eClassroom

A-LEVEL MATHEMATICS • MECHANICS 2

Projectiles

Worked Solutions

Pearson Edexcel International A Level (WME02) • Calculator allowed: take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give answers to 2 or 3 s.f.

Full worked solutions for the free Projectiles worksheet, A-Level and International A-Level Maths

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Tap a section to jump straight to it

Full worked solutions for every question on the Mechanics 2 Projectiles worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Mechanics 2 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $78.4 = 4.9t^2$: $\boxed{t = 4\,\text{s}}$; 9×4: $\boxed{36\,\text{m}}$

A2 $\sin\alpha = \frac{5}{13}$, $\cos\alpha = \frac{12}{13}$: $\boxed{24\,\text{m s}^{-1}\ \textbf{and}\ 10\,\text{m s}^{-1}}$; $0 = 10 - 9.8t$: $\boxed{1.02\,\text{s}}$

A3 Each component is $28\cos 45^\circ \approx 19.80$

$T = \frac{2(19.80)}{9.8}$: $\boxed{4.04\,\text{s}}$; $\frac{19.80^2}{19.6}$: $\boxed{20\,\text{m}}$; 19.80×4.041: $\boxed{80\,\text{m}}$

STRETCH

A4 $v_x = 8$, $v_y = 14 - 19.6 = -5.6$

$\sqrt{8^2 + 5.6^2}$: $\boxed{9.77\,\text{m s}^{-1}}$; $\tan^{-1}\frac{5.6}{8}$: $\boxed{35.0^\circ\ \textbf{below the horizontal}}$

A5 $\frac{9.8}{2(441)} \times \frac{25}{9} = \frac{5}{162}$: $\boxed{y = \frac{4}{3}x - \frac{5}{162}x^2}$; $24 - 10$: $\boxed{14\,\text{m}}$

A6 $u_y = -7$: $-15 = -7t - 4.9t^2$, so $4.9t^2 + 7t - 15 = 0$

positive root: $\boxed{1.18\,\text{s}}$; 12.12×1.176: $\boxed{14.3\,\text{m}}$

A7 $u_y^2 = 2(9.8)(4.9)$, $u_y = 9.8$; $T = 2$, $u_x = \frac{39.2}{2} = 19.6$

$\boxed{U = 21.9\,\text{m s}^{-1},\ \alpha = 26.6^\circ}$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $4.9 = 4.9t^2$: $\boxed{1\,\text{s}}$ **(b)** $\boxed{7\,\text{m}}$ **(c)** $19.6 = 4.9t^2$: $\boxed{2\,\text{s}}$ **(d)** $\boxed{14\,\text{m}}$

Four times the height gives twice the time, because the drop grows with t^2.

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B2 **(a)** **17.3 and 10 m s^{-1}** **(b)** **10 and 17.3 m s^{-1}** **(c)** **16 and 12 m s^{-1}** **(d)** **12 and 16 m s^{-1}**

Complementary angles swap the horizontal and vertical components.

B3 **(a)** $\frac{9.8}{9.8}$: **1 s** **(b)** $\frac{9.8^2}{19.6}$: **4.9 m** **(c)** **2 s** **(d)** 6×2: **12 m**

On level ground the time of flight is twice the time to the top.

B4 **(a)** **20 m** **(b)** **17.3 m** **(c)** **17.3 m** **(d)** $\frac{(14\sin 60°)^2}{19.6} = \frac{147}{19.6}$: **7.5 m**

$R = \frac{U^2 \sin 2\alpha}{g}$: *30° and 60° give the same range, and 45° gives the greatest.*

B5 **(a)** **$(5\mathbf{i} + 4.9\mathbf{j})$ m s^{-1} and $(5\mathbf{i} - 4.9\mathbf{j})$ m s^{-1}**

(b) The horizontal components are equal and the vertical components have the same size, so the speeds are equal: the path is symmetric about the top, at $t = 1.5$

(c) $\sqrt{5^2 + 14.7^2}$: **15.5 m s^{-1}**

B6 **(a)** **$x = 10t$, $y = 20t - 4.9t^2$** **(b)** $t = \frac{x}{10}$: **$y = 2x - 0.049x^2$**

(c) $40 - 19.6$: **20.4 m** **(d)** $2x = 0.049x^2$: **40.8 m**

Eliminating t turns the two equations of motion into the equation of the path.

