



eClassroom

A-LEVEL MATHEMATICS • PURE 1

Straight Line Graphs

Worked Solutions

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Straight Line Graphs worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 1 Straight Line Graphs worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $m = \frac{5 - (-1)}{-1 - 3} = -\frac{3}{2}$; $y + 1 = -\frac{3}{2}(x - 3)$, so $2y + 2 = -3x + 9$: $3x + 2y - 7 = 0$

A2 $y = -\frac{5}{2}x + \frac{7}{2}$, so $m = -\frac{5}{2}$ and the perpendicular gradient is $\frac{2}{5}$
 $y - 3 = \frac{2}{5}(x + 5)$, so $5y - 15 = 2x + 10$: $2x - 5y + 25 = 0$

A3 $AB = \sqrt{4^2 + 8^2} = \sqrt{80} = 4\sqrt{5}$; midpoint $\left(\frac{-1+3}{2}, \frac{5-3}{2}\right) = (1, 1)$

STRETCH

A4 Midpoint $(-1, 1)$; $m_{AB} = \frac{10}{-6} = -\frac{5}{3}$, so the bisector has gradient $\frac{3}{5}$
 $y - 1 = \frac{3}{5}(x + 1)$, so $5y - 5 = 3x + 3$: $3x - 5y + 8 = 0$

A5 $x + 3(3x - 2) = 24$, so $10x = 30$, $x = 3$, and $y = 3x - 2$: $P(3, 7)$
 The y intercepts are $(0, -2)$ and $(0, 8)$: base 10, height 3: $\frac{1}{2} \times 10 \times 3 = 15$

A6 $m_{PQ} = -\frac{2}{3}$, $m_{QR} = \frac{6}{4} = \frac{3}{2}$, and $-\frac{2}{3} \times \frac{3}{2} = -1$, so angle $PQR = 90^\circ$
 $PQ = \sqrt{13}$, $QR = \sqrt{52} = 2\sqrt{13}$: area $= \frac{1}{2} \times \sqrt{13} \times 2\sqrt{13} = 13$

A7 $C(c, c)$: $(c - 3)^2 + (c - 1)^2 = 10$, so $2c^2 - 8c = 0$, so $c = 0$ or $c = 4$, on $y = x$
 $(0, 0)$ or $(4, 4)$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $\boxed{3}$, $\boxed{-5}$ (b) $y = 3x + \frac{1}{2}$: $\boxed{3}$, $\boxed{\frac{1}{2}}$ (c) $y = -2x + \frac{3}{2}$: $\boxed{-2}$, $\boxed{\frac{3}{2}}$ (d) $y = \frac{3}{5}x - 3$: $\boxed{\frac{3}{5}}$, $\boxed{-3}$

Always make y the subject first; the sign of the x term changes when it moves across.

B2 (a) $\boxed{2x - y + 2 = 0}$ (b) $\boxed{2x + y - 6 = 0}$ (c) $2y - 8 = x - 1$: $\boxed{x - 2y + 7 = 0}$ (d) $3y - 3 = -2x - 6$:
 $\boxed{2x + 3y + 3 = 0}$

With a fractional gradient, multiply through by the denominator before collecting terms.

B3 (a) $m = 2$: $\boxed{2x - y + 1 = 0}$ (b) $m = -2$: $\boxed{2x + y - 1 = 0}$ (c) $m = \frac{1}{2}$: $\boxed{x - 2y + 8 = 0}$ (d) $m = -\frac{2}{3}$:
 $\boxed{2x + 3y - 12 = 0}$

Check by substituting both points into your answer.

B4 (a) 3 and 3: $\boxed{\text{parallel}}$ (b) 3 and $-\frac{1}{3}$: $\boxed{\text{perpendicular}}$ (c) $-\frac{2}{5}$ and $\frac{5}{2}$: $\boxed{\text{perpendicular}}$ (d) $\frac{2}{3}$ and $\frac{3}{2}$:
 $\boxed{\text{neither}}$

In (d) the gradients are reciprocals but not negative reciprocals: their product is 1, not -1

B5 (a) $\sqrt{9+16} = \boxed{5}$ (b) $\sqrt{16+64} = \sqrt{80} = \boxed{4\sqrt{5}}$ (c) $\sqrt{36+64} = \boxed{10}$

Square the differences: a negative difference becomes positive.

