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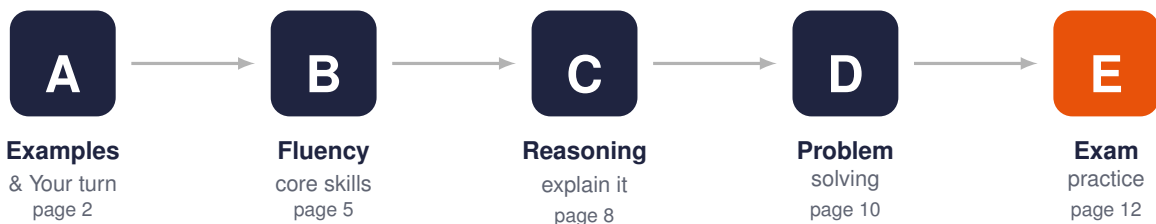
GCSE MATHEMATICS • GEOMETRY

Circles & Circle Theorems

Worksheet

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • No calculator

Free Circles and Circle Theorems worksheet for GCSE and iGCSE Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: name the parts of a circle • use the fact that radii are equal, so triangles with two radii are isosceles • find angles in regular polygons drawn inside a circle • *Higher:* use and prove the circle theorems, giving a reason for every step.

Which questions are for me?

FOUNDATION Everyone starts here. These skills appear on both tiers.

HIGHER Higher only: the circle theorems, reasons for every step, and proofs. Grades 6 to 9.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Foundation marks	23	10	13	15
Higher marks	16	14	17	15
My score				

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Class

Date



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A**Worked examples & Your turn**

Read each example carefully. Cover it up, then answer the question beside it using the same method.

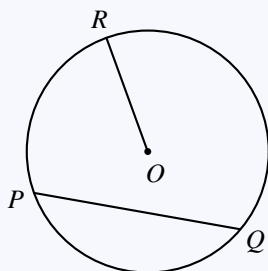
FOUNDATION

Everyone starts here

Worked example A1

O is the centre.

Write down the mathematical name of the line PQ and the line OR

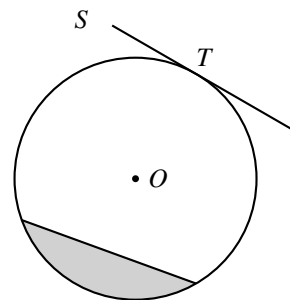


PQ chord; OR radius

Your turn A1

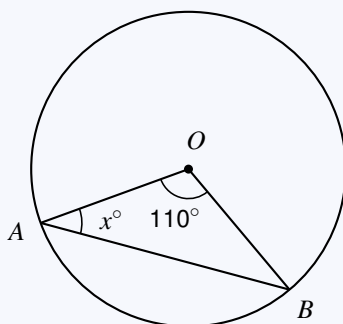
O is the centre.

Write down the name of the line ST and of the shaded region.

**Worked example A2**

O is the centre.

Work out the value of x .



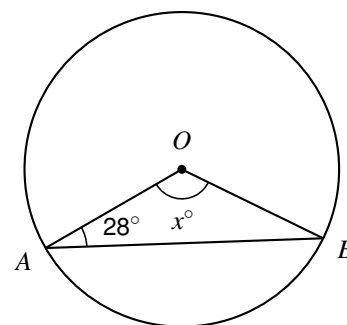
$OA = OB$ (radii), so the triangle is isosceles: $(180 - 110) \div 2$

$x = 35$

Your turn A2

O is the centre.

Work out the value of x .

**Worked example A3**

A regular hexagon is drawn inside a circle with centre O , and each vertex is joined to O . Work out the angle at O in each triangle, and a base angle. $360 \div 6$ at the centre; each triangle is isosceles.

60° ; 60°

Your turn A3

A regular octagon is drawn inside a circle with centre O , and each vertex is joined to O . Work out the angle at O in each triangle, and a base angle.

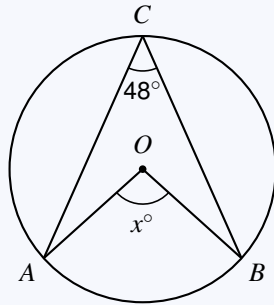


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HIGHER*Higher only: the circle theorems***Worked example A4: the angle at the centre**

O is the centre.
Work out the value of x . Give a reason.

