



eClassroom

A-LEVEL MATHEMATICS • PURE 4

# Binomial Expansion

Worked Solutions

Pearson Edexcel International A Level (WMA14) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Binomial Expansion worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

**Full worked solutions** for every question on the Pure 4 Binomial Expansion worksheet, in the same order and with the same question numbers.

**How to mark your work.** Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

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**A****Your turn answers**

Compare each line of your working with the solution.

**CORE**

$$\text{A1} \quad 1 + \frac{1}{2}(-3x) + \frac{\frac{1}{2}(-\frac{1}{2})}{2}(-3x)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{6}(-3x)^3$$

$$1 - \frac{3}{2}x - \frac{9}{8}x^2 - \frac{27}{16}x^3$$

$$\text{A2} \quad 1 + \left(-\frac{1}{2}\right)\left(-\frac{2x}{5}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}\left(-\frac{2x}{5}\right)^2, \text{ valid for } \left|\frac{2x}{5}\right| < 1$$

$$1 + \frac{x}{5} + \frac{3x^2}{50}, |x| < \frac{5}{2}$$

$$\text{A3} \quad \sqrt{9-2x} = 3\left(1 - \frac{2x}{9}\right)^{\frac{1}{2}} = 3\left(1 - \frac{x}{9} - \frac{x^2}{162}\right)$$

$$3 - \frac{x}{3} - \frac{x^2}{54}, |x| < \frac{9}{2}$$

**STRETCH**

$$\text{A4} \quad (4+3x)^{-\frac{1}{2}} = \frac{1}{2}\left(1 + \frac{3x}{4}\right)^{-\frac{1}{2}} = \frac{1}{2}\left(1 - \frac{3x}{8} + \frac{27x^2}{128}\right)$$

$$\frac{1}{2} - \frac{3x}{16} + \frac{27x^2}{256}, |x| < \frac{4}{3}$$

$$\text{A5} \quad g(x) = \frac{1}{1+x} + \frac{2}{3-x} = (1-x+x^2) + \frac{2}{3}\left(1 + \frac{x}{3} + \frac{x^2}{9}\right)$$

$$\frac{5}{3} - \frac{7x}{9} + \frac{29x^2}{27}, |x| < 1$$

$$\text{A6} \quad (1+x)^{\frac{1}{3}} \approx 1 + \frac{x}{3} - \frac{x^2}{9}, \text{ and } \sqrt[3]{1.008} = \sqrt[3]{\frac{126}{125}} = \frac{\sqrt[3]{126}}{5}$$

$$\sqrt[3]{126} \approx 5(1 + 0.0026667 - 0.0000071): \quad 5.0133$$



A7  $\frac{\frac{1}{2}(-\frac{1}{2})}{2}k^2 = -\frac{k^2}{8} = -2$ , so  $k = 4$

$x^3$ :  $\frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{6}(-4)^3 = \frac{1}{16} \times (-64)$ :  $k = 4, -4$

## B

## Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

## CORE

B1 (a)  $1 - x + x^2$  (b)  $1 + \frac{x}{2} - \frac{x^2}{8}$  (c)  $1 + 2x + 3x^2$  (d)  $1 - \frac{x}{3} + \frac{2x^2}{9}$

The same formula each time; only  $n$  and the sign of  $x$  change.

B2 (a)  $1 - 2x + 4x^2$  (b)  $1 - x - x^2$  (c)  $1 - x + \frac{3}{4}x^2$  (d)  $1 - 6x + 6x^2$

Square the whole term, coefficient included:  $(-3x)^2 = 9x^2$ , not  $3x^2$

B3 (a)  $|x| < \frac{1}{3}$  (b)  $|x| < 4$  (c)  $|x| < 2$  (d)  $|x| < \frac{9}{4}$

Write the bracket as  $(1 + y)^n$  first, then solve  $|y| < 1$

B4 (a)  $\frac{1}{4} - \frac{x}{16} + \frac{x^2}{64}$  (b)  $3 + \frac{x}{6} - \frac{x^2}{216}$  (c)  $\frac{1}{8} + \frac{3x}{16} + \frac{3x^2}{16}$  (d)  $2 + \frac{x}{6} - \frac{x^2}{72}$

Take out  $a^n$  (here  $4^{-1}$ ,  $9^{\frac{1}{2}}$ ,  $2^{-3}$ ,  $8^{\frac{1}{3}}$ ) before expanding.

B5 (a)  $\frac{1}{1-x} + \frac{1}{1+x}$ :  $2 + 2x^2$  (b)  $\frac{2}{1-2x} + \frac{1}{1+x}$ :  $3 + 3x + 9x^2$

(c) (a)  $|x| < 1$ ; (b)  $|x| < \frac{1}{2}$

Each partial fraction has its own validity; the whole is valid only where both are.

