



eClassroom

A-LEVEL MATHEMATICS • PURE 1

Radians

Worked Solutions

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Radians worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 1 Radians worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

$$\mathbf{A1} \quad 210 \times \frac{\pi}{180} = \boxed{\frac{7\pi}{6}}; \quad \frac{5\pi}{12} \times \frac{180}{\pi} = \boxed{75^\circ}$$

$$\mathbf{A2} \quad s = 9 \times \frac{2\pi}{3} = \boxed{6\pi \text{ cm}}; \quad A = \frac{1}{2} \times 81 \times \frac{2\pi}{3} = \boxed{27\pi \text{ cm}^2}$$

$$\mathbf{A3} \quad \frac{1}{2} \times 100 \times (1.4 - \sin 1.4) = 50(1.4 - \sin 1.4) \approx \boxed{20.7 \text{ cm}^2} \quad (\text{calculator in radian mode})$$

STRETCH

$$\mathbf{A4} \quad 2r + r\theta = 28, \text{ so } r\theta = 28 - 2r. \text{ Area: } \frac{1}{2}r(28 - 2r) = 48, \text{ so } r^2 - 14r + 48 = 0, \text{ so } (r - 6)(r - 8) = 0$$

$$\boxed{r = 6, \theta = \frac{8}{3}} \text{ or } \boxed{r = 8, \theta = 1.5} \quad (\text{both have } 0 < \theta < 2\pi)$$

$$\mathbf{A5} \quad AB = 2 \times 12 \sin 1.1 \approx \boxed{21.4 \text{ cm}} \quad (\text{or } AB^2 = 288 - 288 \cos 2.2)$$

$$\mathbf{A6} \quad \text{The graph of } \cos x \text{ moves } \frac{\pi}{4} \text{ to the right. } y \text{ intercept } \cos\left(-\frac{\pi}{4}\right): \boxed{\left(0, \frac{\sqrt{2}}{2}\right)}$$

$$x - \frac{\pi}{4} = \frac{\pi}{2} \text{ or } \frac{3\pi}{2}: \boxed{x = \frac{3\pi}{4}}, \boxed{x = \frac{7\pi}{4}}$$

$$\mathbf{A7} \quad \cos x = -\frac{1}{2}: \cos \frac{\pi}{3} = \frac{1}{2}, \text{ and the graph gives the negative values at } \pi \mp \frac{\pi}{3}: \boxed{x = \frac{2\pi}{3}}, \boxed{x = \frac{4\pi}{3}}$$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $\frac{\pi}{4}$ (b) $\frac{2\pi}{3}$ (c) $\frac{11\pi}{6}$ (d) $\frac{2\pi}{5}$

Multiply by $\frac{\pi}{180}$ and simplify the fraction.

B2 (a) 20° (b) 300° (c) $1.2 \times \frac{180}{\pi} \approx 68.8^\circ$ (d) 67.5°

A radian angle without π , like 1.2, still converts by $\times \frac{180}{\pi}$

B3 (a) $2\pi \text{ cm}$ (b) 9 cm (c) $\frac{9\pi}{2} \text{ cm}$ (d) $150^\circ = \frac{5\pi}{6}$: $5\pi \text{ cm}$

$s = r\theta$ only works in radians, so (d) is converted first.

B4 (a) $2\pi \text{ cm}^2$ (b) 4.8 cm^2 (c) $\frac{20\pi}{3} \text{ cm}^2$ (d) $12\pi \text{ cm}^2$

A reflex angle works in the same formula: (d) is three quarters of the circle.

B5 (a) $5+5+6 = 16 \text{ cm}$ (b) $10+5\pi \approx 25.7 \text{ cm}$ (c) $10+5(2\pi-1.2) = 4+10\pi \approx 35.4 \text{ cm}$

A perimeter always includes both radii, and the major arc uses the angle $2\pi - \theta$

B6 (a) $18\left(\frac{\pi}{2} - 1\right) = 9\pi - 18 \approx 10.3 \text{ cm}^2$ (b) $18\left(\frac{\pi}{6} - \frac{1}{2}\right) = 3\pi - 9 \approx 0.425 \text{ cm}^2$

(c) $18(2 - \sin 2) \approx 19.6 \text{ cm}^2$ (d) $\sin \pi = 0$, so the segment is a semicircle: $18\pi \text{ cm}^2$

$\frac{1}{2}r^2(\theta - \sin \theta)$ in every case; at $\theta = \pi$ the triangle has no area.