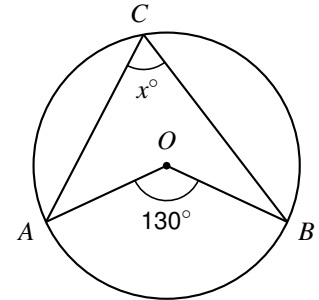


The angle at the centre is twice the angle at the circumference.

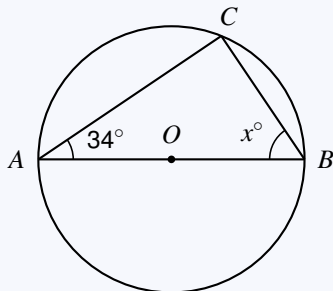
$$x = 96$$

Your turn A4

O is the centre.
Work out the value of x . Give a reason.

**Worked example A5: the angle in a semicircle**

AB is a diameter.
Work out the value of x . Give reasons.

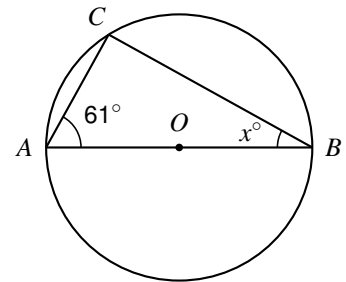


The angle in a semicircle is 90° : $180 - 90 - 34$

$$x = 56$$

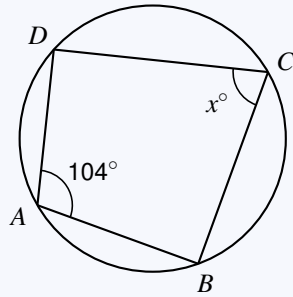
Your turn A5

AB is a diameter.
Work out the value of x . Give reasons.



Worked example A6: cyclic quadrilaterals

$ABCD$ is a cyclic quadrilateral. Work out the value of x . Give a reason.

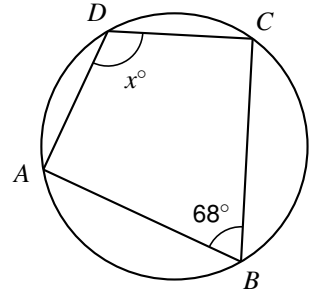


Opposite angles of a cyclic quadrilateral add up to 180°

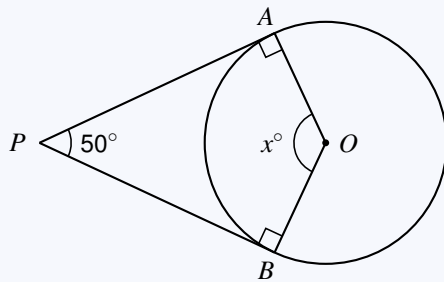
$$x = 76$$

Your turn A6

$ABCD$ is a cyclic quadrilateral. Work out the value of x . Give a reason.

**Worked example A7: tangents**

PA and PB are tangents. Work out the value of x . Give reasons.

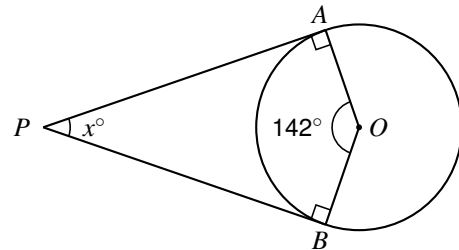


A tangent meets a radius at 90° . Angles in a quadrilateral: $360 - 90 - 90 - 50$

$$x = 130$$

Your turn A7

PA and PB are tangents. Work out the value of x . Give reasons.



B**Fluency**

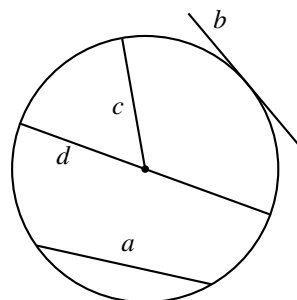
Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

FOUNDATION

Everyone starts here

- B1** O is the centre. Write down the mathematical name of each line.

- (a) line a (b) line b
(c) line c (d) line d



[4 marks]

- B2** OAB is a triangle where O is the centre of a circle and A and B are on the circle.

- (a) Angle $AOB = 40^\circ$. Find angle OAB (b) Angle $AOB = 90^\circ$. Find angle OAB
(c) Angle $OAB = 25^\circ$. Find angle AOB (d) Angle $OAB = 64^\circ$. Find angle AOB

[4 marks]

- B3** A regular polygon is drawn inside a circle with centre O , and each vertex is joined to O .

- (a) For a pentagon, find the angle at O in each triangle. (b) Find a base angle of each triangle.
(c) For a decagon, find the angle at O in each triangle. (d) Find a base angle of each triangle.