B6 (a) $2x - 3 = x + 1$, and $y = x + 1$: $\boxed{(4, 5)}$ (b) $x + 2x - 3 = 6$, and $y = 2x - 3$: $\boxed{(3, 3)}$
(c) $2x + 6x = 12$, and $y = 2x$: $\boxed{\left(\frac{3}{2}, 3\right)}$ (d) $8x + 12y = 48$ and $9x - 12y = 3$, so $17x = 51$: $\boxed{(3, 2)}$

Substitute when one equation gives y directly; eliminate when neither does.

STRETCH

B7 (a) midpoint (2, 1), gradient -2 : $\boxed{2x + y - 5 = 0}$ (b) midpoint (3, -1), gradient -1 : $\boxed{x + y - 2 = 0}$
(c) midpoint (1, 3), gradient $\frac{3}{2}$: $\boxed{3x - 2y + 3 = 0}$ (d) a vertical segment, so the bisector is horizontal:
 $\boxed{y = 4}$

In (d) the gradient of AB is undefined, so use the picture: the bisector is the horizontal line through the midpoint.



B8 $l: y = -\frac{k-1}{2}x + \frac{5}{2}$, so its gradient is $-\frac{k-1}{2}$

(a) $-\frac{k-1}{2} = 3: k = -5$ (b) $-\frac{k-1}{2} = -\frac{1}{3}: k = \frac{5}{3}$ (c) $3(k-1)+2=5: k=2$ (d) gradient 0: $k=1$

One line, four conditions: each gives a different equation in k .

B9 (a) $\frac{1}{2} \times 6 \times 5 = 15$ (b) $\frac{1}{2} \times 6 \times 5 = 15$ (c) $\frac{1}{2} \times 5 \times 2 = 5$

(d) $m_{AB} = \frac{1}{2}, m_{BC} = -2$: a right angle at B , with $AB = BC = \sqrt{20}$: $\frac{1}{2} \times \sqrt{20} \times \sqrt{20} = 10$

Look for a side along a grid line, or a right angle, to give a perpendicular height.

B10 (a) $(x-2)^2 + (y-3)^2 = 8$ with $y = x+1$: $2(x-2)^2 = 8$: $(0, 1), (4, 5)$

(b) $(x-2)^2 + (y-3)^2 = 2$ with $y = x+1$: $2(x-2)^2 = 2$: $(1, 2), (3, 4)$

(c) $(x-3)^2 + (y-5)^2 = 25$ with $y = 1$: $(x-3)^2 = 9$: $(0, 1), (6, 1)$

(d) $(x-7)^2 + (y-12)^2 = 169$ with $y = 0$: $(x-7)^2 = 25$: $(2, 0), (12, 0)$

A point at a given distance from a line's point usually has two positions, one on each side.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 Ana used the reciprocal. Perpendicular gradients multiply to -1 , so she needs the *negative* reciprocal:

$$-\frac{1}{4}$$

C2 Raj's answer has fractions and is not written as $ax + by + c = 0$. Multiply by 3 and collect: $2x + 3y - 11 = 0$

C3 $m_{AB} = \frac{6}{3} = 2$ and $m_{BC} = \frac{4}{2} = 2$: equal gradients, and the lines share the point B , so A, B and C are collinear.

Through $B(2, 4)$ with gradient 2: $y = 2x$

C4 Joe subtracted the coordinates instead of adding them. The midpoint is $\left(\frac{-3+7}{2}, \frac{5-1}{2}\right) = (2, 2)$

Length = $\sqrt{10^2 + 6^2} = \sqrt{136} = 2\sqrt{34}$



STRETCH

- C5** $m_{AB} = \frac{1}{4} = m_{DC}$ and $m_{BC} = 2 = m_{AD}$: both pairs of opposite sides are parallel, so $ABCD$ is **a parallelogram**
 $\frac{1}{4} \times 2 = \frac{1}{2} \neq -1$, so AB is not perpendicular to BC : **not a rectangle**

- C6** $m_{PR} = \frac{3}{5}$, $m_{QS} = \frac{5}{-3}$, and $\frac{3}{5} \times \left(-\frac{5}{3}\right) = -1$: perpendicular. Both midpoints are $\left(\frac{7}{2}, \frac{7}{2}\right)$: they bisect each other.

