



eClassroom

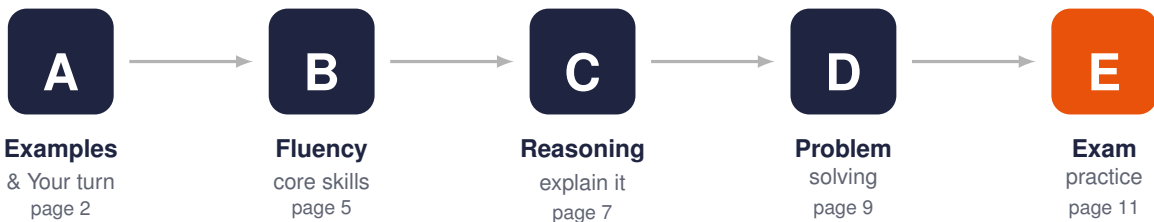
GCSE MATHEMATICS • NUMBER

Powers and Roots

Worksheet

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • Calculator only where shown

Free Powers and Roots worksheet for GCSE and iGCSE Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: work out squares, cubes, higher powers and their roots • solve equations such as $x^2 = 49$ • use a calculator for powers and roots • *Higher:* simplify and calculate with surds, rationalise denominators, and give exact answers.

Which questions are for me?

FOUNDATION Everyone starts here. These skills appear on both tiers.

HIGHER Surds. This is Higher-only content: simplifying, adding and multiplying surds, rationalising denominators, and using surds in geometry and proof.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Foundation marks	25	11	15	13
Higher marks	20	16	18	15
My score				

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

FOUNDATION

Everyone starts here

Worked example A1

Work out $\sqrt[3]{125} + 2^4$

$\sqrt[3]{125}$ asks: which number cubed gives 125?

$$5 \times 5 \times 5 = 125, \text{ so } \sqrt[3]{125} = 5$$

2^4 means four 2s multiplied together.

$$2 \times 2 \times 2 \times 2 = 16 \quad 5 + 16 = 21$$

21

Your turn A1

Work out $\sqrt[3]{64} + 3^4$

Worked example A2: equations with squares and cubes

Solve (a) $x^2 = 49$ (b) $x^3 = -8$

A square has **two** roots, because a negative times a negative is positive.

$$(a) 7^2 = 49 \text{ and } (-7)^2 = 49$$

A cube has only **one** root. A negative number cubed is negative.

$$(b) (-2) \times (-2) \times (-2) = -8$$

$$(a) x = 7 \text{ or } x = -7 \quad (b) x = -2$$

Your turn A2

Solve (a) $x^2 = 121$ (b) $x^3 = -27$

Worked example A3: using a calculator

Work out $\sqrt[3]{4.8^2 + 3.7}$

Give your answer correct to 3 significant figures.

Work out the inside of the root first and write it down.

$$4.8^2 + 3.7 = 23.04 + 3.7 = 26.74$$

Then take the cube root. Write more figures than you need before rounding.

$$\sqrt[3]{26.74} = 2.9903 \dots$$

2.99

Your turn A3

Work out $\sqrt[3]{7.3^3 - 12.5}$

Give your answer correct to 3 significant figures.



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HIGHER

Higher only: surds

Worked example A4: simplifying a surdSimplify $\sqrt{72}$ *Find the largest square number that is a factor of 72*

$$72 = 36 \times 2$$

Use $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

$$\sqrt{72} = \sqrt{36} \times \sqrt{2} = 6\sqrt{2}$$

$$6\sqrt{2}$$

Your turn A4Simplify $\sqrt{180}$ **Worked example A5: adding surds**Write $\sqrt{27} + \sqrt{48}$ in the form $k\sqrt{3}$ *Simplify each surd first. You can only add surds with the same root.*

$$\sqrt{27} = 3\sqrt{3} \quad \sqrt{48} = 4\sqrt{3}$$

Add like terms.

$$3\sqrt{3} + 4\sqrt{3} = 7\sqrt{3}$$

$$7\sqrt{3}$$

Your turn A5Write $\sqrt{8} + \sqrt{98}$ in the form $k\sqrt{2}$ **Worked example A6: rationalising a denominator**Rationalise the denominator of $\frac{15}{\sqrt{5}}$ *Multiply top and bottom by $\sqrt{5}$. This does not change the value, and $\sqrt{5} \times \sqrt{5} = 5$*

$$\frac{15}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{15\sqrt{5}}{5}$$

Simplify.

$$3\sqrt{5}$$

Your turn A6Rationalise the denominator of $\frac{22}{\sqrt{11}}$ 

Worked example A7: squaring a bracket with a surd

Write $(2 + \sqrt{5})^2$ in the form $a + b\sqrt{5}$

Write the bracket twice and expand all four terms.

$$(2 + \sqrt{5})(2 + \sqrt{5}) = 4 + 2\sqrt{5} + 2\sqrt{5} + 5$$

$\sqrt{5} \times \sqrt{5} = 5$. *Collect like terms.*

$9 + 4\sqrt{5}$

Your turn A7

Write $(3 - \sqrt{2})^2$ in the form $a + b\sqrt{2}$

Worked example A8: a denominator with two terms

Rationalise the denominator of $\frac{6}{3 - \sqrt{3}}$

Multiply top and bottom by $3 + \sqrt{3}$. Changing the sign makes the surds cancel.

