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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

Pure Mathematics 4

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA14 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	own PDF	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	2	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	own PDF	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	own PDF	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	own PDF	
6 Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	own PDF	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	own PDF	

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CHAPTER 2 • KEY SKILLS

Partial Fractions

Specification P4: Algebra and functions • 10 questions • 78 marks • about 94 minutes

1 The identity

$$\frac{5x+1}{(x-1)(x+2)} \equiv \frac{A}{x-1} + \frac{B}{x+2}$$

Multiply by the whole denominator:

$$5x+1 \equiv A(x+2) + B(x-1)$$

2 Finding constants

Substitute the value of x that makes each factor zero. Compare coefficients (e.g. of x^2) for any constant left over.

3 Factorise first

Use the factor theorem on a cubic, or $x^2 - 4 = (x-2)(x+2)$.

$(x^2 - 4)(x+2) = (x-2)(x+2)^2$ has a *repeated* factor.

4 Repeated factors

$(x-a)^2$ needs two fractions:

$$\frac{A}{x-b} + \frac{B}{x-a} + \frac{C}{(x-a)^2}$$

5 Improper fractions

Degree of numerator $\geq$ degree of denominator: divide first, or start the identity with A (or $Ax+B$).

6 Unknown coefficients

If some constants are given, compare coefficients: the x^2 terms give $A+B+C=1$ when the numerator is $x^2 + \dots$

7 Differentiating

Differentiate term by term:

$$k(ax+b)^{-1} \rightarrow -ka(ax+b)^{-2} \text{ and}$$

$$k(ax+b)^{-2} \rightarrow -2ka(ax+b)^{-3}$$

8 Series

Split each term, write out the first and last few terms of the sum, and cancel:

$$\frac{1}{r} - \frac{1}{r+1} \text{ telescopes.}$$

9 Graphs

$A + \frac{B}{x-a} + \dots$ has asymptotes $x=a$

and $y=A$. With $u = \frac{1}{x-a}$ a repeated factor gives a quadratic in u .

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	6	5	5	5	10	10	10	11	11
1 The identity	•			•			•		•	
2 Finding constants	•	•	•	•	•	•	•	•	•	•
3 Factorise first	•				•	•				
4 Repeated factors		•	•		•					•
5 Improper fractions			•			•		•		•
6 Unknown coefficients		•	•	•						
7 Differentiating		•				•			•	
8 Series							•			
9 Graphs								•		•
Interleaved: factor theorem and sigma notation (P2)	•						•			
Interleaved: inequalities and completing the square (P1)								•		•
Interleaved: differentiation, e and ln, number of solutions (P3)		•				•			•	•
Interleaved: proof by contradiction (Chapter 1)								•		

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write the identity with the whole denominator, and give the final answer as the full expression, not just A , B and C . Watch the signs: write $-\frac{3}{x+3}$, not $+\frac{-3}{x+3}$. Substituting values of x is quicker than simultaneous equations.



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1.

$$f(x) = \frac{4x^2 + 3x - 25}{x^3 - 2x^2 - 5x + 6}$$

- (a) Show that $(x - 1)$ is a factor of $x^3 - 2x^2 - 5x + 6$, and hence factorise $x^3 - 2x^2 - 5x + 6$ completely.

(2)

- (b) Hence express $f(x)$ in partial fractions.

(3)

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Question 1 continued

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(Total for Question 1 is 5 marks)



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2.

$$f(t) = \frac{15t^2 - 10t + 7}{(t+1)(3t-1)^2} \quad t \neq -1, t \neq \frac{1}{3}$$

(a) Given that

$$f(t) \equiv \frac{A}{t+1} + \frac{B}{3t-1} + \frac{C}{(3t-1)^2}$$

find the values of the constants A , B and C .

(4)

(b) Hence find the value of $f'(0)$

(2)

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Question 2 continued

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(Total for Question 2 is 6 marks)



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3.

$$g(x) = \frac{2x^3 - 2x^2 + x + 3}{x(x-1)^2} \quad x \neq 0, x \neq 1$$

(a) Show that $g(x) = 2 + \frac{2x^2 - x + 3}{x(x-1)^2}$ (2)

(b) Hence express $g(x)$ in partial fractions. (3)

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Question 3 continued

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(Total for Question 3 is 5 marks)



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4. Given that

$$\frac{x^2 + px + q}{(x+1)(x-2)(x+4)} \equiv \frac{2}{x+1} + \frac{B}{x-2} + \frac{3}{x+4}$$

where p , q and B are constants, find the value of B , the value of p and the value of q .

(5)

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Question 4 continued

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(Total for Question 4 is 5 marks)



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5.

$$h(x) = \frac{3x^2 + x + 2}{(x^2 - 4)(x + 2)} \quad x \neq \pm 2$$

Express $h(x)$ in partial fractions.

(5)

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Question 5 continued

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(Total for Question 5 is 5 marks)



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$$g(x) = \frac{2x^3 + 7x^2 + 3x + 9}{x(x+3)} \quad x > 0$$

$$g(x) \equiv Ax + B + \frac{C}{x} + \frac{D}{x+3}$$

(2)

(3)

(1)

Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)



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(3)

$$\sum_{r=1}^n \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$$

(4)

(2)

(d) Explain why $\sum_{r=1}^n \frac{2}{r(r+1)(r+2)} < \frac{1}{2}$ for every positive integer n .