STRETCH

B7 **(a)** $24.5 = 4.9t^2$: **2.24 s** **(b)** $-24.5 = 14.7t - 4.9t^2$: **4.19 s** **(c)** $24.5 = 14.7t + 4.9t^2$: **1.19 s**

(d) In each case the vertical speed at the ground satisfies $v_y^2 = u_y^2 + 2(9.8)(24.5)$ and the horizontal part is unchanged, so speed$^2 = 14.7^2 + 480.2$ every time: **26.4 m s^{-1}**

The direction changes the time, but not the landing speed.

B8 **(a)** $16 - 9.8t = 12$: **0.408 s** **(b)** $16 - 9.8t = -12$: **2.86 s**

(c) $12\sqrt{2}$: **17.0 m s^{-1}** both times **(d)** **2.45 s**

At 45° the vertical component equals the horizontal one; the two instants are symmetric about the top.

B9 **(a)** $\frac{9.8}{2(196)} = \frac{1}{40}$: **$y = x - \frac{x^2}{20}$** **(b)** **$y = 2x - \frac{x^2}{8}$** **(c)** **20 m and 16 m**

(d) $5 = 10T - 2.5(1 + T^2)$, so $T^2 - 4T + 3 = 0$: **$\tan\alpha = 1$ or $\tan\alpha = 3$**

The path equation is a quadratic in $\tan\alpha$, so a point in range is usually reached at two angles.

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B10 **(a)** $\dfrac{2U\sin 30^\circ}{9.8}$: $\boxed{\dfrac{U}{9.8}\ \mathbf{s}}$ **(b)** $U\cos 30^\circ \times \dfrac{U}{9.8}$: $\boxed{\dfrac{\sqrt{3}U^2}{19.6}\ \mathbf{m}}$

(c) $U^2 = \dfrac{40 \times 19.6}{\sqrt{3}}$: $\boxed{U = 21.3}$ **(d)** $\dfrac{(0.5U)^2}{19.6}$: $\boxed{\mathbf{5.77\ m}}$

Write each result in terms of U first, then use the one fact you know.

C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 At the top only the *vertical* component is zero; the horizontal component $20\cos 30^\circ$ is unchanged: $\boxed{\textbf{least speed } 17.3\,\text{m s}^{-1}}$

C2 25 is the speed, not the vertical component. $u_y = 25 \times \frac{3}{5} = 15$: $0 = 15 - 9.8t$: $\boxed{\mathbf{1.53\ s}}$

C3 The ground is 15 m *below* the start, so the displacement is -15: $-15 = 9.8t - 4.9t^2$

$4.9t^2 - 9.8t - 15 = 0$, positive root: $\boxed{\mathbf{3.02\ s}}$

C4 $R = U\cos\alpha \times \dfrac{2U\sin\alpha}{g} = \dfrac{2U^2\sin\alpha\cos\alpha}{g}$

$\sin 30^\circ\cos 30^\circ = \sin 60^\circ\cos 60^\circ = \dfrac{\sqrt{3}}{4}$, so $\boxed{\textbf{both land } \dfrac{\sqrt{3}U^2}{2g} \textbf{ away}}$: the longer flight is cancelled by the smaller horizontal component.

