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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 2 (WFM02)

# Further Pure 2 Practice Paper 3

Mark Scheme

**AIMING FOR A/A***

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM02/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

**Mark every line, not just the answer.** Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

| Code | Meaning |
|---|---|
| **M1** | Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark. |
| **dM1** | Dependent method mark: can only be given if the method mark before it has been given. |
| **A1** | Accuracy mark: for a correct answer or line of working. It depends on the method mark before it. |
| **B1** | Independent mark: for a correct result or statement, whatever method was used. |
| * | The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso). |
| **cao / cso** | Correct answer only / correct solution only, with no errors anywhere in the working. |
| **ft / oe** | Follow through from your own earlier answer / or any correct equivalent form. |

**"Solutions relying on calculator technology are not acceptable."** Questions 1 and 3 say this. There, a correct final answer with no algebraic working scores **no marks**. Elsewhere a calculator may be used, but the method must still be seen.

**Rough grade guide (out of 75):** A* **58** • A **52** • B **45** • C **38**

Based on recent WFM02 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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# What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

| Q | What it tested | Marks | Practise it | Score |
|---|---|---|---|---|
| 1 | A modulus against a rational function, split into cases, with a root outside its case | 10 | Ch 1 Inequalities | |
| 2 | A telescoping sum of logarithms; the least $n$ for a strict inequality | 10 | Ch 2 Series | |
| 3 | $\sin 5\theta$ by de Moivre's theorem; exact roots of a quartic; a product of sines | 11 | Ch 3 Complex Numbers | |
| 4 | The image of a region under $w = \frac{z-1}{z+1}$; an open range of $\lvert w - 1 \rvert$ | 11 | Ch 4 Further Argand Diagrams | |
| 5 | A second-order equation reduced by $x = \mathrm{e}^{u}$ | 11 | Ch 6 Second-Order DEs<br>Ch 5 First-Order DEs | |
| 6 | A product of Maclaurin series; a limit; the range of validity | 11 | Ch 7 Maclaurin and Taylor | |
| 7 | Tangents perpendicular to the initial line; the area inside a limaçon | 11 | Ch 8 Polar Coordinates | |
| | **Total** | **75** | | |

**Where did the marks go?** Every question on this paper has a trap, and at A* the lost marks are nearly always there: a root kept that does not belong to its case, an equality case counted as a strict inequality, a boundary included that the region does not contain, an answer left in $u$, or a polar area that also counts an inner loop. After marking, write down the check that would have caught each one.

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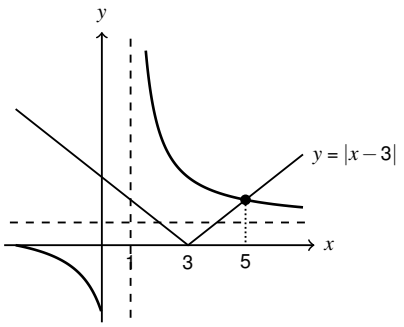
## Question 1

10 marks

| Part | Scheme | Marks |
|---|---|---|
| **(a)** | $y = \frac{x+3}{x-1}$: two branches, asymptotes $x = 1$ and $y = 1$, through $(-3, 0)$ and $(0, -3)$ | **B1** |
| | $y = \lvert x-3 \rvert$: a V with vertex $(3, 0)$, through $(0, 3)$ | **B1** |
| | Both on one diagram, meeting once, for $x > 3$   | **B1** |
| | | **(3)** |
| **(b)** | $x \geqslant 3$: $x+3 = (x-3)(x-1)$ | **M1** |
| | $x^2 - 5x = 0$, $x = 0$ or $x = 5$ | **A1** |
| | $x = 0$ is not in $x \geqslant 3$, so the only critical value here is $x = 5$ | **B1** |
| | $x < 3$: $x+3 = (3-x)(x-1)$, $x^2 - 3x + 6 = 0$ | **M1** |
| | Discriminant $9 - 24 < 0$: no real roots, so no other intersection | **A1** |
| | Critical values 1 (asymptote) and 5; the curve is on or above the V between them | **dM1** |
| | $\boxed{1 < x \leqslant 5}$ | **A1** |
| | *$x = 1$ must be excluded (the curve is not defined there) and $x = 5$ included. Keeping $x = 0$ as a critical value is the main trap.* | |
| | | **(7)** |
| | | **Total for Question 1: 10 marks** |