$PQ = QR = RS = SP = \sqrt{17}$, and $m_{PQ} = -\frac{1}{4}$, $m_{QR} = 4$ give a right angle at Q : **$PQRS$ is a square**

- C7** $\frac{1}{2} \times AB \times AC$ needs a right angle at A , but $m_{AB} = \frac{1}{3}$ and $m_{AC} = 4$, and $\frac{1}{3} \times 4 \neq -1$

Rectangle 6 by 4 minus three right-angled triangles: $24 - 6 - 2 - 5 =$ **11**

- C8** (a) Gradient $-\frac{b}{a}$, so $y = -\frac{b}{a}x + b$; divide by b : $\frac{y}{b} = -\frac{x}{a} + 1$, so $\frac{x}{a} + \frac{y}{b} = 1$ **as required**

The triangle has base a along the x -axis and height b along the y -axis: area $\frac{1}{2}ab$

- (b) $\frac{3}{a} + \frac{4}{b} = 1$ and $ab = 48$. Multiply the first by ab : $3b + 4a = 48$. With $b = \frac{48}{a}$: $\frac{144}{a} + 4a = 48$

$4a^2 - 48a + 144 = 0$, so $a^2 - 12a + 36 = 0$, $(a - 6)^2 = 0$: $a = 6$, $b = 8$

$\frac{x}{6} + \frac{y}{8} = 1$: **$4x + 3y - 24 = 0$**

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

- D1** (a) $m = \frac{6}{8} = \frac{3}{4}$; $y + 1 = \frac{3}{4}(x + 2)$, so $4y + 4 = 3x + 6$: **$3x - 4y + 2 = 0$**

(b) $y = 0$: $3x + 2 = 0$: **$C\left(-\frac{2}{3}, 0\right)$**



D2 (a) Midpoint $(4, 3)$; $m_{AB} = \frac{-8}{6} = -\frac{4}{3}$, so l has gradient $\frac{3}{4}$

$$y - 3 = \frac{3}{4}(x - 4), \text{ so } 4y - 12 = 3x - 12: \boxed{3x - 4y = 0}$$

(b) $3(0) - 4(0) = 0$, so O lies on l . Every point on the perpendicular bisector is the same distance from A and B , so $\boxed{OA = OB}$ (both are $\sqrt{50}$)

D3 (a) $m = \frac{8-2}{3\sqrt{3}-\sqrt{3}} = \frac{6}{2\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$ as required

(b) Perpendicular gradient $-\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$$y - 8 = -\frac{\sqrt{3}}{3}(x - 3\sqrt{3}) = -\frac{\sqrt{3}}{3}x + 3: \boxed{y = -\frac{\sqrt{3}}{3}x + 11}$$

D4 $x + 2(2x + 2) = 14$, so $5x = 10$, $x = 2$, and $y = 2x + 2$: $P(2, 6)$

$A(-1, 0)$ and $B(14, 0)$: base 15, height 6: $\frac{1}{2} \times 15 \times 6 = \boxed{45}$

STRETCH

D5 (a) $AB = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$ as required

(b) $C(c, 1)$: $(c - 5)^2 + 6^2 = 100$, so $(c - 5)^2 = 64$, $c = 13$ or $c = -3$. But $c = -3$ gives A itself: $C(13, 1)$

(c) Base $AC = 16$ along $y = 1$, height $7 - 1 = 6$: $\frac{1}{2} \times 16 \times 6 = \boxed{48}$

D6 (a) $m_{AB} = -\frac{4}{k}$ and $m_{BC} = \frac{3}{k}$; a right angle needs $-\frac{4}{k} \times \frac{3}{k} = -1$, so $k^2 = 12$ as required