STRETCH

C5 Perpendicular to $(3\mathbf{i}+4\mathbf{j})$ means a zero scalar product, so the velocity is parallel to $(4\mathbf{i}-3\mathbf{j})$, not $(3\mathbf{i}-4\mathbf{j})$

$v_x = 3$, so $v_y = -2.25$: $4 - 9.8t = -2.25$: $\boxed{\mathbf{0.638\ s}}$

C6 They are also 3 m apart horizontally (6 m and 3 m from the start): $\sqrt{3^2+4^2}$: $\boxed{\mathbf{5\ m\ apart}}$

C7 The ball does not land at the level it started from. After the top it falls $4.9 + 14.7 = 19.6$ m, not 4.9 m

$-14.7 = 9.8t - 4.9t^2$, so $t^2 - 2t - 3 = 0$: $\boxed{\mathbf{3\ s}}$

C8 **(a)** At the top $v_y = 0$: $0 = U^2 \sin^2\alpha - 2gH$, so $\boxed{H = \dfrac{U^2 \sin^2\alpha}{2g}}$

It lands when $U \sin\alpha\, t - \frac{1}{2}gt^2 = 0$, $t = \dfrac{2U\sin\alpha}{g}$; $R = U\cos\alpha \times \dfrac{2U\sin\alpha}{g}$: $\boxed{R = \dfrac{U^2 \sin 2\alpha}{g}}$

(b) $\dfrac{4H}{R} = \dfrac{2U^2\sin^2\alpha}{g} \times \dfrac{g}{2U^2 \sin\alpha\cos\alpha} = \dfrac{\sin\alpha}{\cos\alpha}$: $\boxed{\tan\alpha = \dfrac{4H}{R}}$

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 **(a)** At C the motion is at 45°, so $v_y = v_x = U$. Time to C: $t = \dfrac{12}{U}$

$v_y = 9.8t$: $U = \dfrac{9.8 \times 12}{U}$, so $U^2 = 117.6$: $\boxed{U = \sqrt{117.6}\text{, as required}}$

(b) $h = \dfrac{U^2}{2(9.8)} = \dfrac{117.6}{19.6}$: $\boxed{h = 6}$

D2 **(a)** $u_x = 14$, $u_y = 10.5$; over the crossbar when $t = \dfrac{19.6}{14} = 1.4$

$y = 10.5(1.4) - 4.9(1.4)^2 = 5.096 > 2.44$: $\boxed{\text{clears it by 2.66 m}}$

(b) $v_y = 10.5 - 13.72 = -3.22 < 0$: $\boxed{\text{falling}}$

D3 **(a)** Time of flight $\dfrac{8u}{9.8}$; $OA = 3u \times \dfrac{8u}{9.8} = \dfrac{24u^2}{9.8} = 30$

$u^2 = 12.25$: $\boxed{u = 3.5}$

(b) $u_y = 14$: $\dfrac{14^2}{19.6}$: $\boxed{\text{10 m}}$

D4 P is highest when $9.8 - 9.8t = 0$, $t = 1$: P at $(8,\ 4.9)$

Q at $t = 1$: $(6,\ 14.7 - 4.9) = (6,\ 9.8)$

$\sqrt{2^2 + 4.9^2}$: $\boxed{\text{5.29 m}}$

STRETCH

D5 **(a)** $-40 = 7t - 4.9t^2$, so $4.9t^2 - 7t - 40 = 0$

$t = \dfrac{7 + \sqrt{833}}{9.8}$: **3.66 s**

(b) $v_y = 7 - 9.8(3.659) \approx -28.86$, $v_x = 7$

$\tan^{-1}\dfrac{28.86}{7}$: **76.4° below the horizontal**

D6 Horizontally: $u_x = \dfrac{24}{2.5} = 9.6$

Vertically: $-20 = 2.5u_y - 4.9(2.5)^2$, so $u_y = 4.25$

$U = \sqrt{9.6^2 + 4.25^2}$: $U = 10.5$; $\tan^{-1}\dfrac{4.25}{9.6}$: $\theta = 23.9$

D7 More than 30° when $|v_y| > 8\tan 30° \approx 4.619$

$12 - 9.8t = 4.619$ gives $t \approx 0.753$; by symmetry the same length of time on the way down

2×0.753: **1.51 s**

D8 **(a)** $\tan 45° = 1$: $y = x - \dfrac{9.8x^2}{2U^2}(1+1) = x - \dfrac{9.8x^2}{U^2}$

