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INTERNATIONAL A LEVEL MATHEMATICS • PURE 3 (WMA13)

Pure 3 Practice Paper 3

Edexcel IAL Pure Mathematics 3

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA13/01) • Calculator allowed

A full 75-mark paper set above the hardest recent series: long questions, unfamiliar settings and very few steps given

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 8 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	Inverse tangent equation and proof	9	
2	Stationary points in a geometric series	10	
3	Quadratic in e^x, inverse, modulus	10	
4	Compound angles and area between curves	9	
5	Differentiating $x = e^y - e^{-y}$	10	
6	Harmonic form and double angles	9	
7	Roots by iteration and sign change	9	
8	Algebraic division, area and proof	9	
	Total	**75**	

Rough grade guide (out of 75): A* **56** • A **49** • B **43** • C **37**

Based on recent WMA13 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Solve the equation

$$\arctan x + \arctan 2x = \frac{\pi}{4}$$

(6)

(b) Prove that

$$2\arctan\left(\frac{1}{3}\right) + \arctan\left(\frac{1}{7}\right) = \frac{\pi}{4}$$

(3)

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Question 1 continued

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(Total for Question 1 is 9 marks)

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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

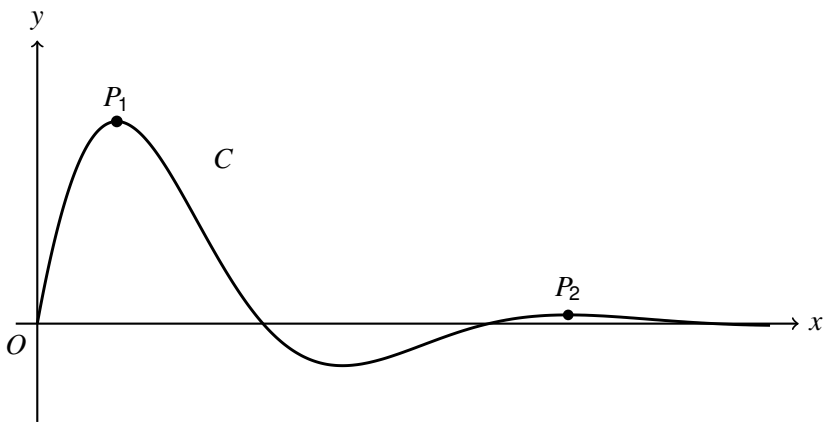

Figure 1

Figure 1 shows part of the curve C with equation

$$y = e^{-\frac{1}{2}x} \sin x \qquad x \geqslant 0$$

The maximum points on C, in order of increasing x, are $P_1, P_2, P_3, \ldots$

(a) Show that the x coordinate of every stationary point on C satisfies the equation $\tan x = 2$

(3)

(b) Prove that the y coordinates of $P_1, P_2, P_3, \ldots$ form a geometric sequence, and find its common ratio.

(4)

(c) Find the exact value of the sum of the y coordinates of all the maximum points on C.

(3)

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Question 2 continued

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(Total for Question 2 is 10 marks)

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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = e^{2x} - 6e^{x} + 5 \qquad x \in \mathbb{R}$$

(a) Sketch the curve with equation $y = f(x)$, showing the coordinates of the minimum point and of each point where the curve meets the coordinate axes, and the equation of the asymptote.

(3)

(b) Explain why f does not have an inverse.

The function g is defined by $g(x) = f(x)$, $x \geqslant k$, and g has an inverse.

Find the least possible value of k and, for this value of k, find $g^{-1}(x)$, stating its domain.

(4)

(c) Find the exact solutions of the equation $|f(x)| = 3$

(3)

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Question 3 continued

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(Total for Question 3 is 10 marks)

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4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

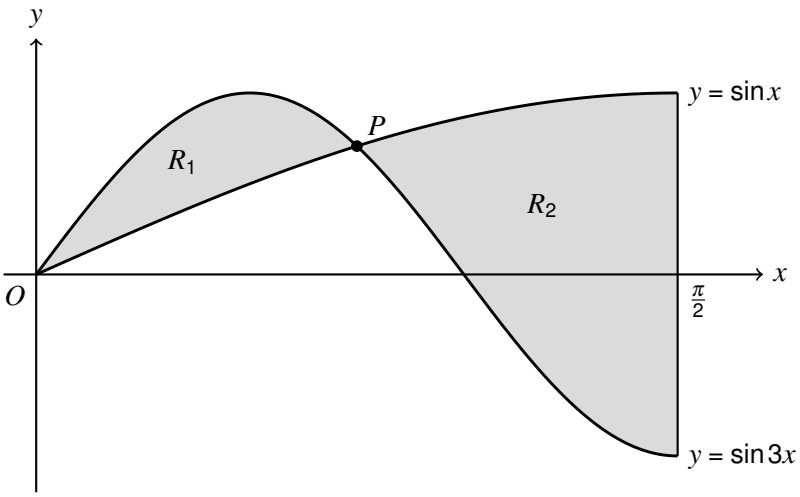

Figure 2

Figure 2 shows the curves with equations $y = \sin 3x$ and $y = \sin x$, for $0 \leqslant x \leqslant \frac{\pi}{2}$

The curves meet at O and at the point P.