STRETCH

B7 (a) $\theta = \frac{15}{6} = 2.5$ (b) $\frac{1}{2}r^2 \times 0.8 = 40$, so $r^2 = 100$: $r = 10 \text{ cm}$

(c) $\frac{1}{2}r \times 8 = 24$, so $r = 6 \text{ cm}$, $\theta = \frac{8}{6} = \frac{4}{3}$ (d) $16+8\theta = 30$: $\theta = 1.75$

Write what you know as equations in r and θ ; in (c), $\text{area} = \frac{1}{2}r \times \text{arc}$



- B8 (a) 2π (b) π (c) π (d) 6π

A number outside does not change the period; $\sin bx$ has period $\frac{2\pi}{b}$

- B9 (a) 2 (b) 4 (c) 2 (d) 1

Count the cycles in 0 to 2π : two for $\cos 2x$; one maximum for (d), at $x = \frac{3\pi}{4}$

- B10 (a) $\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{2}}{2}$ (c) $\frac{\sqrt{3}}{3}$ (d) $-\frac{\sqrt{3}}{2}$

Use the related acute angle, then the graph for the sign.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 $s = r\theta$ needs θ in radians. $70^\circ = \frac{7\pi}{18}$, so $s = \frac{35\pi}{18} \approx 6.11 \text{ cm}$

C2 Lily has found only the arc, $6 \times 1.5 = 9$. The perimeter also includes the two radii: $9 + 6 + 6 = 21 \text{ cm}$

C3 $r\theta = r$, so $\theta = 1$ radian. Area = $\frac{1}{2}r^2 \times 1 = \frac{1}{2}r^2$

π radians = 180° , so 1 radian = $\frac{180}{\pi} \approx 57.3^\circ$

C4 Angles on a straight line add up to π , not 2π , so angle $COB = \pi - 2.1$

Arc $CB = 4(\pi - 2.1) \approx 4.17 \text{ cm}$

STRETCH

C5 OB is inside the badge, where the triangle and sector are joined, so it is not part of the perimeter. Mo added it.

Arc $BC = 5 \times 0.8 = 4$; perimeter = $OA + AB + \text{arc } BC + CO = 6 + 4 + 4 + 5 = 19 \text{ cm}$



C6 Sector area $\frac{1}{2}r^2\theta$; triangle area $\frac{1}{2}r \times r \times \sin \theta$

$$\text{Segment} = \text{sector} - \text{triangle} = \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin \theta = \frac{1}{2}r^2(\theta - \sin \theta) \quad \text{as required}$$

C7 $\sin \frac{\pi}{6} = \frac{1}{2}$ and $\frac{3}{\pi} \times \frac{\pi}{6} = \frac{1}{2}$, so $x = \frac{\pi}{6}$ is a solution.

The line $y = \frac{3x}{\pi}$ passes through O , where it also meets the curve, and both graphs have rotational symmetry about O , so $-\frac{\pi}{6}$ is a solution too. For $|x| > \frac{\pi}{3}$ the line is beyond ± 1 , so there are no others.

$$x = 0 \quad \text{and} \quad x = -\frac{\pi}{6}$$

C8 (a) Triangle OAB is isosceles; the line from O to the midpoint M of AB bisects angle AOB and is perpendicular to AB . So $AM = r \sin \frac{\theta}{2}$, and $AB = 2r \sin \frac{\theta}{2}$ as required

(b) $\theta = \frac{12}{10} = 1.2$; chord $= 20 \sin 0.6 \approx 11.3 \text{ cm}$, shorter than the arc because a straight line is the shortest path between two points.

(c) $2r \sin \frac{\theta}{2} = r$, so $\sin \frac{\theta}{2} = \frac{1}{2}$, $\frac{\theta}{2} = \frac{\pi}{6}$: $\theta = \frac{\pi}{3}$

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $8 \times 0.9 = 7.2 \text{ cm}$ (b) $\frac{1}{2} \times 64 \times 0.9 = 28.8 \text{ cm}^2$

(c) $28.8 - \frac{1}{2} \times 64 \times \sin 0.9 \approx 28.8 - 25.07 = 3.73 \text{ cm}^2$

D2 (a) Large sector – small sector $= \frac{1}{2} \times 1.4 \times (100 - 36) = 44.8 \text{ cm}^2$

(b) Arcs $10 \times 1.4 = 14$ and $6 \times 1.4 = 8.4$, plus $AB + CD = 8$: $14 + 8.4 + 8 = 30.4 \text{ cm}$