At P: $2.5 = 10 - \dfrac{980}{U^2}$, so $\dfrac{980}{U^2} = 7.5$: **$U^2 = \dfrac{392}{3}$, as required**

(b) $u_x = u_y = \sqrt{\dfrac{196}{3}} \approx 8.083$; $t = \dfrac{10}{8.083} \approx 1.237$; $v_y \approx 8.083 - 12.124 = -4.041$

$\sqrt{8.083^2 + 4.041^2}$: $9.04\,\text{m s}^{-1}$; $\tan^{-1}\dfrac{4.041}{8.083}$: **26.6° below the horizontal**

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $0 = 9.8t - 4.9t^2$ *M1*

$t = 2$ *A1*

(b) 14×2: **28 m** *B1*

(c) $0 = 9.8^2 - 2(9.8)h$ *M1*

4.9 m *A1*

(d) $v_y = 9.8 - 9.8(1.5) = -4.9$, $v_x = 14$ *M1*

$\sqrt{14^2 + 4.9^2}$: $14.8\,\text{m s}^{-1}$ *A1*

$\tan^{-1}\frac{4.9}{14}$: **19.3° below the horizontal** *A1*

(e) $3.675 = 9.8t - 4.9t^2$, so $t^2 - 2t + 0.75 = 0$, $t = 0.5$ or $t = 1.5$ *M1*

$1.5 - 0.5$: **1 second** *A1*

E2 **(a)** Vertically from B to C: $(-24.5)^2 = y^2 + 2(9.8)(19.6)$ *M1 A1*

$y^2 = 600.25 - 384.16 = 216.09$: $y = 14.7$ *A1**

(b) $-24.5 = 14.7 - 9.8t$ *M1*

$t = 4$ *A1*

(c) $x = 7$ (the horizontal component never changes): 7×4: **28 m** *B1*

(d) Higher than B while $14.7t - 4.9t^2 > 0$, that is $0 < t < 3$ *M1*

3 seconds *A1*

(e) At 45°, $|v_y| = v_x = 7$: $14.7 - 9.8t = 7$ or $14.7 - 9.8t = -7$ *M1*

$t = 0.786$ *A1*

$t = 2.21$ *A1*

STRETCH

E3 **(a)** $x = U\cos\alpha\, t$, $y = U\sin\alpha\, t - \frac{1}{2}gt^2$ *M1*

$t = \dfrac{x}{U\cos\alpha}$, so $y = x\tan\alpha - \dfrac{gx^2}{2U^2\cos^2\alpha}$ *M1 A1*

$\dfrac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$: $\boxed{y = x\tan\alpha - \dfrac{gx^2}{2U^2}(1+\tan^2\alpha)}$ *A1**

(b) Comparing: $\boxed{\tan\alpha = \dfrac{4}{3}}$ *B1*

$\dfrac{9.8}{2U^2}\left(1 + \dfrac{16}{9}\right) = \dfrac{2}{45}$, so $U^2 = 306.25$ *M1*

$\boxed{U = 17.5}$ *A1*

(c) $u_y = 17.5 \times \dfrac{4}{5} = 14$: $\dfrac{14^2}{19.6}$ *M1*

$\boxed{\textbf{10 m}}$ *A1*

(d) $6.4 = \frac{4}{3}x - \frac{2}{45}x^2$, so $x^2 - 30x + 144 = 0$, $x = 6$ or $x = 24$ *M1*

$u_x = 17.5 \times \dfrac{3}{5} = 10.5$: $\dfrac{24-6}{10.5}$: $\boxed{\textbf{1.71 s}}$ *A1*

(e) At $x = 24$: $t = \dfrac{24}{10.5} \approx 2.286$, $v_y = 14 - 9.8(2.286) \approx -8.4$ *M1*

$\sqrt{10.5^2 + 8.4^2}$: $\boxed{13.4\,\text{m s}^{-1}}$ *A1*

$\tan^{-1}\dfrac{8.4}{10.5}$: $\boxed{38.7^\circ \textbf{ below the horizontal}}$ *A1*

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