eClassroom

A-LEVEL MATHEMATICS • MECHANICS 2

Centres of Mass

Worked Solutions

Pearson Edexcel International A Level (WME02) • Calculator allowed: take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give answers to 2 or 3 s.f.

Full worked solutions for the free Centres of Mass worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

B **Fluency** core skills page 2

C **Reasoning** explain it page 4

D **Problem** solving page 5

E **Exam** practice page 6

Tap a section to jump straight to it

Full worked solutions for every question on the Mechanics 2 Centres of Mass worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $\bar{x} = \dfrac{8-1+15}{8}$, $\bar{y} = \dfrac{0+5+9}{8}$: $\boxed{(2.75,\ 1.75)}$

A2 Rectangle 24 at height 2; triangle 9 at height $4+1=5$

$\bar{y} = \dfrac{24(2)+9(5)}{33} = \dfrac{31}{11}$: $\boxed{\textbf{2.82 cm}}$

A3 Base 6 at $(3a, 0)$; sides 2 at $(0, a)$ and $(6a, a)$: $\bar{y} = \dfrac{2a+2a}{10}$: $\boxed{(3a,\ 0.4a)}$

STRETCH

A4 Disc 36π at O; hole 4π at 3 cm: $\bar{x} = \dfrac{0-4\pi(3)}{36\pi-4\pi} = -\dfrac{3}{8}$

$\boxed{\textbf{0.375 cm from } O}$, on the side away from the hole

A5 From A: lamina M at $(4,3)$, particle $2M$ at $(8,0)$: $G = \left(\frac{20}{3},\ 1\right)$

From $D(0,6)$: across $\frac{20}{3}$, down 5; $\tan\theta = \dfrac{20}{15}$: $\boxed{53.1°}$

A6 $\tan\alpha = \dfrac{2.5}{6}$: $\boxed{\alpha = 22.6°}$

A7 Moments about A, horizontal distances: $F(6a) = W(3a) + \frac{1}{2}W(6a)$: $\boxed{F = W}$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $\dfrac{48}{8}$: $\boxed{6}$ **(b)** $\dfrac{48+8}{12}$: $\boxed{\frac{14}{3}}$ **(c)** $48+4p=60$: $\boxed{p=3}$ **(d)** $48+12m=8(8+m)$: $\boxed{m=4}$

The same equation $\sum mx = \bar{x}\sum m$ works forwards and backwards.

B2 **(a)** $\frac{8+6}{6}$: $\boxed{\frac{7}{3}}$ **(b)** $\frac{2+15}{6}$: $\boxed{\frac{17}{6}}$ **(c)** $8+3p=18$: $\boxed{p=\frac{10}{3}}$ **(d)** $2+5m=3(3+m)$: $\boxed{m=3.5}$

Each coordinate is found on its own.

B3 **(a)** $\boxed{(2,\ 3)}$ **(b)** $\frac{4(6)}{3\pi}$: $\boxed{\textbf{2.55 cm}}$ **(c)** $\frac{2(6)}{\pi}$: $\boxed{\textbf{3.82 cm}}$ **(d)** $\boxed{(3,\ 3)}$

The arc's centre of mass is further out than the lamina's: all of its mass is on the rim.

B4 **(a)** $\frac{36(3)-4(5)}{32}$: $\boxed{(2.75,\ 2.75)}$ **(b)** $\boxed{(3,\ 2.75)}$ **(c)** $\frac{36(3)+4(7)}{40}$: $\boxed{(3,\ 3.4)}$ **(d)** $\boxed{(3,\ 3)}$

A removed part is a negative mass: the centre of mass moves away from it, and a symmetric hole moves nothing.

B5 **(a)** $\frac{0+2\left(\frac{3\sqrt{3}}{2}\right)}{3}$: $\boxed{\textbf{1.73 cm}}$ **(b)** $\frac{2\left(\frac{3\sqrt{3}}{2}\right)}{4}$: $\boxed{\textbf{1.30 cm}}$ **(c)** $\frac{4(0)+4(2)+4(2)}{12}$: $\boxed{\textbf{1.33 cm}}$

For rods, mass goes with length (and mass per unit length), acting at each midpoint.

B6 **(a)** $\tan\theta=\frac{3}{4}$: $\boxed{36.9^\circ}$ **(b)** $\boxed{53.1^\circ}$ **(c)** $\boxed{\textbf{}AB\textbf{ horizontal}}$ **(d)** $G=(2,\ 4.5)$, $\tan\theta=\frac{4.5}{2}$: $\boxed{66.0^\circ}$

The line from the point of suspension to G is vertical; the angle is between that line and the side.

STRETCH

B7 **(a)** $-\frac{4\pi(2)}{12\pi}$: $\boxed{-\frac{2}{3}}$ **(b)** $-\frac{\pi(2)}{15\pi}$: $\boxed{-\frac{2}{15}}$ **(c)** $\boxed{\left(-\frac{1}{7},\ -\frac{1}{7}\right)}$ **(d)** $\frac{-8+2.5}{11}$: $\boxed{-\frac{1}{2}}$

Mass is proportional to area, so π cancels: use masses 16, 4 and 1.