D3 (a) $2 \cos\left(-\frac{\pi}{3}\right) = 2 \times \frac{1}{2} = 1$: $(0, 1)$

(b) maximum when $x - \frac{\pi}{3} = 0$: $P\left(\frac{\pi}{3}, 2\right)$

(c) $x - \frac{\pi}{3} = \frac{\pi}{2}$ or $\frac{3\pi}{2}$: $x = \frac{5\pi}{6}$, $x = \frac{11\pi}{6}$

D4 (a) $\frac{1}{2} \times 144 \times \theta = 90$, so $72\theta = 90$: $\theta = 1.25$

(b) Arc = $12 \times 1.25 = 15$; perimeter = $12 + 12 + 15 = 39$ cm

STRETCH

D5 (a) $OC = 9 \cos 1.1 \approx 4.082$ and $AC = 9 \sin 1.1 \approx 8.021$

Area $R = \frac{1}{2} \times 81 \times 1.1 - \frac{1}{2} \times OC \times AC \approx 44.55 - 16.37$: 28.2 m^2

(b) Arc $AB = 9.9$; $CB = 9 - OC \approx 4.918$; perimeter $\approx 9.9 + 8.021 + 4.918$: 22.8 m

D6 (a) Triangle OAQ has sides 5, 5 and 6. The line OQ bisects angle AOB , and $\cos(\text{angle } AOQ) = \frac{3}{5}$

Angle $AOB = 2 \cos^{-1} \frac{3}{5} \approx 1.8546$, which is 1.855 to 4 s.f. as required

(b) The shaded region is two equal segments: $2 \times \frac{1}{2} \times 25 \times (1.8546 - \sin 1.8546) \approx 22.4 \text{ cm}^2$

D7 (a) $x + \frac{\pi}{3} = \frac{\pi}{2}$ or $\frac{3\pi}{2}$: $x = \frac{\pi}{6}$ and $x = \frac{7\pi}{6}$

(b) $\tan \frac{\pi}{3} = \sqrt{3}$: $(0, \sqrt{3})$

(c) $x + \frac{\pi}{3} = \pi$ or 2π : $x = \frac{2\pi}{3}$, $x = \frac{5\pi}{3}$

D8 (a) $2r + r\theta = 2r\theta$, so $2r = r\theta$, and $r \neq 0$, so $\theta = 2$ as required

(b) $\frac{1}{2}r^2 \times 2 = 49$, so $r^2 = 49$: $r = 7$

(c) $\frac{1}{2} \times 49 \times (2 - \sin 2) \approx 26.7 \text{ cm}^2$



E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) Sector = $\frac{1}{2} \times 144 \times 0.9 = 64.8$; triangle = $\frac{1}{2} \times 12 \times 5 = 30$

M1: both areas

$$94.8 \text{ m}^2$$

A1

(b) Arc $AB = 12 \times 0.9 = 10.8$

B1

$$OC = \sqrt{12^2 + 5^2} = 13$$

M1: Pythagoras

$$12 + 10.8 + 5 + 13 = 40.8 \text{ m}$$

A1: OB not included

E2 (a) The maximum value is 2, so $a = 2$

B1

(0, 1): $2 \cos b = 1$, so $\cos b = \frac{1}{2}$

M1

$0 < b < \pi$: $b = \frac{\pi}{3}$; check: the maximum is where $x + \frac{\pi}{3} = 2\pi$, at $x = \frac{5\pi}{3}$

A1

(b) $x + \frac{\pi}{3} = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

M1

$$x = \frac{\pi}{6}, x = \frac{7\pi}{6}$$

A1

(c) 2: the curve falls from 1 to -2 and rises to 2 at $x = \frac{5\pi}{3}$, then falls to 1 at $x = 2\pi$, so $y = 1.5$ is met once on the way up and once on the way down

B1

STRETCH

E3 (a) $AB^2 = 36 + 36 - 72 \cos \frac{2\pi}{3} = 72 + 36 = 108$

M1: cosine rule

$$AB = \sqrt{108} = 6\sqrt{3} \quad \text{as required}$$

A1*

(b) Sector = $\frac{1}{2} \times 36 \times \frac{2\pi}{3} = 12\pi$

B1

$$\text{Triangle} = \frac{1}{2} \times 36 \times \sin \frac{2\pi}{3} = 9\sqrt{3}$$

M1

$$12\pi - 9\sqrt{3} \text{ cm}^2$$

A1

(c) Semicircle radius $3\sqrt{3}$: area $\frac{1}{2} \pi \times 27 = \frac{27\pi}{2}$

B1

$$L = \frac{27\pi}{2} - (12\pi - 9\sqrt{3}) = \frac{3\pi}{2} + 9\sqrt{3} \quad \text{as required}$$

M1 A1*