[4 marks]

- B4** (a) A circle has radius 7 cm. Write down its diameter. (b) A circle has diameter 23 cm. Write down its radius.

- (c) A and B are on a circle of radius 6 cm with centre O , and chord $AB = 8$ cm. Work out the perimeter of triangle OAB

- (d) Angle $AOB = 60^\circ$ in a circle of radius 9 cm. Explain why chord AB is 9 cm.

[4 marks]



FOUNDATION

continued: facts and algebra with radii

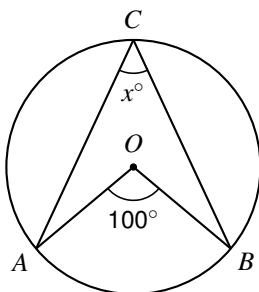
- B5** True or false? (a) A diameter is a chord. (b) A tangent crosses a circle at two points.
 (c) A segment is the region between a chord and an arc. [3 marks]
- B6** O is the centre of a circle and A, B are on the circle. Angle $OAB = x^\circ$
 (a) Find angle AOB when $x = 40$ (b) Write an expression for angle AOB in terms of x
 (c) Angle AOB is 3 times angle OAB . Find x . (d) Angle AOB is 30° more than angle OAB . Find x . [4 marks]

HIGHER

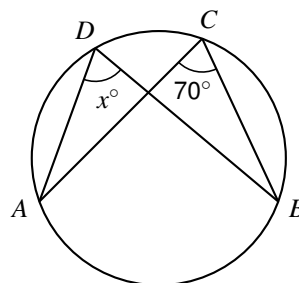
Higher tier only

- B7** Work out the value of x in each diagram. O is the centre.

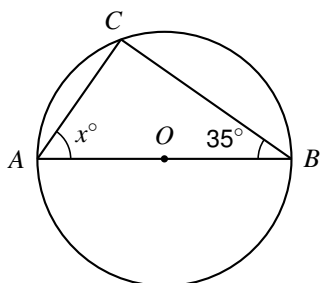
(a)



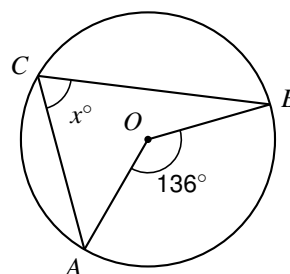
(b)



(c)



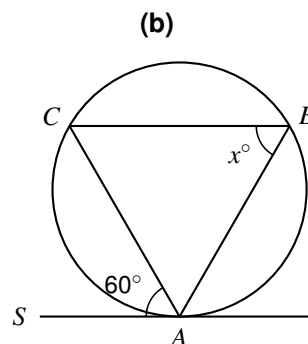
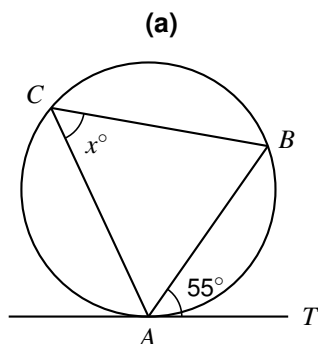
(d)



[4 marks]

- B8** $ABCD$ is a cyclic quadrilateral.
 (a) Angle $A = 72^\circ$. Find angle C (b) Angle $B = 95^\circ$. Find angle D
 (c) Angle $A = 3x^\circ$ and angle $C = (x + 20)^\circ$. Find x . (d) The exterior angle at C is 80° . Find angle A [4 marks]



HIGHER*continued: tangents and the alternate segment theorem***B9** PA and PB are tangents from P to a circle with centre O **(a)** Write down angle OAP **(b)** Angle $APO = 30^\circ$. Find angle AOP **(c)** $PA = 7$ cm. Write down PB **(d)** Angle $APB = 70^\circ$. Find angle AOB **[4 marks]****B10** Each diagram shows a tangent at A . Work out the value of x .**(c)** The angle between a tangent at A and a chord AB is 62° . Find the angle that AB makes at a point C in the alternate segment.**(d)** A tangent at A makes 48° with chord AB and 71° with chord AC , on opposite sides. Find angle BAC **[4 marks]**

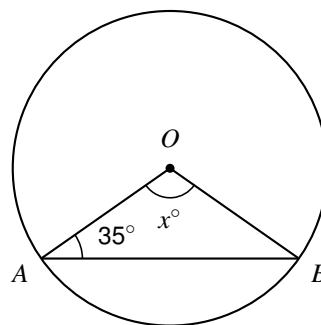
C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

FOUNDATION

Everyone starts here

- C1** Tom says, "The longest chord of a circle is a radius." Explain why Tom is wrong. [2 marks]
- C2** Sam calls the region between a chord and an arc a sector. Explain Sam's mistake. [2 marks]
- C3** O is the centre of a circle and A, B are points on the circle. Explain why triangle OAB is always isosceles. [3 marks]
- C4** O is the centre. Show that angle $AOB = 110^\circ$.
Give reasons.