## Question 2

10 marks

| Part | Scheme | Marks |
|---|---|---|
| (a) | $\ln\frac{r+1}{r+2} - \ln\frac{r}{r+1} = \ln\left(\frac{r+1}{r+2} \times \frac{r+1}{r}\right)$<br>$= \ln\frac{(r+1)^2}{r(r+2)}$ | M1<br>A1* <br>(2) |
| (b) | $r=1$: $\ln\frac{2}{3} - \ln\frac{1}{2}$; $r=2$: $\ln\frac{3}{4} - \ln\frac{2}{3}$; …; $r=n$: $\ln\frac{n+1}{n+2} - \ln\frac{n}{n+1}$<br>All terms cancel except $\ln\frac{n+1}{n+2} - \ln\frac{1}{2}$<br>$= \ln\left(\frac{n+1}{n+2} \div \frac{1}{2}\right)$<br>$\boxed{\ln\frac{2(n+1)}{n+2}}$ | M1<br>A1<br>M1<br>A1<br>(4) |
| (c) | ln is increasing: $\frac{2(n+1)}{n+2} > 1.99$<br>$2n+2 > 1.99n + 3.98,\ 0.01n > 1.98$<br>$n > 198$<br>$\boxed{n = 199}$<br>*At $n = 198$ the sum equals $\ln 1.99$ exactly, so 198 does not satisfy the strict inequality.* | M1<br>M1<br>A1<br>A1<br>(4) |
| | | **Total for Question 2: 10 marks** |

## Question 3

11 marks

| Part | Scheme | Marks |
|---|---|---|
| **(a)** | $\sin 5\theta = \text{Im}(\cos\theta + \text{i}\sin\theta)^5$ | **B1** |
| | Binomial expansion, imaginary terms: $5c^4\text{i}s + 10c^2(\text{i}s)^3 + (\text{i}s)^5$ | **M1** |
| | $\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$ | **A1** |
| | $c^2 = 1 - s^2$: $5(1-s^2)^2 s - 10(1-s^2)s^3 + s^5$ | **M1** |
| | $= 16s^5 - 20s^3 + 5s$ | **A1*** |
| | | **(5)** |
| **(b)** | $\sin 5\theta = \sin\theta\left(16\sin^4\theta - 20\sin^2\theta + 5\right)$; $\sin 5\theta = 0$ with $\sin\theta \neq 0$ | **M1** |
| | $5\theta = k\pi$: $\theta = \pm\frac{\pi}{5}, \pm\frac{2\pi}{5}$ ($\theta = 0$ gives $\sin\theta = 0$, rejected) | **M1** |
| | Two roots correct | **A1** |
| | $\boxed{x = \sin\frac{\pi}{5},\ \sin\left(-\frac{\pi}{5}\right),\ \sin\frac{2\pi}{5},\ \sin\left(-\frac{2\pi}{5}\right)}$ | **A1** |
| | $\sin\frac{3\pi}{5}$ *and* $\sin\frac{4\pi}{5}$ *repeat* $\sin\frac{2\pi}{5}$ *and* $\sin\frac{\pi}{5}$*, so they are not new roots. The form asked for needs* $-\frac{1}{2} < k < \frac{1}{2}$. | |
| | | **(4)** |
| **(c)** | Product of the four roots $= \frac{5}{16}$: $\sin^2\frac{\pi}{5}\sin^2\frac{2\pi}{5} = \frac{5}{16}$ | **M1** |
| | Both sines positive: $\boxed{\frac{\sqrt{5}}{4}}$ | **A1** |
| | | **(2)** |
| | | **Total for Question 3: 11 marks** |

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## Question 4

11 marks

| Part | Scheme | Marks |
|---|---|---|
| (a) | $z = \dfrac{1+w}{1-w}$, $\lvert z\rvert = 1$: $\lvert 1+w\rvert = \lvert 1-w\rvert$ | M1 |
| | $(1+u)^2+v^2 = (1-u)^2+v^2$ | M1 |
| | $u = 0$: the imaginary axis | A1* |
| | *Or* $z = e^{i\theta}$: $w = \dfrac{e^{i\theta}-1}{e^{i\theta}+1} = i\tan\frac{\theta}{2}$. | |
| | | (3) |
| (b) | $z = \dfrac{(1+w)(1-w^*)}{\lvert 1-w\rvert^2} = \dfrac{1-\lvert w\rvert^2+2iv}{\lvert 1-w\rvert^2}$ | M1 |
| | Re$z > 0$: $\lvert w\rvert < 1$ | A1 |
| | Im$z > 0$: $v > 0$; $\lvert z\rvert < 1$: $u < 0$ | A1 |
| | Image: $\lvert w\rvert < 1$, $u < 0$, $v > 0$, a quarter disc in the second quadrant | A1 |
| | Quarter disc shaded, with its boundary not included | B1 |
| | *Boundary images: the segment* $0 < x < 1$ *goes to* $-1 < u < 0$*; the segment* $0 < y < 1$ *of the imaginary axis goes to the arc* $\lvert w\rvert = 1$*; the arc of* $\lvert z\rvert = 1$ *goes to* $0 < v < 1$ *on the imaginary axis.* | |
| | | (5) |
| (c) | $\lvert w-1\rvert$ is the distance from 1: nearest point of the region is 0, furthest is $-1$ | M1 |
| | Neither 0 nor $-1$ is in the image | A1 |
| | $\boxed{1 < \lvert w-1\rvert < 2}$ | A1 |
| | | (3) |
| | | **Total for Question 4: 11 marks** |