B8 **(a)** $\frac{6(0)+8(4)}{14}$: $\boxed{\textbf{}\frac{16}{7}\textbf{ cm}}$ **(b)** $\frac{6(3)+8(0)}{14}$: $\boxed{\textbf{}\frac{9}{7}\textbf{ cm}}$ **(c)** $\tan\theta=\frac{16/7}{6-9/7}$: $\boxed{25.9^\circ}$

(d) Equal angles: $\bar{x}=\bar{y}$; $\frac{16}{7}=\frac{9}{7}+6k$: $\boxed{k=\frac{1}{6}}$

From A the vertical distance is $6-\frac{9}{7}$, not $\frac{9}{7}$

B9 **(a)** $\tan\alpha=\frac{3}{4}$: $\boxed{36.9^\circ}$ **(b)** $\frac{4}{3}$: $\boxed{53.1^\circ}$ **(c)** $\frac{3}{3}$: $\boxed{45^\circ}$ **(d)** G is 2 from the lower corner and 3 up: $\boxed{33.7^\circ}$

Always $\tan\alpha=\dfrac{\textit{distance of } G \textit{ up the slope from the lower corner}}{\textit{height of } G}$

B10 **(a)** $\boxed{45^\circ}$ **(b)** $G=(3,1)$, $\tan\theta=\frac{1}{3}$: $\boxed{18.4^\circ}$ **(c)** $\frac{2}{2+4k}=\frac{1}{\sqrt{3}}$: $\boxed{k=\frac{\sqrt{3}-1}{2}}$

(d) $\boxed{\textbf{the angle decreases towards }0^\circ}$: G moves towards B, so AB approaches the vertical.

With A as origin, $G=\left(\frac{2+4k}{1+k},\ \frac{2}{1+k}\right)$

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C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 25.1 cm is more than the radius, so G would be outside the lamina. The formula is $\frac{4r}{3\pi}$: **2.55 cm**

C2 The rods have different lengths, so different masses: use 3 and 5. $\frac{3(0)+5(2.5)}{8}$: **1.56 cm**

C3 The triangle is removed, so its mass is negative. Square 36 at 3; triangle 4.5 at 5

$\frac{108-22.5}{31.5}=\frac{19}{7}$: **2.71 cm**

C4 $G=(5,2)$ from A. $\tan\theta=\frac{5}{2}$ gives the angle between AD and the vertical. For AB: $\tan\theta=\frac{2}{5}$: 21.8°

STRETCH

C5 A trapezium has more area near its longer side. Rectangle 2×4 at height 2; two triangles of total area 8 at height $\frac{4}{3}$

$\frac{16+\frac{32}{3}}{16}=\frac{5}{3}$: **1.67 cm from the longer side**

C6 Mass = (mass per unit area)×(area): 16(1) : 4(3) = 16 : 12, not 1 : 3

$\frac{16(2)+12(5)}{28}=\frac{23}{7}$: **3.29 cm**

C7 The moment of W uses the *perpendicular* distance from A to its line of action, the horizontal distance 2, not AG

$W(2)=F(4)$: $F=\frac{1}{2}W$

C8 **(a)** Square $4a^2$ at a; removed x^2 at $2a - \frac{1}{2}x$: $\bar{x} = \dfrac{4a^3 - x^2\left(2a - \frac{1}{2}x\right)}{4a^2 - x^2}$

$\times\dfrac{2}{2}$: $\boxed{\bar{x} = \dfrac{8a^3 - 4ax^2 + x^3}{2(4a^2 - x^2)}}$

(b) $\bar{x} = 2a - x$: $8a^3 - 4ax^2 + x^3 = 2(4a^2 - x^2)(2a - x)$, so $x^3 - 8a^2x + 8a^3 = 0$

$(x - 2a)(x^2 + 2ax - 4a^2) = 0$; $x \neq 2a$, so $x = -a + \sqrt{5a^2}$: $\boxed{x = (\sqrt{5} - 1)a}$

D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 **(a)** Rectangle 72 at $6a$; triangle 18 at $12a + 2a$: $\dfrac{72(6a) + 18(14a)}{90}$: $\boxed{7.6a}$

(b) $\bar{y} = 3a$ by symmetry; $\tan\theta = \dfrac{3}{7.6}$: $\boxed{21.5°}$

D2 **(a)** $AEFD$: 24 at $3a$; folded square: 16, now between $2a$ and $6a$, at $4a$

$\dfrac{24(3a) + 16(4a)}{40}$: $\boxed{3.4a}$

(b) $G = (3.4a,\ 2a)$; from D: across $3.4a$, down $2a$; $\tan\theta = \dfrac{3.4}{2}$: $\boxed{59.5°}$

D3 **(a)** Arc: mass πr at $\dfrac{2r}{\pi}$; diameter: mass $2r$ at 0: $\dfrac{\pi r \times \frac{2r}{\pi}}{\pi r + 2r}$: $\boxed{\dfrac{2r}{\pi + 2}}$