[3 marks]

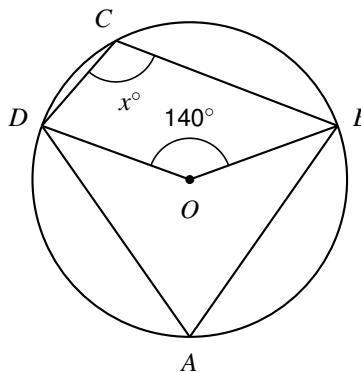


HIGHER*Higher tier only*

C5 Kai says, "The angle at the circumference is twice the angle at the centre." Explain Kai's mistake, and give an example. [3 marks]

C6 Use the theorem about the angle at the centre to explain why the angle in a semicircle is 90° [3 marks]

C7 O is the centre. Work out the value of x . Give a reason for each step.



[3 marks]

C8 $ABCD$ is a cyclic quadrilateral with centre O . Angle $BAD = a$

(a) Write down the angle BOD (the one not containing A) in terms of a

(b) Prove that the opposite angles of a cyclic quadrilateral add up to 180°

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

FOUNDATION

Everyone starts here

- D1** O is the centre. Work out the value of x . Give reasons.

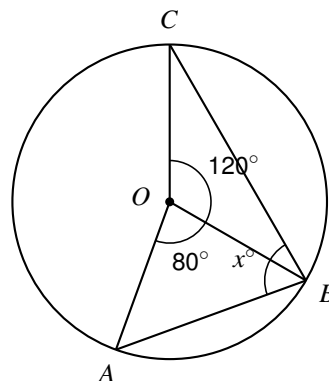


Diagram NOT accurately drawn

[3 marks]

- D2** A regular pentagon $ABCDE$ is drawn inside a circle with centre O

- (a) Work out angle AOB (b) Work out angle OAB (c) Work out angle ABC (d) Work out angle ACD

[4 marks]

- D3** O is the centre and AB is a diameter. Work out the value of x . Give reasons.

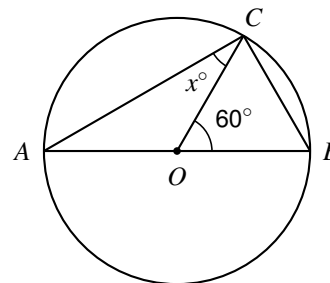


Diagram NOT accurately drawn

[3 marks]

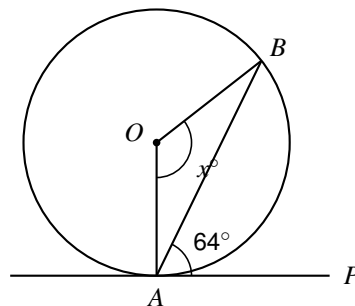
- D4** The centre of a clock face is joined to the 12 and to the 4. Work out the angle between the two lines, and the other two angles of the triangle they make with the line from 12 to 4

[3 marks]

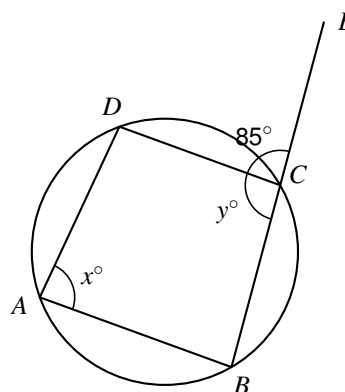


HIGHER*Higher tier only*

- D5** PA is a tangent at A and O is the centre. Work out the value of x . Give reasons.

*Diagram NOT accurately drawn***[4 marks]**

- D6** $ABCD$ is a cyclic quadrilateral and BCE is a straight line. Work out the values of x and y . Give reasons.

*Diagram NOT accurately drawn***[4 marks]**

- D7** A , B and C are on a circle with centre O . Angle $ACB = (x + 10)^\circ$ and angle $AOB = 3x^\circ$. Work out the size of angle AOB .