B6 (a)  $x = 0.04$ :  $1.0198$  (b)  $(1-x)^{-1}$ ,  $x = 0.02$ :  $1.0204$  (c)  $(1-x)^{\frac{1}{3}}$ ,  $x = 0.03$ :  $0.9899$  (d)  $(1+x)^{-2}$ ,  $x = 0.01$ :  $0.9803$

Match each value to  $(1 \pm x)^n$  with a small  $x$



## STRETCH

B7 (a)  $\frac{1}{9} - \frac{2x}{27} + \frac{x^2}{27}$  (b)  $5 - x - \frac{x^2}{10}$

(c)  $(x-4)^2 = (4-x)^2: \frac{1}{16} + \frac{x}{32} + \frac{3x^2}{256}$  (d)  $(2+x)(1+2x+3x^2): 2+5x+8x^2$

In (c) squaring removes the sign, so rewrite as  $(4-x)^{-2}$  before taking out  $4^{-2}$

B8 (a)  $|x| < \frac{3}{2}$  (b)  $|x| < 1$  (c)  $|x| < 1$  (d)  $|x| < \frac{1}{3}$

For a product, the narrower range wins: in (d)  $|x| < 4$  and  $|x| < \frac{1}{3}$  give  $|x| < \frac{1}{3}$

B9 (a)  $\frac{6}{1+3x} - \frac{1}{1+x}: 5 - 17x + 53x^2$  (b)  $\frac{1}{1+x} + \frac{1}{1-x} + \frac{2}{(1-x)^2}: 4 + 4x + 8x^2$

(c)  $-1 + \frac{2}{1-x^2}: 1 + 2x^2$  (d)  $\frac{4}{1-2x} - \frac{2}{1-x}: 4 \times 8 - 2: 30$

Split first, then expand each simple term; in (c) divide first because the fraction is improper.

B10 (a)  $1 - \frac{x}{2} - \frac{x^2}{8}$  (b)  $1 - 0.005 - 0.0000125: 0.9949875$

(c)  $\sqrt{\frac{99}{100}} = \frac{\sqrt{9 \times 11}}{10}$ , so  $\sqrt{11} \approx \frac{10}{3} \times 0.9949875: 3.3166$

(d) **the terms left out are then very small**, so three terms are accurate.

Find a small  $x$  that turns the number into a simple multiple of the root you want.

## C

## Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

## CORE

C1 The factor taken out must also be raised to the power  $\frac{1}{2}: 4^{\frac{1}{2}} = 2$ , not 4

$$(4+x)^{\frac{1}{2}} = 2 \left(1 + \frac{x}{4}\right)^{\frac{1}{2}}$$

C2 Validity needs  $|3x| < 1$ , not  $|x| < 3: |x| < \frac{1}{3}$



**C3** He used  $4x^2$  instead of  $(4x)^2 = 16x^2$ : the term is  $\frac{\frac{1}{2}(-\frac{1}{2})}{2} \times 16x^2 = -2x^2$

$$1 + 2x - 2x^2$$

**C4**  $(1-x)^{-\frac{1}{2}} = \frac{1}{\sqrt{1-x}}$ , so it estimates  $\frac{1}{\sqrt{0.96}} \approx 1 + 0.02 + 0.0006 = 1.0206$

$$\sqrt{0.96} \approx \frac{1}{1.0206} \approx 0.9798$$

### STRETCH

**C5** His series is in powers of  $\frac{1}{x}$  and is valid only for  $|x| > 2$ , not ascending powers of  $x$

$$(x-2)^{-1} = -\frac{1}{2} \left(1 - \frac{x}{2}\right)^{-1}: -\frac{1}{2} - \frac{x}{4} - \frac{x^2}{8}, |x| < 2$$

**C6**  $\frac{2}{1-2x}$  is valid only for  $|2x| < 1$ ; both expansions must be valid

$$|x| < \frac{1}{2}$$

**C7** He is wrong: the expansion is valid for  $\left|\frac{x}{4}\right| < 1$ , i.e.  $|x| < 4$ , so  $x = 1$  may be used.

$$2 + \frac{1}{4} - \frac{1}{64} = 2.2344 \text{ against } \sqrt{5} = 2.2361: \text{ valid, but less accurate, since } \frac{x}{4} = \frac{1}{4} \text{ is not very small}$$

**C8 (a)**  $\frac{(-2)(-3)\cdots(-2-r+1)}{r!}(-1)^r = \frac{(r+1)!}{r!}: r+1$

**(b)**  $x = \frac{1}{2}$  (inside  $|x| < 1$ ): the sum is  $\left(1 - \frac{1}{2}\right)^{-2}: 4$

**(c)**  $x = 2$  is outside  $|x| < 1$ , so the series does not converge to  $(1-x)^{-2}$



**D****Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

**CORE**

$$\text{D1} \quad 1 + \left(-\frac{1}{3}\right)(6x) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2}(6x)^2$$

$$1 - 2x + 8x^2, \quad |x| < \frac{1}{6}$$

$$\text{D2} \quad (\text{a}) \quad 1 + 3x + 6x^2$$

$$(\text{b}) \quad (1 + 4x + 4x^2)(1 + 3x + 6x^2): \quad 1 + 7x + 22x^2, \quad |x| < 1$$

