (2)
(c)	Parallel tangents have equal gradients: $3x^2 - 12x + 5 = 20$ $x^2 - 4x - 5 = 0 \Rightarrow (x - 5)(x + 1) = 0$, so $x = 5$ $P(5, 2)$ <i>The common error is to find the equation of the tangent at R and intersect it with C. That finds a different point and scores M0.</i>	M1 M1 A1 (3)
(d)	Midpoint of RP is $(2, -4)$ $2^3 - 6 \times 2^2 + 5 \times 2 + 2 = 8 - 24 + 10 + 2 = -4$, so the midpoint lies on C *	M1 A1* (2)
Total for Question 7: 9 marks		



Question 8

9 marks

Part	Scheme	Marks
(a)	$f'(x) = 6x^2 + 2kx - 12$ and $f''(x) = 12x + 2k$ $f'(2) = f''(2)$: $24 + 4k - 12 = 24 + 2k$ $2k = 12$, so $k = 6$ *	M1 M1 A1* (3)
(b)	$6x^2 + 12x - 12 = 12x + 12 \Rightarrow 6x^2 = 24$ $x = -2$ $f(-2) = -16 + 24 + 24 + 1 = 33$ <i>The y coordinate comes from $f(x)$, not from f' or f''.</i>	M1 A1 A1 (3)
(c)	$f(1) = -3$ and $f'(1) = 6 + 12 - 12 = 6$ Normal gradient $-\frac{1}{6}$: $y + 3 = -\frac{1}{6}(x - 1)$ $x + 6y + 17 = 0$	M1 M1 A1 (3)
Total for Question 8: 9 marks		

Question 9

8 marks

Part	Scheme	Marks
(a)	Writes $\frac{2}{x}$ as $2x^{-1}$ and differentiates $3x^2 - 2x^{-2}$	M1 A1 (2)
(b)	Gradient of the tangent is 5: $3x^2 - \frac{2}{x^2} = 5$ Multiplies by x^2 : $3x^4 - 5x^2 - 2 = 0$ *	M1 A1* (2)
(c)	$(3x^2 + 1)(x^2 - 2) = 0$, so $x^2 = 2$ (as $x^2 = -\frac{1}{3}$ is impossible) $x = \sqrt{2}$ (as $x > 0$) $y = 2\sqrt{2} + \frac{2}{\sqrt{2}} = 3\sqrt{2}$, and substitutes into $y = 5x + c$ $c = -2\sqrt{2}$	M1 A1 M1 A1 (4)
Total for Question 9: 8 marks		



Question 10

10 marks

Part	Scheme	Marks
(a)	Expands: $y = x^{\frac{3}{2}} - 4x^{\frac{1}{2}}$ Differentiates: $\frac{3}{2}x^{\frac{1}{2}} \dots$ $\frac{3}{2}x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$	M1 M1 A1 (3)
(b)	At $x = 4$: $y = 0$ and gradient $= \frac{3}{2} \times 2 - 2 \times \frac{1}{2} = 2$ $y - 0 = 2(x - 4)$, so $y = 2x - 8$ * $4^{\frac{1}{2}} = 2$ and $4^{-\frac{1}{2}} = \frac{1}{2}$ must be seen.	M1 A1* (2)
(c)	Normal gradient $-\frac{1}{2}$ through $(4, 0)$ $y = -\frac{1}{2}x + 2$	M1 A1 (2)
(d)	$A(0, -8)$ and $B(0, 2)$ Area $= \frac{1}{2} \times AB \times (x \text{ coordinate of } P) = \frac{1}{2} \times 10 \times 4$ 20	B1 M1 A1 (3)
Total for Question 10: 10 marks		



9

Chapter 9: Integration

Mark scheme. Marks are shown on the right of each line.

Question 1

4 marks

Part	Scheme	Marks
	Raises a power by one on any term: $x^n \rightarrow x^{n+1}$	M1
	Two of x^6 , $-2x^{-2}$, $-6x^{\frac{1}{2}}$	A1
	All three terms correct and simplified	A1
	$x^6 - 2x^{-2} - 6x^{\frac{1}{2}} + c$	A1
	<i>The last A1 needs +c. Common errors: $+2x^{-2}$ (from $\frac{4x^{-2}}{-2}$) and $-2x^{\frac{1}{2}}$ (from dividing by 2 instead of $\frac{1}{2}$).</i>	
		(4)
Total for Question 1: 4 marks		

