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A-LEVEL MATHEMATICS • STATISTICS 2

Continuous Random Variables

Worksheet

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Free Continuous Random Variables worksheet for A-Level and International A-Level Maths, with full worked solutions

A **Examples** & Your turn page 2

B **Fluency** core skills page 4

C **Reasoning** explain it page 6

D **Problem** solving page 7

E **Exam** practice page 9

Tap a section to jump straight to it

By the end of this worksheet you can: use a probability density function • find and use the cumulative distribution function • find unknown constants • find the mean, variance and E(g(X)) • *Stretch:* piecewise functions, the median and quartiles from the correct piece, the mode, skewness and locating a value.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Piecewise functions, unknown constants from two conditions, the median from the right piece and the mode at an end point.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	22
Stretch marks	16	14	17	14
My score				

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Name ..... Class ..... Date .....

A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE *Secure the method*

Worked example A1: a constant and a probability

$f(x) = kx(4-x)$ for $0 \leqslant x \leqslant 4$, and 0 otherwise. Find k and $P(X < 1)$

Total area 1

$k\left[2x^2 - \frac{1}{3}x^3\right]_0^4 = \frac{32}{3}k = 1;\ \frac{3}{32}\left(2 - \frac{1}{3}\right)$

$k = \frac{3}{32},\ \frac{5}{32}$

Your turn A1

$f(x) = k(x^2 + 2x)$ for $0 \leqslant x \leqslant 3$, and 0 otherwise. Find k and $P(X > 2)$

Worked example A2: the cdf and the median

$f(x) = \dfrac{3x^2}{64}$ for $0 \leqslant x \leqslant 4$, and 0 otherwise. Find $F(x)$ and the median.

$F(x) = \int_0^x f(t)\,dt$, *with 0 and 1 outside*

$F(x) = \dfrac{x^3}{64};\ m^3 = 32$

$m = 3.17$

Your turn A2

$f(x) = \dfrac{2x+1}{12}$ for $0 \leqslant x \leqslant 3$, and 0 otherwise. Find $F(x)$ and the median.

Worked example A3: mean and variance

$f(x) = \dfrac{x}{2}$ for $0 \leqslant x \leqslant 2$, and 0 otherwise. Find $E(X)$ and $\text{Var}(X)$

$E(X) = \int_0^2 \frac{x^2}{2}\,dx = \frac{4}{3};\ E(X^2) = \int_0^2 \frac{x^3}{2}\,dx = 2$

$\frac{4}{3},\ 2 - \frac{16}{9} = \frac{2}{9}$

Your turn A3

$f(x) = \dfrac{2(3-x)}{9}$ for $0 \leqslant x \leqslant 3$, and 0 otherwise. Find $E(X)$ and $\text{Var}(X)$

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STRETCH

Worked example A4: the pdf from the cdf

$F(x) = \frac{x^2 - 1}{8}$ for $1 \leqslant x \leqslant 3$. Find $f(x)$ and $P(X > 2)$

Differentiate: $f(x) = F'(x)$

$f(x) = \frac{x}{4}$; $1 - F(2) = 1 - \frac{3}{8}$

$\frac{x}{4}$, $\frac{5}{8}$

Your turn A4

$F(x) = \frac{x^3 + x}{10}$ for $0 \leqslant x \leqslant 2$. Find $f(x)$ and $P(1 < X < 1.5)$

Worked example A5: the mode

$f(x) = \frac{4}{81}x(9 - x^2)$ for $0 \leqslant x \leqslant 3$, and 0 otherwise. Find the mode.

Solve $f'(x) = 0$, *not* $f(x) = 0$

$\frac{4}{81}(9 - 3x^2) = 0$, $x^2 = 3$

$\sqrt{3} = 1.73$

Your turn A5

$f(x) = \frac{1}{64}x(16 - x^2)$ for $0 \leqslant x \leqslant 4$, and 0 otherwise. Find the mode.

Worked example A6: the median in the right piece

$f(x) = \frac{x}{5}$ for $0 \leqslant x \leqslant 2$, $f(x) = \frac{2(5 - x)}{15}$ for $2 < x \leqslant 5$. Find the median.