$$\text{D3} \quad (\text{a}) \quad 1 + \frac{x}{3} - \frac{x^2}{9}$$

$$(\text{b}) \quad \sqrt[3]{\frac{9}{8}} = \frac{\sqrt[3]{9}}{2}, \text{ so } \sqrt[3]{9} \approx 2 \left(1 + \frac{1}{24} - \frac{1}{576}\right) = 2.07986: \quad 2.080$$

$$\text{D4} \quad (3+x)(1+2x+4x^2) = 3 + 6x + 12x^2 + x + 2x^2$$

$$3 + 7x + 14x^2, \quad |x| < \frac{1}{2}$$

**STRETCH**

$$\text{D5} \quad x^2: \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}a^2 = \frac{3}{8}a^2 = 6, \text{ so } a = \pm 4; \text{ the } x \text{ coefficient } -\frac{a}{2} \text{ is negative, so } a = 4$$

$$x^3: -\frac{5}{16} \times 4^3 = -20: \quad a = 4, \text{ coefficient } -20, \quad |x| < \frac{1}{4}$$

$$\text{D6} \quad (\text{a}) \quad \sqrt{1 \pm 2x} = 1 \pm x - \frac{x^2}{2} \pm \frac{x^3}{2}; \text{ the } x^2 \text{ terms are equal, so they cancel}$$

$$g(x) \approx 2x + x^3$$

$$(\text{b}) \quad x = 0.1: 0.2 + 0.001: \quad 0.201$$

$$\text{D7} \quad (\text{a}) \quad 1 + 2x + 6x^2$$

$$(\text{b}) \quad (2+x)(1+2x+6x^2) = 2 + 4x + 12x^2 + x + 2x^2: \quad 2 + 5x + 14x^2, \quad |x| < \frac{1}{4}$$



D8 (a)  $\frac{2}{1-x} - \frac{1}{1+x} + \frac{2}{(1+x)^2}$

(b)  $2(1+x+x^2+x^3) - (1-x+x^2-x^3) + 2(1-2x+3x^2-4x^3)$

$3-x+7x^2$ ;  $x^3$ :  $2+1-8$ :  $-5$

## E

## Exam practice solutions

Mark-scheme notes are shown on the right of each line.

## CORE

Exam questions for everyone

E1 (a)  $(27-9x)^{\frac{1}{3}} = 3\left(1-\frac{x}{3}\right)^{\frac{1}{3}}$

B1: factor 3

$3\left(1+\frac{1}{3}\left(-\frac{x}{3}\right)+\frac{\frac{1}{3}\left(-\frac{2}{3}\right)}{2}\left(-\frac{x}{3}\right)^2\right)$

M1

$3-\frac{x}{3}-\frac{x^2}{27}, |x| < 3$

A1

(b)  $27-9x=26$  when  $x=\frac{1}{9}$

M1

$3-\frac{1}{27}-\frac{1}{2187}$ :  $2.9625$

A1

E2 (a)  $4+3x \equiv A(1+2x)+B(2-x)$ :  $x=2$ :  $A=2$ ;  $x=-\frac{1}{2}$ :  $B=1$

M1

$\frac{2}{2-x} + \frac{1}{1+2x}$

A1

(b)  $\left(1-\frac{x}{2}\right)^{-1} = 1+\frac{x}{2}+\frac{x^2}{4}$ ;  $(1+2x)^{-1} = 1-2x+4x^2$

M1 M1

$2-\frac{3x}{2}+\frac{17x^2}{4}$

A1

(c)  $\frac{2}{2-x}$  needs  $|x| < 2$  but  $\frac{1}{1+2x}$  needs  $|x| < \frac{1}{2}$ : **both must hold**

B1



## STRETCH

- E3** (a)  $1 + \frac{1}{2}(8x) + \frac{\frac{1}{2}(-\frac{1}{2})}{2}(8x)^2 + \frac{\frac{1}{2}(-\frac{1}{2})(-\frac{3}{2})}{6}(8x)^3$  M1 A1
- $1 + 4x - 8x^2 + 32x^3$  A1
- (b)  $\sqrt{1.08} = \sqrt{\frac{108}{100}} = \frac{6\sqrt{3}}{10} = \frac{3\sqrt{3}}{5}$ ;  $x = 0.01$  gives 1.039232 M1
- $\sqrt{3} \approx \frac{5}{3} \times 1.039232$ :  $1.732053$  A1
- (c)  $1 + 8\left(-\frac{1}{32}\right) = \frac{3}{4}$ , and  $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$ ; the series gives  $1 - \frac{1}{8} - \frac{1}{128} - \frac{1}{1024}$  M1
- $\sqrt{3} \approx 2 \times 0.8662109$ :  $1.732422$  A1
- (d) the first, since  $x = 0.01$  is much smaller than  $\frac{1}{32}$ , so the neglected terms are smaller B1

