



eClassroom

A-LEVEL MATHEMATICS • PURE 3

Differentiation

Worked Solutions

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Differentiation worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 3 Differentiation worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $\ln 5x = \ln 5 + \ln x$: $4 \cos 4x - \frac{2}{x}$

A2 $(4x^2 + 1)^{\frac{1}{2}}: \frac{1}{2}(4x^2 + 1)^{-\frac{1}{2}} \times 8x = \frac{4x}{\sqrt{4x^2 + 1}}$

A3 $3x^2 \ln x + x^3 \cdot \frac{1}{x} = x^2(3 \ln x + 1)$

STRETCH

A4 $\frac{2x(x+2) - x^2}{(x+2)^2} = \frac{x(x+4)}{(x+2)^2}$

A5 $(\sin 3x)^{-1} \rightarrow -(\sin 3x)^{-2} \times 3 \cos 3x = -3 \operatorname{cosec} 3x \cot 3x$

A6 $\frac{dx}{dy} = 2e^{2y} - 1 = 1$ at $y = 0$: $\frac{dy}{dx} = 1$

A7 $\frac{dy}{dx} = \ln x + 1 = 2$ at $x = e$, and $y = e$: $y - e = 2(x - e)$: $y = 2x - e$



B

Fluency solutions

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $5e^{5x}$ (b) $\frac{1}{x}$ (c) $-3\sin 3x$ (d) $-2e^{-\frac{x}{2}}$

The coefficient of x comes out in front; $\ln kx$ always gives $\frac{1}{x}$

B2 (a) $12(2x+5)^5$ (b) $\frac{2x}{x^2+1}$ (c) $2xe^{x^2}$ (d) $3\sin^2 x \cos x$

Each time: the outer derivative times the derivative of the inside.

B3 (a) $e^{2x}(1+2x)$ (b) $2x\sin x + x^2\cos x$ (c) $e^x(\cos x - \sin x)$ (d) $\ln 3x + 1$

Factorise the common factor, usually the exponential.

B4 (a) $\frac{1}{(x+1)^2}$ (b) $\frac{e^x(x-1)}{x^2}$ (c) $\frac{1-2\ln x}{x^3}$ (d) $\frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$

In (d), $\sin^2 x + \cos^2 x = 1$ makes the numerator collapse.

B5 (a) $e^{-x}(1-x)$ (b) $-e^{-2}$ (c) $1-x=0: (1, e^{-1})$

e^{-x} is never zero, so only the bracket gives the stationary point.

B6 (a) $3\sec^2 3x$ (b) $2\sec 2x \tan 2x$ (c) $-\operatorname{cosec}^2 x$ (d) $-2x\operatorname{cosec}(x^2)\cot(x^2)$

These follow from the quotient or chain rule on $\frac{1}{\cos x}$, $\frac{1}{\sin x}$ and $\frac{\cos x}{\sin x}$

STRETCH

B7 (a) $xe^{-x}(2-x)$ (b) $e^{2x}(2\sin 3x + 3\cos 3x)$ (c) $\frac{3(x-1)^2(x+1) - (x-1)^3}{(x+1)^2} = \frac{2(x-1)^2(x+2)}{(x+1)^2}$
(d) $\cot x$

In (c), take out $(x-1)^2$ before expanding anything.

B8 (a) $\frac{1}{2y+3}$ (b) $\frac{1}{2\cos 2y}$ (c) $y+1=e^x$ (d) $\cos^2 y = \frac{1}{\sec^2 y} = \frac{1}{1+x^2}$

Find $\frac{dx}{dy}$, take the reciprocal, then use the equation (or an identity) to change to x



B9 (a) $\frac{dy}{dx} = \frac{2}{2x-1}$: $\boxed{2}$ (b) $\boxed{y = 2x - 2}$ (c) $\boxed{y = -\frac{1}{2}x + \frac{1}{2}}$ (d) $\frac{2}{2x-1} = \frac{1}{2}$: $\boxed{x = \frac{5}{2}}$

The normal's gradient is the negative reciprocal of the tangent's.

B10 (a) $e^{-x^2} + x(-2xe^{-x^2}) = e^{-x^2}(1 - 2x^2)$ **as required** (b) $\boxed{x = \pm \frac{\sqrt{2}}{2}}$
 (c) $\boxed{\frac{\sqrt{2}}{2}e^{-\frac{1}{2}}}$ (d) $\boxed{e^{-x^2} > 0 \text{ for all } x, \text{ so only } 1 - 2x^2 = 0 \text{ gives solutions}}$

Never set an exponential factor equal to zero.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 Sam forgot to multiply by the derivative of $3x$: $\boxed{3 \cos 3x}$

C2 The derivative of the inside, $2x$, is missing: $\boxed{\frac{2x}{x^2 + 4}}$

C3 The quotient rule is $\frac{vu' - uv'}{v^2}$; Kai subtracted the wrong way round: $\frac{(x+2) - x}{(x+2)^2} = \boxed{\frac{2}{(x+2)^2}}$

C4 e^x is never zero, but x can be. Setting each factor to zero: $\boxed{x = 0 \text{ and } x = -2}$