$F(2) = 0.4 < 0.5$, *so the median is in the second piece*

$1 - \frac{(5 - m)^2}{15} = 0.5$, $(5 - m)^2 = 7.5$

$m = 2.26$

Your turn A6

$f(x) = \frac{2x}{5}$ for $0 \leqslant x \leqslant 1$, $f(x) = \frac{5 - x}{10}$ for $1 < x \leqslant 5$. Find the median.

Worked example A7: $E(g(X))$

For $f(x) = \frac{x}{2}$ on $[0, 2]$, $E(X^2) = 2$. Find $E(3X^2 - 1)$

$3E(X^2) - 1$

5

Your turn A7

$f(x) = \frac{3x^2}{8}$ for $0 \leqslant x \leqslant 2$, and 0 otherwise. Find $E(4X - X^2)$

B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE *Secure the method*

B1 $f(x) = k(1+x)$ on an interval, and 0 otherwise. Find k when the interval is

(a) $[0,2]$

(b) $[0,4]$

(c) $[1,3]$

(d) $[0,a]$ and $k = \frac{2}{15}$: find a

[4 marks]

B2 $f(x) = \frac{3x^2}{125}$ for $0 \leqslant x \leqslant 5$, and 0 otherwise. Find

(a) $P(X < 2)$

(b) $P(X > 4)$

(c) $P(2 < X < 4)$

(d) $P(X = 3)$

[4 marks]

B3 $f(x) = \frac{1}{4}$ for $0 \leqslant x \leqslant 2$, $f(x) = \frac{4-x}{4}$ for $2 < x \leqslant 4$, and 0 otherwise.

(a) Find $F(x)$ for $0 \leqslant x \leqslant 2$

(b) Find $F(x)$ for $2 < x \leqslant 4$

(c) State $F(x)$ for $x < 0$ and for $x > 4$

(d) Find the median.

[4 marks]

B4 $f(x) = \frac{3(4-x^2)}{16}$ for $0 \leqslant x \leqslant 2$, and 0 otherwise. Find

(a) $E(X)$

(b) $E(X^2)$

(c) $\mathrm{Var}(X)$

(d) $\mathrm{Var}(4X-1)$

[4 marks]

B5 $F(x) = \frac{x^3}{27}$ for $0 \leqslant x \leqslant 3$, with $F(x) = 0$ for $x < 0$ and $F(x) = 1$ for $x > 3$

(a) Find $f(x)$

(b) Find $P(1 < X < 2)$

(c) Find the median.

[3 marks]

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B6 $F(x) = \frac{x^2}{25}$ for $0 \leqslant x \leqslant 5$. Find

(a) the median

(b) the lower quartile

(c) the upper quartile

(d) the 90th percentile

[4 marks]

STRETCH

B7 Find the mode of X when $f(x)$ is a positive multiple of the given function on $[0, 3]$, and 0 otherwise:

(a) $x(3-x)$

(b) $x^2(3-x)$

(c) $x(3-x)^2$

(d) x

[4 marks]

B8 $f(x) = a + bx$ for $0 \leqslant x \leqslant 2$, and 0 otherwise. The median of X is 1.2

(a) Show that $a + b = \frac{1}{2}$

(b) Find a and b

(c) Find $E(X)$

(d) State the mode.

[4 marks]

B9 $f(x) = \frac{3}{64}x(4-x)^2$ for $0 \leqslant x \leqslant 4$, and 0 otherwise.

(a) Find the mode.

(b) Find $E(X)$

(c) Show that the median is 1.54, to 3 s.f.

(d) Describe the skewness, giving a reason.