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## Question 5

11 marks

| Part | Scheme | Marks |
|---|---|---|
| (a) | $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{x}\frac{dy}{du}$ | B1 |
| | $\frac{d^2y}{dx^2} = -\frac{1}{x^2}\frac{dy}{du} + \frac{1}{x}\frac{d^2y}{du^2}\frac{du}{dx}$ (product and chain rules) | M1 |
| | $= \frac{1}{x^2}\left(\frac{d^2y}{du^2} - \frac{dy}{du}\right)$ | A1 |
| | $\left(\frac{d^2y}{du^2} - \frac{dy}{du}\right) - 2\frac{dy}{du} + 2y = e^{3u}$ | M1 |
| | $\frac{d^2y}{du^2} - 3\frac{dy}{du} + 2y = e^{3u}$ | A1* |
| | | (5) |
| (b) | $m^2 - 3m + 2 = 0,\ m = 1, 2:\ Ae^u + Be^{2u}$ | M1 |
| | $y = \lambda e^{3u}:\ 9\lambda - 9\lambda + 2\lambda = 1,\ \lambda = \frac{1}{2}$ | M1 |
| | $y = Ae^u + Be^{2u} + \frac{1}{2}e^{3u}$ | A1 |
| | $y = Ax + Bx^2 + \frac{1}{2}x^3$ | A1 |
| | *The final A1 needs $y$ in terms of $x$.* | |
| | | (4) |
| (c) | $A + B + \frac{1}{2} = 0,\ A + 2B + \frac{3}{2} = 1$ | M1 |
| | $y = \frac{1}{2}x^3 - \frac{1}{2}x$ | A1 |
| | | (2) |
| | | **Total for Question 5: 11 marks** |

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## Question 6

11 marks

| Part | Scheme | Marks |
|---|---|---|
| (a) | Replace $x$ by $2x$ in $x - \frac{x^2}{2} + \frac{x^3}{3}$ | M1 |
| | $2x - 2x^2$ | A1 |
| | $\boxed{\ln(1+2x) = 2x - 2x^2 + \frac{8}{3}x^3}$ | A1 |
| | $\boxed{-\frac{1}{2} < x \leqslant \frac{1}{2}}$ | B1 |
| | | (4) |
| (b) | $e^x = 1 + x + \frac{1}{2}x^2 + \ldots$ | B1 |
| | $\left(1 + x + \frac{1}{2}x^2\right)\left(2x - 2x^2 + \frac{8}{3}x^3\right)$ | M1 |
| | $x^2$: $-2 + 2 = 0$ | A1 |
| | $\boxed{2x + \frac{5}{3}x^3}$ | A1 |
| | | (4) |
| (c) | $\frac{2x + \frac{5}{3}x^3 + \ldots - 2x}{x^3} = \frac{5}{3} + \ldots$ | M1 |
| | $\boxed{\frac{5}{3}}$ | A1 |
| | | (2) |
| (d) | e ln 3 needs $x = 1$, which is outside $-\frac{1}{2} < x \leqslant \frac{1}{2}$, so the series for $\ln(1+2x)$ does not converge there | B1 |
| | | (1) |
| | | **Total for Question 6: 11 marks** |

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## Question 7

11 marks

| Part | Scheme | Marks |
|---|---|---|
| (a) | $x = r\cos\theta = \cos\theta + 2\cos^2\theta$ | M1 |
| | $\frac{dx}{d\theta} = -\sin\theta - 4\sin\theta\cos\theta$ | M1 |
| | $= -\sin\theta(1 + 4\cos\theta) = 0$ | A1 |
| | $\sin\theta = 0$ or $\cos\theta = -\frac{1}{4}$ | M1 |
| | $(3, 0)$ | A1 |
| | $\left(\frac{1}{2}, 1.82\right)$ and $\left(\frac{1}{2}, -1.82\right)$ | A1 |
| | *$\theta = 0$ is easy to lose when dividing by $\sin\theta$. $\theta = \pm 1.82$ is inside the domain $\lvert\theta\rvert \leqslant \frac{2\pi}{3} = 2.09$.* | (6) |
| (b) | $\frac{1}{2}\int (1 + 2\cos\theta)^2 d\theta$ | M1 |
| | $= \frac{1}{2}\int (3 + 4\cos\theta + 2\cos 2\theta) d\theta$ | M1 |
| | $= \frac{1}{2}[3\theta + 4\sin\theta + \sin 2\theta]$ | A1 |
| | Limits $-\frac{2\pi}{3}$ to $\frac{2\pi}{3}$ (or twice 0 to $\frac{2\pi}{3}$): $2\pi + 2\sqrt{3} - \frac{\sqrt{3}}{2}$ | M1 |
| | $2\pi + \frac{3\sqrt{3}}{2}$ | A1 |
| | *Integrating from 0 to $2\pi$ gives $3\pi$: that also counts the inner loop where $1 + 2\cos\theta < 0$, which is not part of $C$.* | (5) |
| | | **Total for Question 7: 11 marks** |

**TOTAL FOR PAPER: 75 MARKS**