Bottom: $(3 - \sqrt{3})(3 + \sqrt{3}) = 9 - 3 = 6$

Top: $6(3 + \sqrt{3})$

Simplify.

$$\frac{6(3 + \sqrt{3})}{6} = 3 + \sqrt{3}$$

$3 + \sqrt{3}$

Your turn A8

Rationalise the denominator of $\frac{10}{\sqrt{6} + 1}$

Give your answer in the form $a\sqrt{6} + b$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer? No calculator except B6.

FOUNDATION*Everyone starts here***B1** Work out

(a) 7^2

(b) $(-7)^2$

(c) -7^2

(d) 0.7^2

[4 marks]**B2** Work out

(a) 2^3

(b) $(-2)^3$

(c) 10^3

(d) $\left(\frac{1}{2}\right)^3$

[4 marks]**B3** Work out

(a) $\sqrt{64}$

(b) $\sqrt[3]{64}$

(c) $\sqrt{\frac{1}{4}}$

(d) $\sqrt{0.09}$

[4 marks]**B4** Work out

(a) 2^5

(b) 3^4

(c) 10^6

(d) 1^{100}

[4 marks]**B5** Solve

(a) $x^2 = 36$

(b) $x^3 = 125$

(c) $x^2 = 2.25$

(d) $x^3 = -1000$

[4 marks]**B6** *(calculator)* Work out

(a) 3.7^3

(b) $\sqrt{200}$ correct to 2 decimal places

(c) $\sqrt[3]{-50}$ correct to 3 decimal places

[3 marks]**B7** Write these in order of size. Start with the smallest.

2^5

5^2

3^3

$\sqrt{900}$

[2 marks]

HIGHER*Higher only: surds***B8** Simplify

(a) $\sqrt{50}$

(b) $\sqrt{48}$

(c) $\sqrt{63}$

(d) $3\sqrt{8}$

[4 marks]

B9 Simplify

(a) $\sqrt{3} \times \sqrt{12}$

(b) $\sqrt{5} \times \sqrt{10}$

(c) $\frac{\sqrt{72}}{\sqrt{2}}$

(d) $(2\sqrt{3})^2$

[4 marks]

B10 Simplify

(a) $\sqrt{12} + \sqrt{27}$

(b) $\sqrt{50} - \sqrt{18}$

(c) $3\sqrt{20} + \sqrt{45}$

(d) $\sqrt{8} + \sqrt{32} - \sqrt{2}$

[4 marks]

B11 Rationalise the denominator. Give each answer in its simplest form.

(a) $\frac{6}{\sqrt{3}}$

(b) $\frac{10}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{5}}$

(d) $\frac{9}{2\sqrt{3}}$

[4 marks]

B12 Expand and simplify

(a) $\sqrt{2}(3 + \sqrt{2})$

(b) $(1 + \sqrt{5})(3 - \sqrt{5})$

(c) $(4 - \sqrt{3})^2$

(d) $(\sqrt{7} + 2)(\sqrt{7} - 2)$

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

FOUNDATION

Everyone starts here

- C1** Sam says, " $(-5)^2 = -25$ "
Explain why Sam is wrong. [2 marks]
- C2** Is $\sqrt{9+16}$ equal to $\sqrt{9} + \sqrt{16}$?
You **must** show your working. [2 marks]
- C3** Which is greater, 2^6 or 6^2 ?
You **must** show your working. [2 marks]
- C4** Find a whole number between 2 and 100 that is both a square number and a cube number. Show that it is both. [2 marks]
- C5** (a) Between which two consecutive whole numbers does $\sqrt{40}$ lie? Give a reason.
(b) Hana says, "The cube root of a number is always smaller than the number."
Give an example to show that Hana is wrong. [3 marks]



HIGHER*Higher tier only*

C6 Leo says, " $\sqrt{18} + \sqrt{2} = \sqrt{20}$ "

(a) Show that Leo is wrong. (b) Write $\sqrt{18} + \sqrt{2}$ as a single surd in the form $\sqrt{n}$.

[3 marks]

C7 Ali writes $\frac{6}{\sqrt{3}} = \frac{6}{3} = 2$

Explain Ali's mistake, and write down the correct simplified answer.

[2 marks]

C8 Show that $(\sqrt{12} - \sqrt{3})^2$ is a whole number.