Question 2

5 marks

Part	Scheme	Marks
	Splits into separate powers: $\frac{3x^4}{x^{\frac{5}{2}}} - \frac{2}{x^{\frac{5}{2}}}$	M1
	$3x^{\frac{3}{2}} - 2x^{-\frac{5}{2}}$	A1
	Raises a power by one on either term	M1
	One term correct: $\frac{6}{5}x^{\frac{5}{2}}$ or $+\frac{4}{3}x^{-\frac{3}{2}}$	A1
	$\frac{6}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{-\frac{3}{2}} + c$	A1
	<i>Integrating the numerator and denominator separately scores 0.</i>	
		(5)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	Expands: $\frac{4x - 12\sqrt{x} + 9}{\sqrt{x}}$	M1
	$4x^{\frac{1}{2}} - 12 + 9x^{-\frac{1}{2}}$, so $A = -12$, $B = 9$	A1
		(2)
(b)	Raises a power by one on any term	M1
	Two terms correct	A1
	$\frac{8}{3}x^{\frac{3}{2}} - 12x + 18x^{\frac{1}{2}} + c$	A1
		(3)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
	Integrates: $x^{\frac{1}{2}} \rightarrow x^{\frac{3}{2}}$ or $x^{-2} \rightarrow x^{-1}$	M1
	$2x^{\frac{3}{2}}$	A1
	$+4x^{-1} + c$	A1
	Substitutes (4, 10): $2 \times 8 + \frac{4}{4} + c = 10$	M1
	$f(x) = 2x^{\frac{3}{2}} + \frac{4}{x} - 7$	A1
The final answer must be written out as $f(x) = \dots$, not just $c = -7$.		(5)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
(a)	Reduces a power by one on either term $6x^2 + 3x^{-\frac{3}{2}}$ <i>No +c after differentiating.</i>	M1 A1 (2)
(b)	Raises a power by one on either term One term correct: $\frac{1}{2}x^4$ or $-12x^{\frac{1}{2}}$ $\frac{1}{2}x^4 - 12x^{\frac{1}{2}} + c$	M1 A1 A1 (3)
Total for Question 5: 5 marks		

Question 6

9 marks

Part	Scheme	Marks
(a)	Integrates: $x^3 - 4x^2 + 5x + c$ All three terms correct Uses (3, 4): $27 - 36 + 15 + c = 4$ $f(x) = x^3 - 4x^2 + 5x - 2$	M1 A1 M1 A1 (4)
(b)	Expands: $(x - 1)^2(x - 2) = (x^2 - 2x + 1)(x - 2)$ $= x^3 - 4x^2 + 5x - 2$, which is $f(x)$ *	M1 A1* (2)
(c)	Positive cubic shape Touches the x -axis at (1, 0) and crosses at (2, 0) Crosses the y -axis at (0, -2)	B1 B1 B1 (3)
Total for Question 6: 9 marks		



Question 7

8 marks

Part	Scheme	Marks
(a)	Normal gradient $-\frac{1}{10}$, so the tangent gradient is 10 $f'(4) = 4k - 3 \times 2 = 10$ $4k = 16$, so $k = 4$ *	B1 M1 A1* (3)
(b)	Integrates $4x - 3x^{\frac{1}{2}}$ $2x^2$ $-2x^{\frac{3}{2}} + c$ Uses (4, 20): $32 - 16 + c = 20$ $f(x) = 2x^2 - 2x^{\frac{3}{2}} + 4$	M1 A1 A1 M1 A1 (5)
Total for Question 7: 8 marks		

Question 8

8 marks

Part	Scheme	Marks
(a)	From the tangent: $f'(1) = 5$, so $a + b = 5$ * Normal gradient $-\frac{1}{16}$, so $f'(4) = 16$: $4a + 2b = 16$ $2a + b = 8$ *	B1* M1 A1* (3)
(b)	Solves simultaneously: $a = 3$, $b = 2$ Integrates $3x + 2x^{\frac{1}{2}}$: $\frac{3}{2}x^2 + \frac{4}{3}x^{\frac{3}{2}} + c$ Both terms correct The tangent gives $f(1) = 5 \times 1 - 2 = 3$: $\frac{3}{2} + \frac{4}{3} + c = 3$ $f(x) = \frac{3}{2}x^2 + \frac{4}{3}x^{\frac{3}{2}} + \frac{1}{6}$ <i>The point (1, 3) comes from the tangent equation. Using (1, 5) is the common error.</i>	B1 M1 A1 M1 A1 (5)
Total for Question 8: 8 marks		



Question 9

8 marks

Part	Scheme	Marks
(a)	P is $(1, 5)$ and the normal gradient is $-\frac{1}{2}$: $y - 5 = -\frac{1}{2}(x - 1)$ $x + 2y - 11 = 0$	M1 A1 (2)
(b)	Integrates $f''(x)$: $f'(x) = 6x^2 + 2x^{-2} + c_1$ Correct $f'(x)$, including $+c_1$ Uses $f'(1) = 2$: $6 + 2 + c_1 = 2$, so $c_1 = -6$ Integrates again: $f(x) = 2x^3 - 2x^{-1} - 6x + c_2$ Uses $f(1) = 5$: $2 - 2 - 6 + c_2 = 5$ $f(x) = 2x^3 - \frac{2}{x} - 6x + 11$ <i>Integrating twice and adding a constant only at the end, or using $(1, 5)$ to find c_1, are the most common errors.</i>	M1 A1 M1 A1ft M1 A1 (6)
Total for Question 9: 8 marks		

Question 10

10 marks

Part	Scheme	Marks
(a)	Splits: $f'(x) = 1 - 9x^{-2}$ Integrates: $x + 9x^{-1} + c$ Uses $(3, 2)$: $3 + 3 + c = 2$ $f(x) = x + \frac{9}{x} - 4$	M1 A1 M1 A1 (4)
(b)	$f(1) = 6$ and $f'(1) = 1 - 9 = -8$ $y - 6 = -8(x - 1)$ $y = -8x + 14$	M1 M1 A1 (3)
(c)	$x + \frac{9}{x} - 4 = -8x + 14$ and multiplies by x: $9x^2 - 18x + 9 = 0$ $9(x - 1)^2 = 0$, a repeated root (or $b^2 - 4ac = 324 - 324 = 0$) So $x = 1$ is the only solution: the tangent touches C at $x = 1$ and meets it nowhere else *	M1 M1 A1* (3)
Total for Question 10: 10 marks		

