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INTERNATIONAL A LEVEL MATHEMATICS • STATISTICS 2

Statistics 2

Chapter 2: Poisson Distributions

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WST02/01) • Calculator and Statistical Tables allowed

Multi-step exam questions at the difficulty of the real WST02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Binomial Distributions** Conditions, probabilities and tables, mean and variance, unknown n and p	10	80	separate file	
2	**Poisson Distributions** Probabilities, modelling conditions, scaling the rate, adding Poisson variables	10	80	2	
3	**Approximations** Poisson and normal approximations, continuity corrections	10	80	separate file	
4	**Continuous Random Variables** Probability density and cumulative distribution functions, mean, variance, median, mode	10	81	separate file	
5	**Continuous Uniform Distribution** Rectangular distribution, mean and variance, modelling	10	80	separate file	
6	**Sampling and Sampling Distributions** Populations and samples, statistics, sampling distributions	10	80	separate file	
7	**Hypothesis Testing** Critical regions, significance levels, binomial and Poisson tests	10	80	separate file	

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CHAPTER 2 • KEY SKILLS

2 Poisson Distributions

Specification S2: Poisson distribution • 10 questions • 80 marks • about 96 minutes

1 The model

Events in an interval of time or space, at a mean rate λ: $X \sim \text{Po}(\lambda)$, with the parameter stated.

2 The formula

$\text{P}(X = r) = \dfrac{\text{e}^{-\lambda}\lambda^r}{r!}$; in particular $\text{P}(X = 0) = \text{e}^{-\lambda}$

3 Using the tables

Tables give $\text{P}(X \leqslant x)$: $\text{P}(X > 5) = 1 - \text{P}(X \leqslant 5)$ and $\text{P}(X = 3) = \text{P}(X \leqslant 3) - \text{P}(X \leqslant 2)$

4 Scaling the rate

7 per 10 minutes is 3.5 per 5 minutes and 0.7 per minute. Scale λ in proportion to the length of the interval.

5 Conditions in context

The events occur singly, independently, randomly, at a constant rate. Name the context (calls, customers, defects).

6 Mean and variance

$\text{E}(X) = \text{Var}(X) = \lambda$. If a sample has mean close to variance, a Poisson model is plausible.

7 Adding

If $X \sim \text{Po}(\lambda)$ and $Y \sim \text{Po}(\mu)$ are independent, then $X + Y \sim \text{Po}(\lambda + \mu)$

8 Inside other models

A Poisson probability can become the p of a binomial ("busy minutes"), or part of a conditional probability.

9 Finding λ or t

$\text{e}^{-\lambda t} = k$ gives $t = -\dfrac{\ln k}{\lambda}$. Equations between probabilities cancel $\text{e}^{-\lambda}$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 The model	•					•	•	•		•
2 The formula	•	•		•	•		•	•	•	
3 Using the tables	•	•	•	•		•	•	•	•	•
4 Scaling the rate		•		•	•	•	•	•		
5 Conditions in context	•					•				•
6 Mean and variance			•							
7 Adding				•				•	•	
8 Inside other models						•	•	•		•
9 Finding λ or t					•			•	•	
Interleaved: binomial distribution (Chapter 1)						•				
Interleaved: conditional probability and expectation (S1)			•	•			•	•		•
Interleaved: exponentials and logarithms (P2)					•			•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WST02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write the distribution with its parameter for the interval in the question, e.g. "$Y \sim$ Po(3.5) for calls in 5 minutes". State conditions in the context of the question, with "rate", not "probability".

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1. A shop sells pairs of running shoes at a mean rate of 4 pairs per hour. The number of pairs sold in an hour is X.

(a) Write down a suitable distribution to model X.

(1)

(b) State one condition, in context, needed for this distribution to be a suitable model.

(1)

(c) Find P(X = 6)

(2)

(d) Find the probability that more than 5 pairs are sold in an hour.

(2)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. Telephone calls arrive at a help desk at a mean rate of 7 every 10 minutes.

(a) Find the probability that exactly 3 calls arrive in a 5-minute period.

(2)

(b) Find the probability that at least 10 calls arrive in a 10-minute period.

(2)

(c) Find the probability that no calls arrive in a 1-minute period.

(2)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3. The numbers of flaws in 100 one-metre lengths of fabric were recorded.