[4 marks]

B10 $f(x) = \frac{2x}{9}$ for $0 \leqslant x \leqslant 3$, and 0 otherwise. Find

(a) $E(X)$

(b) $\text{Var}(X)$

(c) $E(X^3)$

(d) $E\left(\frac{1}{X}\right)$

[4 marks]

C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE *Secure the method*

C1 $f(x) = \frac{3x^2}{32}$ for $0 \leqslant x \leqslant 2\sqrt[3]{4}$. Amir says $P(X = 2) = f(2) = 0.375$

Explain his error, and give $P(X = 2)$ **[2 marks]**

C2 $f(x) = \frac{2x}{9}$ for $0 \leqslant x \leqslant 3$, and 0 otherwise. Bea gives the cumulative distribution function as $F(x) = \frac{x^2}{9}$. Explain why her answer is incomplete, and complete it. **[2 marks]**

C3 $F(x) = \frac{x^2 + 2x}{15}$ for $0 \leqslant x \leqslant 3$. To find $f(x)$, Cal integrates $F(x)$

Explain his error, and find $f(x)$ **[3 marks]**

C4 $f(x) = \frac{3x^2}{8}$ for $0 \leqslant x \leqslant 2$. Dev finds $E(X) = 1.5$, then writes $E(X^2) = 1.5^2$ and $Var(X) = 0$

Explain his error, and find $Var(X)$ **[3 marks]**

STRETCH

C5 $f(x) = \frac{3}{32}x(4 - x)$ for $0 \leqslant x \leqslant 4$. Eve finds the mode by solving $f(x) = 0$ and gets 0 and 4

Explain her error, and find the mode. **[3 marks]**

C6 $f(x) = \frac{x}{4}$ for $0 \leqslant x \leqslant 2$ and $f(x) = \frac{1}{2}$ for $2 < x \leqslant 3$. Eve gives $F(x) = \frac{x}{2} - 1$ for $2 < x \leqslant 3$

Explain her error, and find $F(x)$ for $2 < x \leqslant 3$ **[3 marks]**

C7 $f(x) = \frac{x}{6}$ for $0 \leqslant x \leqslant 3$ and $f(x) = \frac{4 - x}{2}$ for $3 < x \leqslant 4$. Fay solves $1 - \frac{(4 - m)^2}{4} = 0.5$ and gives the median as 2.59

Explain her error, and find the median. **[3 marks]**

C8 $f(x) = kx^n$ for $0 \leqslant x \leqslant 1$, and 0 otherwise, where $n > 0$

(a) Show that $k = n + 1$ and $E(X) = \frac{n+1}{n+2}$

(2)

(b) Show that $Var(X) = \frac{n+1}{(n+2)^2(n+3)}$

(3)

[5 marks]

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D Problem solving

You decide the method. Read the whole question first, and show every step.

CORE *Secure the method*

D1 The continuous random variable X has cumulative distribution function $F(x) = \frac{x^3 + 3x}{14}$ for $0 \leqslant x \leqslant 2$, with $F(x) = 0$ for $x < 0$ and $F(x) = 1$ for $x > 2$

Find $P(X < 1.5 \mid X > 1)$ **[3 marks]**

D2 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The continuous random variable X has probability density function given by

$$f(x) = \begin{cases} cx & 0 \leqslant x \leqslant 1 \\ \frac{1}{3}c(4-x) & 1 < x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $c = \frac{1}{2}$

(2)

(b) Use algebraic integration to show that $E(X) = \frac{5}{3}$

(2)

[4 marks]

D3 The continuous random variable X has probability density function $f(x) = \frac{x+2}{6}$ for $0 \leqslant x \leqslant 2$, and 0 otherwise.

(a) Find the cumulative distribution function $F(x)$ for all values of x

(2)

(b) Find the median of X

(1)

[3 marks]

D4 The continuous random variable X has probability density function $f(x) = \frac{4x^3}{81}$ for $0 \leqslant x \leqslant 3$, and 0 otherwise.

Find $E(5X^2 - 2X + 1)$ **[3 marks]**

STRETCH

D5 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The continuous random variable X has probability density function $f(x) = a + bx^2$ for $0 \leqslant x \leqslant 3$, and 0 otherwise, where a and b are constants. Given that $E(X) = 2$,

(a) show that $a = \frac{1}{9}$ and $b = \frac{2}{27}$

(3)

(b) find $P(X > 2)$

(1)

[4 marks]

D6 The continuous random variable X has cumulative distribution function $F(x) = \frac{8x^2 - x^4}{16}$ for $0 \leqslant x \leqslant 2$, with $F(x) = 0$ for $x < 0$ and $F(x) = 1$ for $x > 2$

(a) Show that the median, m, of X satisfies $m^4 - 8m^2 + 8 = 0$

(2)

(b) Find the value of m, giving a reason for rejecting any other root.

