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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

Pure Mathematics 4

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	own PDF	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	3	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	own PDF	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	own PDF	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	own PDF	
6 Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	own PDF	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	own PDF	

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How to use this mark scheme

The same marking rules Pearson examiners use for WMA14.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

“Solutions relying on calculator technology are not acceptable.” When a question says this, a correct final answer with no algebraic working scores **no marks**. The notes under each scheme line say when this applies.

Marking a proof. A proof by contradiction earns its final mark only when it has all three parts: a clear assumption at the start (“Assume there is ...”), correct deductions in the right order, and a statement of the contradiction with a conclusion. A sketch or calculator values are not a proof.



2

Chapter 2: Partial Fractions

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$1^3 - 2(1)^2 - 5(1) + 6 = 0$, so $(x - 1)$ is a factor; divides: $x^2 - x - 6$ $(x - 1)(x + 2)(x - 3)$	M1 A1 (2)
(b)	$4x^2 + 3x - 25 \equiv A(x + 2)(x - 3) + B(x - 1)(x - 3) + C(x - 1)(x + 2)$ Substitutes $x = 1$, $x = -2$ or $x = 3$ to find one constant, e.g. $x = 1$: $-18 = -6A$ $\frac{3}{x - 1} - \frac{1}{x + 2} + \frac{2}{x - 3}$ <i>The final mark needs the whole expression written out, not only $A = 3$, $B = -1$, $C = 2$.</i>	B1 M1 A1 (3)
Total for Question 1: 5 marks		

Question 2

6 marks

Part	Scheme	Marks
(a)	$15t^2 - 10t + 7 \equiv A(3t - 1)^2 + B(t + 1)(3t - 1) + C(t + 1)$ $t = -1$: $32 = 16A$ so $A = 2$; $t = \frac{1}{3}$: $\frac{16}{3} = \frac{4}{3}C$ so $C = 4$ Compares t^2 coefficients: $15 = 9A + 3B$ $A = 2, B = -1, C = 4$ <i>Multiplying through by $(t + 1)(3t - 1)^3$ is a wrong identity (Jan 2024 report). No value of t gives B on its own: compare coefficients, or use $t = 0$: $7 = A - B + C$.</i>	B1 M1 M1 A1 (4)
(b)	$f'(t) = -\frac{2}{(t + 1)^2} + \frac{3}{(3t - 1)^2} - \frac{24}{(3t - 1)^3}$ $f'(0) = -2 + 3 + 24 = 25$ <i>M1 for differentiating at least two terms to the form $\frac{k}{(at + b)^n}$ with n one higher. Using the quotient rule on the original fraction is valid but much longer.</i>	M1 A1 (2)
Total for Question 2: 6 marks		



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Question 3

5 marks

Part	Scheme	Marks
(a)	$x(x-1)^2 = x^3 - 2x^2 + x$; divides: quotient 2 $2x^3 - 2x^2 + x + 3 - 2(x^3 - 2x^2 + x) = 2x^2 - x + 3$, so $g(x) = 2 + \frac{2x^2 - x + 3}{x(x-1)^2}$ *	M1 A1* (2)
(b)	$2x^2 - x + 3 \equiv B(x-1)^2 + Cx(x-1) + Dx$, with $\frac{B}{x} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$ $x = 0: B = 3; x = 1: D = 4; x^2$ coefficients: $B + C = 2$, so $C = -1$ $g(x) = 2 + \frac{3}{x} - \frac{1}{x-1} + \frac{4}{(x-1)^2}$ <i>Leaving out the 2 from part (a) in the final answer is the common error: the answer is $g(x)$, not just its proper part.</i>	B1 M1 A1 (3)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
	$x^2 + px + q \equiv 2(x-2)(x+4) + B(x+1)(x+4) + 3(x+1)(x-2)$ Compares x^2 coefficients: $1 = 2 + B + 3$ $B = -4$ Expands with $B = -4$: $2x^2 + 4x - 16 - 4x^2 - 20x - 16 + 3x^2 - 3x - 6$ $= x^2 - 19x - 38$, so $p = -19, q = -38$ <i>Substituting $x = 2$ gives $18B = 4 + 2p + q$: with p and q unknown this cannot find B on its own.</i>	B1 M1 A1 M1 A1 (5)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
	$(x^2 - 4)(x + 2) = (x - 2)(x + 2)^2$ Correct form: $\frac{A}{x - 2} + \frac{B}{x + 2} + \frac{C}{(x + 2)^2}$, with $3x^2 + x + 2 \equiv A(x + 2)^2 + B(x - 2)(x + 2) + C(x - 2)$ $x = 2: 16 = 16A$, so $A = 1$; $x = -2: 12 = -4C$, so $C = -3$ Compares x^2 coefficients: $3 = A + B$, so $B = 2$ $\frac{1}{x - 2} + \frac{2}{x + 2} - \frac{3}{(x + 2)^2}$ <i>Treating the denominator as $(x - 2)(x + 2)(x + 2)$ with three different single fractions, or using $\frac{A}{x^2 - 4}$, scores MO.</i>	B1 M1 A1 M1 A1 (5)
Total for Question 5: 5 marks		