[3 marks]

C9 (a) $\sqrt{a} = 5\sqrt{3}$ Find the value of a .

(b) $\sqrt{b} + \sqrt{27} = 5\sqrt{3}$ Find the value of b .

[3 marks]

C10 (a) Show that $\frac{1}{\sqrt{n+1} + \sqrt{n}} = \sqrt{n+1} - \sqrt{n}$

(b) Hence work out the exact value of $\frac{1}{\sqrt{2}+1} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}}$

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

FOUNDATION

Everyone starts here

- D1** A square garden has an area of 169 m^2 .
Work out the perimeter of the garden. [2 marks]
- D2** A cube has a volume of 343 cm^3 .
Work out the total surface area of the cube. [3 marks]
- D3** $1^3 + 2^3 = 9 = 3^2$ and $1^3 + 2^3 + 3^3 = 36 = 6^2$
(a) Show that $1^3 + 2^3 + 3^3 + 4^3$ is also a square number.
(b) The pattern continues. Write down the square number that $1^3 + 2^3 + 3^3 + 4^3 + 5^3$ is equal to, and check it. [3 marks]
- D4** (calculator) A ball is dropped from a height of h metres. The time, t seconds, it takes to reach the ground is
$$t = \sqrt{\frac{h}{4.9}}$$

Work out t when $h = 30$. Give your answer correct to 3 significant figures. [3 marks]
- D5** A square floor has an area of 12.96 m^2 . It is covered with square tiles of side 30 cm, with no gaps.
Work out the number of tiles needed. [4 marks]



HIGHER*Higher tier only*

- D6** A rectangle has length $\sqrt{27}$ cm and width $\sqrt{12}$ cm.
(a) Show that the area of the rectangle is a whole number of cm^2 .
(b) Work out the perimeter. Give your answer in the form $k\sqrt{3}$. [4 marks]
- D7** The two shorter sides of a right-angled triangle are $\sqrt{5}$ cm and $2\sqrt{5}$ cm.
Work out the length of the hypotenuse. [3 marks]
- D8** A rectangle has an area of 12 cm^2 . Its width is $(\sqrt{5} - 1)$ cm.
Work out the length of the rectangle. Give your answer in the form $a\sqrt{5} + b$ where a and b are whole numbers. [4 marks]
- D9** The diagonal of a square is 10 cm.
Work out the exact length of one side, and hence the area of the square. [3 marks]
- D10** Solve $x\sqrt{2} + 3 = x + 5$
Give your answer in the form $a\sqrt{2} + b$ where a and b are whole numbers. [4 marks]



E**Exam practice**

Exam-style questions. Write your answers in the spaces. Aim for about one minute per mark.

FOUNDATION*Foundation and Higher crossover***E1** (a) Work out 9^2

.....
(1)

(b) Work out $\sqrt[3]{27}$

.....
(1)

(c) Work out $2^4 \times \sqrt{25}$

.....
(2)

(Total for Question E1 is 4 marks)**E2** (calculator) (a) Work out $\sqrt{18.49} + 2.6^3$

Write down all the figures on your calculator display.

.....
(2)

(b) Write your answer to part (a) correct to 1 decimal place.

.....
(1)

(Total for Question E2 is 3 marks)

E3 Here is a list of numbers.

1 8 15 27 36 50 64

From the list, write down

(a) a square number that is greater than 10 and is **not** a cube number

.....
(1)

(b) a cube number that is an odd number greater than 1

.....
(1)

(c) a number greater than 1 that is both a square number and a cube number.

.....
(1)

(Total for Question E3 is 3 marks)

E4 **(a)** Solve $x^2 = 196$

.....
(2)

(b) Work out $\sqrt[3]{-216}$

.....
(1)

(Total for Question E4 is 3 marks)



HIGHER*Higher tier only*

E5 Write $\sqrt{28} + \sqrt{112}$ in the form $k\sqrt{7}$ where k is an integer.

.....
(Total for Question E5 is 3 marks)

E6 Rationalise the denominator of $\frac{18}{\sqrt{6}}$
Give your answer in its simplest form.

.....
(Total for Question E6 is 2 marks)

E7 Show that $(3 + \sqrt{8})^2 = 17 + 12\sqrt{2}$
Show each stage of your working.

.....
(Total for Question E7 is 3 marks)



- E8** Rationalise the denominator of $\frac{22}{5 - \sqrt{3}}$

Give your answer in the form $a + \sqrt{b}$ where a and b are integers.

.....
(Total for Question E8 is 3 marks)

- E9** Show that $\frac{\sqrt{45} - \sqrt{5}}{\sqrt{5} + 1}$ can be written in the form $\frac{a - \sqrt{a}}{b}$ where a and b are integers.

Show your working clearly.

(Total for Question E9 is 4 marks)

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