(1)

Question 7 continued

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Question 7 continued

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(Total for Question 7 is 10 marks)



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8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

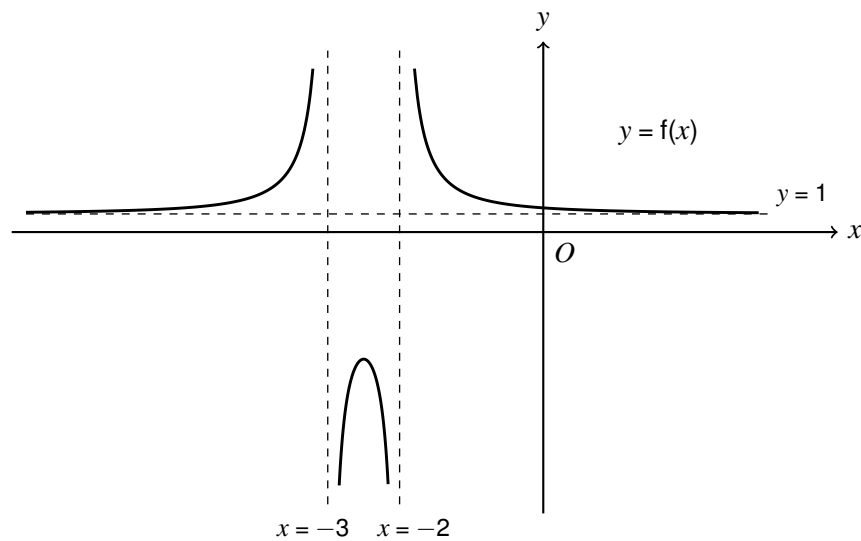


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \frac{x^2 + 5x + 8}{(x+2)(x+3)} \quad x \neq -2, x \neq -3$$

The curve has asymptotes $x = -3$, $x = -2$ and $y = 1$

(a) Express $f(x)$ in the form $A + \frac{B}{x+2} + \frac{C}{x+3}$, where A , B and C are constants.

(4)

(b) Use proof by contradiction to prove that there is no real value of x for which $f(x) = 1$

(3)

(c) Using algebra, find the set of values of x for which $f(x) > 3$, giving your answer in exact form.

(3)



Question 8 continued

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Question 8 continued

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(Total for Question 8 is 10 marks)



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Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{6e^x}{(e^x + 1)(e^x + 3)} \quad x \in \mathbb{R}$$

- (a) Express $\frac{6u}{(u+1)(u+3)}$ in partial fractions.

(3)

- (b)** Hence, or otherwise, show that

$$f'(x) = \frac{3e^x}{(e^x + 1)^2} - \frac{9e^x}{(e^x + 3)^2}$$

(3)

- (c)** Hence find the exact x coordinate of the stationary point of the curve $y = f(x)$, giving your answer in the form $a \ln b$, where a and b are rational.

(3)

- (d) Find the exact maximum value of $f(x)$, giving your answer in the form $p + q\sqrt{3}$, where p and q are integers.

(2)

Question 9 continued

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Question 9 continued

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(Total for Question 9 is 11 marks)



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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

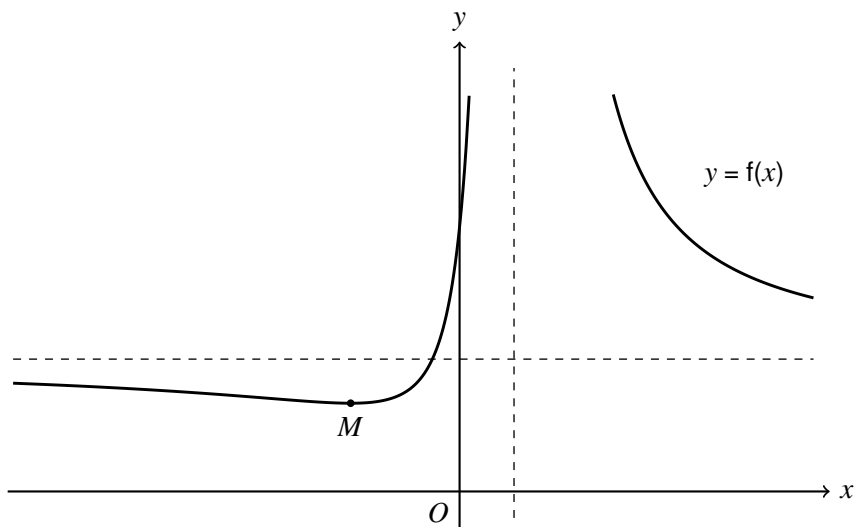


Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \frac{x^2 + 2}{(x - 1)^2} \quad x \neq 1$$

The curve has asymptotes $x = 1$ and $y = 1$, and a minimum point M .

- (a) Express $f(x)$ in the form $A + \frac{B}{x-1} + \frac{C}{(x-1)^2}$, where A , B and C are constants. (4)
- (b) Using $u = \frac{1}{x-1}$, or otherwise, find the exact coordinates of M . (3)
- (c) Show that the equation $f(x) = 1$ has exactly one solution, and find it. (2)
- (d) Find the set of values of the constant k for which the equation $f(x) = k$ has exactly two solutions. (2)



Question 10 continued

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Question 10 continued

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(Total for Question 10 is 11 marks)

TOTAL FOR CHAPTER 2 IS 78 MARKS

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