Question 6

10 marks

Part	Scheme	Marks
(a)	Divides by $x^2 + 3x$: quotient $2x + 1$ Remainder 9, so $\frac{9}{x(x+3)} \equiv \frac{C}{x} + \frac{D}{x+3}$ $A = 2, B = 1$, and one of $C = 3, D = -3$ $g(x) = 2x + 1 + \frac{3}{x} - \frac{3}{x+3}$ <i>The Oct 2021 report: "the most common error was to set the numerator equal to 6 rather than -6". Here $D = -3$, not 3.</i>	M1 M1 A1 A1 (4)
(b)	Differentiates, $\frac{k}{x} \rightarrow -\frac{k}{x^2}$ for at least one term $g'(x) = 2 - \frac{3}{x^2} + \frac{3}{(x+3)^2}$ <i>Differentiating $\frac{3}{x}$ to $3 \ln x$ is integrating, not differentiating (seen in Oct 2021).</i>	M1 A1 (2)
(c)	$g'(x) < 2 \Leftrightarrow \frac{3}{(x+3)^2} < \frac{3}{x^2}$ For $x > 0, x+3 > x > 0$, so $(x+3)^2 > x^2$ so $\frac{3}{(x+3)^2} < \frac{3}{x^2}$, hence $g'(x) < 2$ for all $x > 0$ * <i>Saying only "$g'(x) \rightarrow 2$ as $x \rightarrow \infty$" does not show $g'(x) < 2$: the two fractions must be compared.</i>	M1 M1 A1* (3)
(d)	A tangent parallel to $y = 2x + 5$ needs $g'(x) = 2$, which is impossible by (c)	B1 (1)
Total for Question 6: 10 marks		



Question 7

10 marks

Part	Scheme	Marks
(a)	$2 \equiv A(r+1)(r+2) + Br(r+2) + Cr(r+1)$ and substitutes $r = 0, -1, -2$ Two of $A = 1, B = -2, C = 1$ $\frac{1}{r} - \frac{2}{r+1} + \frac{1}{r+2}$	M1 A1 A1 (3)
(b)	Writes $\left(\frac{1}{r} - \frac{1}{r+1}\right) - \left(\frac{1}{r+1} - \frac{1}{r+2}\right)$, or lists the terms for $r = 1, 2, 3, \dots, n$ Shows the cancelling: $\left(1 - \frac{1}{n+1}\right) - \left(\frac{1}{2} - \frac{1}{n+2}\right)$ $= \frac{1}{2} - \frac{1}{n+1} + \frac{1}{n+2}$ $= \frac{1}{2} - \frac{(n+2) - (n+1)}{(n+1)(n+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)} *$ <i>Listing terms: at least the first three and the last two rows must be shown so the cancelling can be seen.</i>	M1 A1 M1 A1* (4)
(c)	$\sum_{r=1}^{20} - \sum_{r=1}^4 = \left(\frac{1}{2} - \frac{1}{462}\right) - \left(\frac{1}{2} - \frac{1}{30}\right)$ $= \frac{1}{30} - \frac{1}{462} = \frac{12}{385}$ <i>Using $\sum_1^{20} - \sum_1^5$ is the common error: the lower sum must stop at $r = 4$.</i>	M1 A1 (2)
(d)	$\frac{1}{(n+1)(n+2)} > 0$ for every positive integer n , so the sum is less than $\frac{1}{2}$	B1 (1)
Total for Question 7: 10 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	$A = 1$ (the numerator and denominator have equal degree); remainder $x^2 + 5x + 8 - (x^2 + 5x + 6) = 2$ $\frac{2}{(x+2)(x+3)} \equiv \frac{B}{x+2} + \frac{C}{x+3}$ and substitutes $x = -2$ or $x = -3$ One of $B = 2, C = -2$ $f(x) = 1 + \frac{2}{x+2} - \frac{2}{x+3}$	B1 M1 A1 A1 (4)
(b)	Assume there is a real x ($x \neq -2, -3$) for which $f(x) = 1$ Then $\frac{2}{x+2} = \frac{2}{x+3}$, so $2(x+3) = 2(x+2)$ giving $6 = 4$: a contradiction. So $f(x) \neq 1$ for every real x * <i>Equally: $f(x) - 1 = \frac{2}{(x+2)(x+3)}$, and a fraction with numerator 2 is never zero. Pointing at the asymptote on the figure is not a proof.</i>	B1 M1 A1* (3)
(c)	$\frac{2}{(x+2)(x+3)} > 2$ requires $(x+2)(x+3) > 0$ and $(x+2)(x+3) < 1$ $x^2 + 5x + 5 < 0$: critical values $\frac{-5 \pm \sqrt{5}}{2}$ $\frac{-5 - \sqrt{5}}{2} < x < -3 \text{ or } -2 < x < \frac{-5 + \sqrt{5}}{2}$ <i>Multiplying by $(x+2)(x+3)$ without considering its sign includes $-3 < x < -2$, where $f(x) \leq -7$ (the figure shows the curve is below the x-axis there).</i>	M1 M1 A1 (3)
Total for Question 8: 10 marks		