(b) G is r along AB from A; $\tan\theta = \dfrac{2}{\pi + 2}$: $\boxed{21.3°}$

D4 **(a)** Disc 9 at O, hole 1 at $2a$ (per πa^2): $\dfrac{-2a}{8}$: $\boxed{\frac{1}{4}a \text{ from } O}$

(b) $kM(3a) = M\left(\frac{1}{4}a\right)$: $\boxed{k = \frac{1}{12}}$

STRETCH

D5 **(a)** Masses 16 and $2 \times 4 = 8$: $\dfrac{16(2a) + 8(5a)}{24}$: $\boxed{3a}$

(b) Moments about A: $F(6a) = Mg(3a)$: $\boxed{F = \frac{1}{2}Mg}$

D6 **(a)** Square 36 at $3a$; triangle 9 at a: $\dfrac{36(3a)-9(a)}{27}$: $\boxed{\frac{11}{3}a}$

(b) $\bar{y}=3a$; from B: across $6a-\frac{11}{3}a=\frac{7}{3}a$, up $3a$; $\tan\theta=\dfrac{7}{9}$: $\boxed{37.9^\circ}$

D7 **(a)** G is 2 cm up the slope from B and $\frac{8}{3}$ cm from the plane: $\tan\alpha=\dfrac{2}{8/3}$: $\boxed{36.9^\circ}$

(b) Now G is 4 cm from the lower corner A: $\tan\alpha=\dfrac{4}{8/3}$: $\boxed{56.3^\circ}$

D8 **(a)** Rectangle 60 at $5a$; hole 4π at $3a$: $\bar{x}=\dfrac{300-12\pi}{60-4\pi}a$

$\boxed{\dfrac{3(25-\pi)}{15-\pi}a}$ $(\approx 5.53a)$

(b) $\bar{y}=3a$ by symmetry; $\tan\theta=\dfrac{3}{5.530}$: $\boxed{28.5^\circ}$

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** Rectangle (sides $4a$, $6a$): 24 at $2a$; triangle: 12 at $\frac{16}{3}a$ *M1*

$\dfrac{24(2a)+12\left(\frac{16}{3}a\right)}{36}$ *A1*

$\boxed{\frac{28}{9}a}$ *A1**

(b) $\dfrac{24(3a)+12(2a)}{36}$ *M1*

$\boxed{\frac{8}{3}a}$ *A1*

(c) From B: across $\frac{28}{9}a$, down $6a-\frac{8}{3}a=\frac{10}{3}a$ *M1*

$\tan\theta=\dfrac{28/9}{10/3}=\dfrac{14}{15}$ *A1*

$\boxed{43^\circ}$ *A1*

E2 **(a)** Lengths $6a$, $4a$, $6a$, $4a$, $5a$ (total $25a$) at $3a$, $6a$, $3a$, 0, $1.5a$ *M1*

$$\frac{6(3)+4(6)+6(3)+4(0)+5(1.5)}{25}a$$ *A1*

$\boxed{2.7a}$ *A1**

(b) $\dfrac{6(0)+4(2)+6(4)+4(2)+5(2)}{25}a$ *M1*

$\boxed{2a}$ *A1*

(c) From B: across $6a - 2.7a = 3.3a$, up $2a$ *M1*

$\tan\theta = \dfrac{3.3}{2}$ *A1*

$\boxed{58.8^\circ}$ *A1*

(d) $\bar{x} = \dfrac{2.7a}{1+k}$, $\bar{y} = \dfrac{2a+4ka}{1+k}$ *M1*

Equal angles: $\bar{x} = \bar{y}$, $2.7 = 2+4k$ *M1*

$\boxed{k = \frac{7}{40}}$ *A1*

STRETCH

E3 **(a)** Square $36a^2$ at $3a$; triangle $3ah$ at $6a + \frac{1}{3}h$ *M1 A1*

$$\bar{y} = \frac{108a^3 + 3ah\left(6a+\frac{1}{3}h\right)}{36a^2+3ah} = \frac{108a^3+18a^2h+ah^2}{3a(12a+h)}$$ *M1*

$\boxed{\dfrac{h^2+18ah+108a^2}{3(h+12a)}}$ *A1**

(b) $h = 6a$: $\bar{y} = \dfrac{252a^2}{54a} = \dfrac{14}{3}a$; G is $3a$ up the slope from A *M1 A1*

$\tan\alpha = \dfrac{3}{14/3} = \dfrac{9}{14}$: $\boxed{32.7^\circ}$ *A1*

(c) E is on the line of symmetry, so G stays $3a$ up the slope from A *M1*

$$\bar{y} = \frac{M\left(\frac{14}{3}a\right)+kM(12a)}{(1+k)M} = \frac{(14+36k)a}{3(1+k)}$$ *M1*

$\tan\alpha = \dfrac{3a}{\bar{y}}$: $\boxed{\dfrac{9(1+k)}{2(7+18k)}}$ *A1**

(d) $9(1+k) = 2(7+18k)\tan 25^\circ$ *M1*

$\boxed{k = 0.317}$ *A1*