Number of flaws, x	0	1	2	3	4	5
Frequency, f	22	33	25	13	5	2

(a) Find the mean and the variance of the number of flaws per metre.

(3)

(b) Explain how your answers to part (a) support a Poisson model for the number of flaws per metre.

(1)

(c) Using a Poisson distribution with mean 1.5, find the probability that a one-metre length has at least 3 flaws, and compare it with the data.

(2)

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Question 3 continued

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4. Emails arrive in the inbox of Ana at a mean rate of 2.5 per hour and in the inbox of Ben at a mean rate of 1.5 per hour. The numbers of emails arriving in the two inboxes are independent Poisson variables.

(a) Find the probability that exactly 4 emails arrive altogether in a one-hour period.

(2)

(b) Find the probability that no more than 5 emails arrive altogether in a two-hour period.

(2)

(c) Find the probability that, in a one-hour period, exactly 2 emails arrive for Ana and exactly 2 for Ben.

(2)

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Question 4 continued

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(Total for Question 4 is 6 marks)

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5. Faults occur in a computer network at a mean rate of 0.2 per hour, and the number of faults in any time interval has a Poisson distribution.

(a) Find the longest time, in minutes, for which the probability of no faults is at least 0.9

(3)

The random variable $X \sim \text{Po}(\lambda)$ has $\text{P}(X = 0) = 0.0498$

(b) Find the value of λ to the nearest whole number.

(2)

(c) Hence find $\text{P}(X = 2)$

(1)

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Question 5 continued

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6. Cars pass a checkpoint at a mean rate of 2.5 per minute. A minute is called "busy" if at least 5 cars pass the checkpoint in that minute.

(a) Show that the probability that a minute is busy is 0.1088

(2)

(b) Find the probability that at least 2 of 10 separate one-minute periods are busy.

(4)

(c) Find the probability that at least 12 cars pass the checkpoint in a 4-minute period.

(2)

(d) State, in context, one condition needed for the Poisson model, and one assumption needed for the model in part (b).

(2)

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Question 6 continued

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Question 6 continued

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7. The number of goals scored in a football match is modelled by a Poisson distribution with mean 3

(a) Find the probability that more than 3 goals are scored in a match.

(2)

(b) Given that at least one goal is scored in a match, find the probability that exactly 2 goals are scored.

(3)

(c) Find the probability that no more than 6 goals are scored altogether in two matches.

(2)

(d) Show that 2 and 3 are equally likely numbers of goals in a match, and explain why these are the modes.

(2)

(e) Suggest one reason why the Poisson model may not be suitable for goals scored in a match.

(1)

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Question 7 continued

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8. A cable has two types of defect, which occur independently. Type A defects occur at a mean rate of 0.6 per 10 m and type B defects at a mean rate of 0.4 per 10 m, each following a Poisson distribution.

(a) Find the probability that a 10 m length of cable has no defects of either type.

(1)

(b) Find the probability that a 50 m length of cable has at least 3 defects.

(2)

(c) Find the length of cable, in metres, for which the probability of no defects is 0.5

(3)

A 10 m length of cable has exactly 2 defects.

(d) Find the probability that both defects are in the first 5 m of this length.

(4)

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Question 8 continued

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9. The random variable $X \sim \text{Po}(\lambda)$, where $\lambda > 0$, and $\text{P}(X = 2) = 3\,\text{P}(X = 4)$

(a) Show that $\lambda = 2$

(3)

(b) Find $\text{P}(X \geqslant 4)$

(2)

The random variable $Y \sim \text{Po}(\mu)$ is independent of X, and $\text{P}(X + Y = 0) = \text{e}^{-5}$

(c) Find the value of μ.

(3)

(d) Find $\text{P}(X + Y \leqslant 3)$

(2)

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Question 9 continued

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10. A shop hires out boats by the day and has 5 boats. The daily demand for boats, X, is modelled by a Poisson distribution with mean 3.5

(a) Find the probability that, on a given day, the demand cannot all be met.

(2)

(b) Find the expected number of boats hired out on a day.

(4)

(c) Find the smallest number of boats the shop needs so that the probability that the demand cannot all be met is less than 0.05

(2)

(d) Given that the demand on a particular day is more than 3, find the probability that it cannot all be met.

(2)

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TOTAL FOR CHAPTER 2 IS 80 MARKS

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