Question 9

11 marks

Part	Scheme	Marks
(a)	$6u \equiv A(u+3) + B(u+1)$; substitutes $u = -1$ or $u = -3$ $A = -3$ or $B = 9$ $-\frac{3}{u+1} + \frac{9}{u+3}$	M1 A1 A1 (3)
(b)	$f(x) = -\frac{3}{e^x+1} + \frac{9}{e^x+3}$ Chain rule: $\frac{d}{dx} \left(\frac{k}{e^x+c} \right) = -\frac{ke^x}{(e^x+c)^2}$ $f'(x) = \frac{3e^x}{(e^x+1)^2} - \frac{9e^x}{(e^x+3)^2}$ *	B1 M1 A1* (3)
(c)	$f'(x) = 0$ and $e^x \neq 0$: $(u+3)^2 = 3(u+1)^2$ $u > 0$, so $u+3 = \sqrt{3}(u+1)$, giving $u = \frac{3-\sqrt{3}}{\sqrt{3}-1} = \sqrt{3}$ $e^x = \sqrt{3}$, so $x = \frac{1}{2} \ln 3$ <i>Taking $u+3 = -\sqrt{3}(u+1)$ gives a negative u, which is impossible for $u = e^x$.</i>	M1 M1 A1 (3)
(d)	$f = \frac{6\sqrt{3}}{(\sqrt{3}+1)(\sqrt{3}+3)} = \frac{6}{(1+\sqrt{3})^2} = \frac{6}{4+2\sqrt{3}}$ $6-3\sqrt{3}$	M1 A1 (2)
Total for Question 9: 11 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	$x^2 + 2 \equiv A(x-1)^2 + B(x-1) + C$ $x = 1: C = 3; x^2 \text{ coefficients: } A = 1$ $x \text{ coefficients (or } x = 0): 0 = -2A + B, \text{ so } B = 2$ $f(x) = 1 + \frac{2}{x-1} + \frac{3}{(x-1)^2}$	B1 M1 M1 A1 (4)
(b)	$f = 3u^2 + 2u + 1 = 3\left(u + \frac{1}{3}\right)^2 + \frac{2}{3}$ Minimum $\frac{2}{3}$ when $u = -\frac{1}{3}$, i.e. $x - 1 = -3$ $M\left(-2, \frac{2}{3}\right)$ <i>Differentiating $f(x)$ from (a) and solving $-\frac{2}{(x-1)^2} - \frac{6}{(x-1)^3} = 0$ is equally valid.</i>	M1 M1 A1 (3)
(c)	$3u^2 + 2u = 0$, so $u = 0$ or $u = -\frac{2}{3}$; $u = \frac{1}{x-1}$ is never 0, so $u = 0$ is rejected Exactly one solution: $x = -\frac{1}{2}$	M1 A1 (2)
(d)	$3u^2 + 2u + 1 - k = 0$ has two distinct roots when $k > \frac{2}{3}$; each non-zero u gives one x $k = 1$ gives the root $u = 0$ (no x), so $\frac{2}{3} < k < 1 \text{ or } k > 1$ <i>Check with the figure: for $\frac{2}{3} < k < 1$ both solutions have $x < 1$, one on each side of M; for $k > 1$ there is one solution on each branch.</i>	M1 A1 (2)
Total for Question 10: 11 marks		