The regions R_1 and R_2, shown shaded in Figure 2, are bounded by the two curves and the line $x = \frac{\pi}{2}$

(a) Use the compound angle formulae to find the exact x coordinate of P.

(3)

(b) Find the exact total area of R_1 and R_2, giving your answer in the form $a\sqrt{2} + b$, where a and b are rational.

(6)

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Question 4 continued

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(Total for Question 4 is 9 marks)

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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$x = e^{y} - e^{-y} \qquad x \in \mathbb{R}$$

(a) Find $\frac{dy}{dx}$ as a function of x, giving your answer in its simplest form.

(5)

(b) Find y as a function of x.

(3)

(c) Find an equation of the normal to C at the point where $y = \ln 2$, giving your answer in the form $y = mx + c$, where m and c are exact constants.

(2)

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Question 5 continued

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(Total for Question 5 is 10 marks)

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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given that

$$\cos x + \sin x = \frac{6}{5} \qquad 0 < x < \frac{\pi}{2}$$

(a) By writing $\cos x + \sin x$ in the form $R\sin(x+\alpha)$, sketch the curve with equation $y = \cos x + \sin x$ for $0 \leqslant x \leqslant \frac{\pi}{2}$, and explain why there are exactly two values of x for which $\cos x + \sin x = \frac{6}{5}$

(4)

(b) Find the exact value of $\sin 2x$

(2)

(c) Find the exact values of $\cos 2x$, stating which value goes with which value of x.

(3)

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Question 6 continued

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(Total for Question 6 is 9 marks)

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7. In this question you must show all stages of your working.

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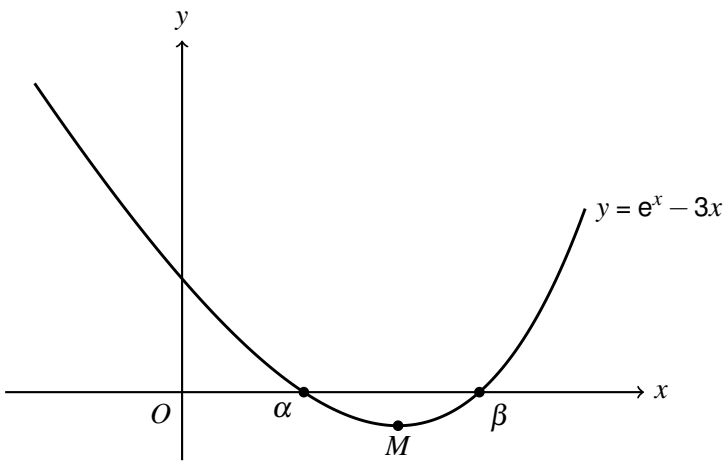

Figure 3

Figure 3 shows part of the curve with equation $y = e^x - 3x$

The curve has a minimum point M and meets the x-axis at the points where $x = \alpha$ and $x = \beta$

(a) Find the exact coordinates of M, and hence explain why the equation $e^x - 3x = 0$ has exactly two roots.

(3)

A student uses the iteration formula

$$x_{n+1} = \frac{1}{3}e^{x_n}$$

with $x_1 = 1.5$ to try to find β.

(b) Find x_2, x_3 and x_4 to 4 decimal places, and explain why this will not give β.

(2)

(c) Find β correct to 3 decimal places, justifying your answer.

(4)

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Question 7 continued

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(Total for Question 7 is 9 marks)

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8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{x^3 + 2x^2 + 3x + 2}{x^2 + 1} \qquad x \in \mathbb{R}$$

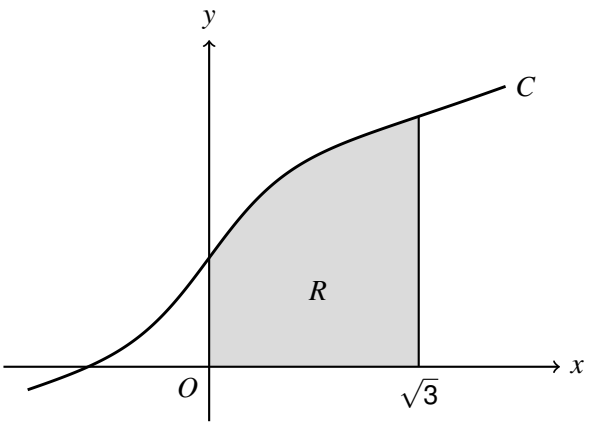

Figure 4

Figure 4 shows part of the curve C with equation $y = f(x)$

The region R, shown shaded in Figure 4, is bounded by C, the coordinate axes and the line $x = \sqrt{3}$

(a) Find the exact area of R, giving your answer in the form $p + q\sqrt{3} + \ln r$, where p, q and r are rational.

(6)

(b) Prove that f is an increasing function.

(3)

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Question 8 continued

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TOTAL FOR PAPER IS 75 MARKS

END

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