STRETCH

C5 Max must say why the derivative is positive, then conclude: $3 > 0$ and $(x+1)^2 > 0$ for $x > -1$, so $\boxed{f'(x) > 0, \text{ so } f \text{ is increasing}}$

C6 $2y$ is $\frac{dx}{dy}$; $\frac{dy}{dx}$ is its reciprocal, $\frac{1}{2y}$. At $(5, 2)$: $\boxed{\frac{1}{4}}$

C7 Perpendicular gradients multiply to -1 , so the normal's gradient is $-\frac{1}{3}$, not -3 $\boxed{-\frac{1}{3}}$



C8 (a) $\frac{\cos x \cos x - \sin x(-\sin x)}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$ as required

(b) $(\cos x)^{-1} \rightarrow -(\cos x)^{-2}(-\sin x) = \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x$ as required

(c) chain rule: $\frac{1}{\sec x} \times \sec x \tan x = \tan x$ as required

D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $\frac{(2x-1) - 2(x+4)}{(2x-1)^2} = \frac{-9}{(2x-1)^2}$, so $k = -9$

(b) $-9 \neq 0$, so $\frac{dy}{dx} \neq 0$; and $\frac{dy}{dx} < 0$ wherever it is defined

D2 (a) $2y - 4$ (b) $\frac{dx}{dy} = 0$ when $y = 2$, $x = 4 - 8 + 1$: $P(-3, 2)$

(c) $\frac{dx}{dy} = 4$ at $y = 4$: $\frac{dy}{dx} = \frac{1}{4}$

D3 $\frac{dy}{dx} = 2 \ln 3x + 2x \cdot \frac{1}{x} = 2 \ln 3x + 2 = 0$, so $\ln 3x = -1$, $x = \frac{1}{3e}$

$y = \frac{2}{3e} \times (-1)$: $Q\left(\frac{1}{3e}, -\frac{2}{3e}\right)$

D4 $\frac{dy}{dx} = 2e^{2x} - 3 = -1$ at $x = 0$, and $y = 1$: tangent $y = 1 - x$; $y = 0$: $R(1, 0)$

STRETCH

D5 (a) $2x(2x+1)^{-\frac{1}{2}} - x^2(2x+1)^{-\frac{3}{2}} = x(2x+1)^{-\frac{3}{2}}(2(2x+1) - x) = \frac{x(3x+2)}{(2x+1)^{\frac{3}{2}}}$ as required

(b) $x = 0$ or $x = -\frac{2}{3}$, but $-\frac{2}{3} < -\frac{1}{2}$ is outside the domain; $x = 0$ gives O



D6 (a) $\frac{dx}{dy} = 2e^{2y} + e^y$; at $y = 0$ it is 3: **gradient $\frac{1}{3}$**

(b) normal gradient -3 : $y = -3(x - 2)$; $x = 0$: **$N(0, 6)$**

D7 (a) $6(3x - k)^{-2} \rightarrow -12(3x - k)^{-3} \times 3$: **$\frac{dy}{dx} = -\frac{36}{(3x - k)^3}$**

(b) $-\frac{36}{(3 - k)^3} = -\frac{9}{2}$, so $(3 - k)^3 = 8$: **$k = 1$**

D8 (a) $-e^{-x} \sin x + e^{-x} \cos x = e^{-x}(\cos x - \sin x)$ **as required**

(b) $\tan x = 1$, $x = \frac{\pi}{4}$: **$M\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}e^{-\frac{\pi}{4}}\right)$** (c) $e^{-\pi}(-1 - 0) =$ **$-e^{-\pi}$**

E**Exam practice solutions**

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $\frac{dy}{dx} = \frac{2x(x - 2) - (x^2 + 5)}{(x - 2)^2}$ M1 A1

$= \frac{x^2 - 4x - 5}{(x - 2)^2} = \frac{(x - 5)(x + 1)}{(x - 2)^2}$: **$a = 5, b = 1$** A1

(b) $(x - 2)^2 > 0$, so the sign is that of $(x - 5)(x + 1)$ M1

$x < -1$ or $x > 5$ A1

E2 (a) $\frac{dy}{dx} = \frac{6}{2x - 3}$; at $x = 2$: **6** M1 A1

(b) normal gradient $-\frac{1}{6}$ M1

$y = -\frac{1}{6}(x - 2)$ M1

$x + 6y - 2 = 0$ A1

(c) **$\left(0, \frac{1}{3}\right)$** B1



STRETCH

- E3** (a) $2xe^x + (x^2 - 3)e^x = e^x(x^2 + 2x - 3) = e^x(x+3)(x-1)$: $p = 3, q = -1$ M1 A1
- (b) $e^x \neq 0$, so $x = -3$ or $x = 1$ M1
- $A(-3, 6e^{-3})$ A1
- $B(1, -2e)$ A1
- (c) $f(x) \geq -2e$ B1
- (d) at $x = 0$: $y = -3$ and $\frac{dy}{dx} = -3$; tangent $y = -3x - 3$ M1
- $T(-1, 0)$ A1