(b) $AB = \sqrt{k^2 + 16} = \sqrt{28} = 2\sqrt{7}$ and $BC = \sqrt{k^2 + 9} = \sqrt{21}$

$$\text{Area} = \frac{1}{2} \times 2\sqrt{7} \times \sqrt{21} = \sqrt{147} = \boxed{7\sqrt{3}}$$

D7 (a) $m = \frac{4}{8} = \frac{1}{2}$; $y - 2 = \frac{1}{2}(x + 2)$: $x - 2y + 6 = 0$

(b) CD has gradient -2 : $y + 1 = -2(x - 5)$, so $y = -2x + 9$. Then $x - 2(-2x + 9) + 6 = 0$, so $5x = 12$:

$$\boxed{D\left(\frac{12}{5}, \frac{21}{5}\right)}$$

(c) $AB = \sqrt{80} = 4\sqrt{5}$ and $CD = \sqrt{\left(\frac{13}{5}\right)^2 + \left(\frac{26}{5}\right)^2} = \frac{13\sqrt{5}}{5}$: area $= \frac{1}{2} \times 4\sqrt{5} \times \frac{13\sqrt{5}}{5} = \boxed{26}$



- D8** (a) $y = 2x + 4$ into $x + 2y - 13 = 0$: $5x - 5 = 0$: $P(1, 6)$. Gradients 2 and $-\frac{1}{2}$ multiply to -1 : perpendicular.
- (b) $\vec{PQ}$ moves 4 right and 2 down, length $\sqrt{20}$. PS is along l_1 (2 up for every 1 across) with the same length, so it moves (2, 4) or $(-2, -4)$. S lies on $2x - y + 4 = 0$:
- $S(3, 10)$, then $R = Q + (2, 4)$: $R(7, 8)$; or $S(-1, 2)$, $R = Q - (2, 4)$: $R(3, 0)$

E**Exam practice solutions**

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

- E1** (a) $m = \frac{7-3}{2-(-4)} = \frac{2}{3}$ M1
- $y - 3 = \frac{2}{3}(x + 4)$: $y = \frac{2}{3}x + \frac{17}{3}$ A1
- (b) Midpoint $(-1, 5)$ and gradient $-\frac{3}{2}$ B1: both
- $y - 5 = -\frac{3}{2}(x + 1)$ M1: their midpoint, their perpendicular gradient
- $2y - 10 = -3x - 3$: $3x + 2y - 7 = 0$ A1: integers, = 0
-
- E2** (a) l_1 has gradient -2 , so $y = \frac{1}{2}x$ B1
- (b) $2x + \frac{1}{2}x - 10 = 0$ M1: substitutes
- $x = 4$, and $y = \frac{1}{2}x$: $P(4, 2)$ A1
- (c) $OP = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$; $A(5, 0)$, $B(0, 10)$, so $AB = \sqrt{125} = 5\sqrt{5}$ B1 B1
- (d) OP is perpendicular to AB , so it is the height: $\frac{1}{2} \times 5\sqrt{5} \times 2\sqrt{5} = 25$ as required B1*



STRETCH

E3 (a) $m = \frac{4}{8} = \frac{1}{2}$, and $y - 2 = \frac{1}{2}(x + 1)$ M1

$$y = \frac{1}{2}x + \frac{5}{2}$$

A1

(b) Gradient -2 through B : $y - 6 = -2(x - 7)$ M1

$$y = -2x + 20$$

A1

(c) $AB = \sqrt{64 + 16} = 4\sqrt{5}$; angle $ABC = 90^\circ$, so area $= \frac{1}{2} \times 4\sqrt{5} \times BC = 30$ M1: area with AB as base

$$BC = \frac{15}{\sqrt{5}} = 3\sqrt{5}$$
 A1

Along l , $C = (7 + t, 6 - 2t)$ with $BC = \sqrt{5}|t|$, so $t = \pm 3$ M1: two positions, on $y = -2x + 20$

$$C(10, 0) \text{ or } C(4, 12)$$

A1