[4 marks]

- D8** PA and PB are tangents from P to a circle with centre O
- (a) Explain why triangles OAP and OBP are congruent.
- (b) Hence prove that $PA = PB$ and that OP bisects angle APB

[5 marks]

E**Exam practice**

Exam-style questions. Write your answers in the spaces. Aim for about one minute per mark.

FOUNDATION*Foundation and Higher crossover***E1** O is the centre. Write down the mathematical name of**(a)** line a

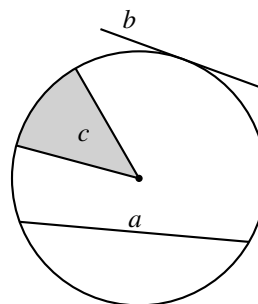
.....

(1)**(b)** line b

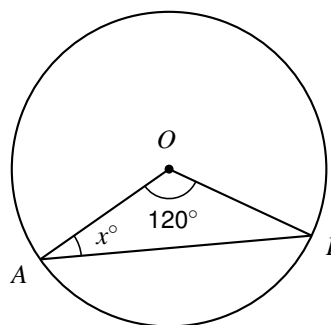
.....

(1)**(c)** the shaded region c

.....

(1)*Diagram NOT accurately drawn***(Total for Question E1 is 3 marks)****E2** O is the centre. Work out the value of x .

.....

*Diagram NOT accurately drawn***(Total for Question E2 is 3 marks)****E3** A regular nine-sided polygon is drawn inside a circle with centre O , and each vertex is joined to O . Work out the angle at O in each triangle, and a base angle of each triangle.

.....

(Total for Question E3 is 3 marks)

E4 O is the centre. Work out the value of x .

.....

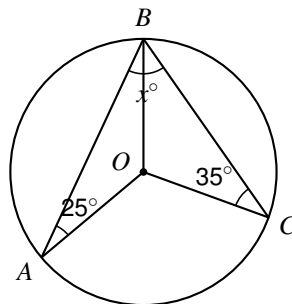


Diagram NOT accurately drawn

(Total for Question E4 is 3 marks)

E5 O is the centre of a circle and A, B are on the circle. Angle $OAB = 38^\circ$

Ben says, "Angle AOB is also 38° , because the triangle is isosceles." Explain Ben's mistake, and find angle AOB

.....

(Total for Question E5 is 3 marks)

HIGHER

Higher tier only

E6 O is the centre. Work out the value of x . Give reasons.

.....

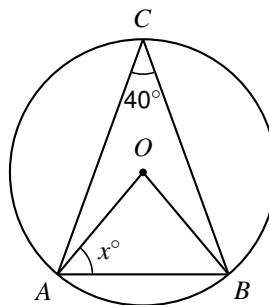


Diagram NOT accurately drawn

(Total for Question E6 is 3 marks)



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- E7** $ABCD$ is a cyclic quadrilateral. Work out the value of x . Give a reason.

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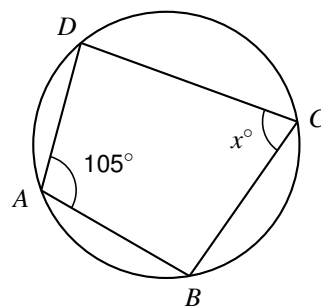


Diagram NOT accurately drawn

(Total for Question E7 is 3 marks)

- E8** TA is a tangent at A . Work out the value of x .
Give reasons for each step.

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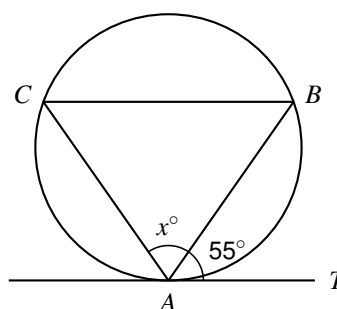


Diagram NOT accurately drawn

(Total for Question E8 is 4 marks)

- E9** A , B and C are points on a circle with centre O .
 COD is a straight line.

Prove that angle AOB is twice angle ACB

Hint: use the isosceles triangles OAC and OBC , and the exterior angles at O .

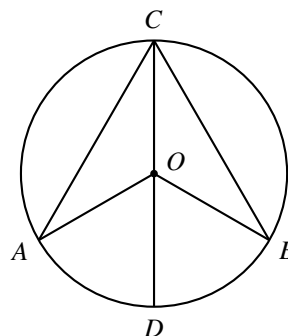


Diagram NOT accurately drawn

(Total for Question E9 is 5 marks)

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