(2)

[4 marks]

D7 The continuous random variable X has cumulative distribution function $F(x) = \frac{x^3 - 3x^2 + 4x}{12}$ for $0 \leqslant x \leqslant 3$, with $F(x) = 0$ for $x < 0$ and $F(x) = 1$ for $x > 3$

(a) Show that the upper quartile of X lies between 2.7 and 2.8

(2)

(b) Find the mode of X, giving a reason.

(2)

[4 marks]

D8 The continuous random variable X has probability density function $f(x) = \frac{3}{8}(x-1)^2$ for $1 \leqslant x \leqslant 3$, and 0 otherwise.

(a) Show that $P(X > 2.5) = \frac{37}{64}$

(2)

(b) Eight independent observations of X are taken. Find the probability that at least 6 of them are greater than 2.5

(3)

[5 marks]

E Exam practice

Full-size exam questions in the style of WST02. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

E1 The time, X hours, that a technician takes to repair a machine is modelled by a continuous random variable with probability density function given by

$$f(x) = \begin{cases} kx & 0 \leqslant x \leqslant 3 \\ 3k(4-x) & 3 < x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. Figure 1 shows a sketch of the graph of $f(x)$, which consists of two straight line segments.

(a) Show that $k = \frac{1}{6}$

(2)

(b) Find $P(X < 2)$

(2)

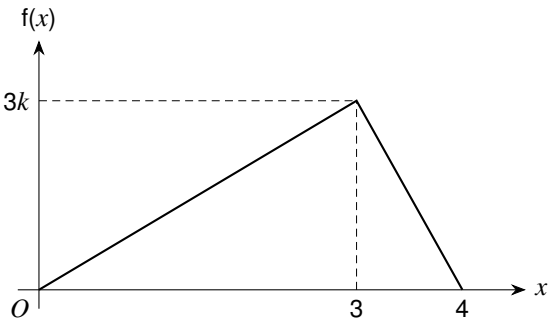


Figure 1

(c) Use algebraic integration to find $E(X)$

(3)

(d) Find the median of X

(2)

(e) State the mode of X and describe the skewness of the distribution, giving a reason.

(2)

(Total for Question E1 is 11 marks)

E2 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The distance, X km, that a delivery driver travels between consecutive drops in a city is modelled by a continuous random variable with probability density function given by

$$f(x) = \begin{cases} k(16 - x^2) & 0 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \dfrac{3}{128}$

(2)

(b) Use algebraic integration to find $E(X)$

(3)

(c) Find $\text{Var}(X)$

(2)

(d) Show that the median, m, of X satisfies $m^3 - 48m + 64 = 0$

(2)

(e) Show that the median lies between 1.3 and 1.4

(2)

(Total for Question E2 is 11 marks)

STRETCH

E3 The length of time, X minutes, that a customer spends at a self-service checkout is modelled by a continuous random variable with cumulative distribution function given by

$$F(x) = \begin{cases} 0 & x < 0 \\ ax^2 + bx^3 & 0 \leqslant x \leqslant 2 \\ 1 & x > 2 \end{cases}$$

where a and b are constants. The mode of X is 1.2

(a) Show that $a = \frac{9}{16}$ and $b = -\frac{5}{32}$

(4)

(b) Find $P(X > 1.5)$

(2)

(c) Show that the lower quartile of X lies between 0.7 and 0.8

(3)

A random sample of ten customers using the self-service checkout is taken, and their times are independent.

(d) Find the probability that exactly 3 of these ten customers spend more than 1.5 minutes at the checkout.

(3)

(e) Find $\text{Var}(X)$

(2)

(Total for Question E3 is 14 marks)

Want more exam practice? The **Exam Questions by Topic** booklets have 10 more full exam questions on Statistics 2 Chapter 4, Continuous Random Variables, with a full mark scheme. **Open the booklet**

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