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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

Pure Mathematics 4

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA14 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	own PDF	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	own PDF	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	2	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	own PDF	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	own PDF	
6 Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	own PDF	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	own PDF	

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Page 1 of 27

CHAPTER 3 • KEY SKILLS

3

Coordinate Geometry in the (x, y) Plane

Specification P4: Parametric equations • 10 questions • 76 marks • about 91 minutes

1 Points from t

Substitute t to find a point. For a condition such as $y = 0$, solve for t first, then find x .

2 Eliminating t

Make t the subject of one equation and substitute into the other, e.g.
 $x(t+1) = 2t \Rightarrow t = \frac{x}{2-x}$. The answer has no t in it.

3 Pythagorean identities

$\sin^2 t + \cos^2 t = 1$, $1 + \tan^2 t = \sec^2 t$.
 $x = a + r \cos t$, $y = b + r \sin t$ is a circle.

4 Double angles and addition

$\sin 2t = 2 \sin t \cos t$, $\cos 2t = 1 - 2 \sin^2 t$.
 Expand $\cos\left(t + \frac{\pi}{6}\right)$ first, then eliminate t .

5 Exponentials and logs

$x = \ln(t+1) \Rightarrow t = e^x - 1$. Write the answer in the form asked for.

6 Domain and range

Domain of f = values of $x(t)$; range = values of $y(t)$. Check turning points inside the interval, not only the ends.

7 Curve sketching

Use key values of t . Two values of t giving the same point means the curve crosses itself.

8 Meeting a line

Substitute $x(t)$ and $y(t)$ into the line to get an equation in t . Reject values of t outside the domain.

9 Modelling

Say what t , x and y mean. Give answers in context with units and state a limitation of the model.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	5	11	9	10	10	11
1 Points from t			•		•	•		•	•	•
2 Eliminating t	•				•	•				
3 Pythagorean identities		•		•			•	•	•	•
4 Double angles and addition			•	•					•	•
5 Exponentials and logs					•					
6 Domain and range	•	•			•		•		•	
7 Curve sketching			•			•	•			•
8 Meeting a line	•					•	•			
9 Modelling								•		
Interleaved: trigonometric identities and R forms (P2, P3)		•	•	•			•	•	•	•
Interleaved: circles, quadratics and factorising (P1, P2)						•	•			
Interleaved: e, ln and algebraic division (P3)					•				•	
Interleaved: proof by contradiction (Chapter 1)									•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. A Cartesian equation must contain only x and y . Work in radians, keep the interval for t in mind for every domain, range and intersection, and find any constant the question asks for (e.g. state $k = 3$).



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1. The curve C has parametric equations

$$x = \frac{2t}{t+1} \quad y = \frac{t-3}{t+1} \quad t > -1$$

- (a) Show that all points on C lie on a straight line, and find the equation of this line in the form $y = mx + c$ (3)
- (b) Determine whether C meets the line with equation $x + y = 13$, giving a reason for your answer. (2)

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Question 1 continued

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(Total for Question 1 is 5 marks)



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2. The curve C has parametric equations

$$x = 2 + \tan t \quad y = 4 \sec^2 t - 3 \quad -\frac{\pi}{4} \leq t \leq \frac{\pi}{3}$$

The curve C has equation $y = f(x)$

- (a) Find $f(x)$ in the form $a(x+b)^2 + c$, where a , b and c are integers.

(2)

- (b) Find the domain and the range of f .

(3)

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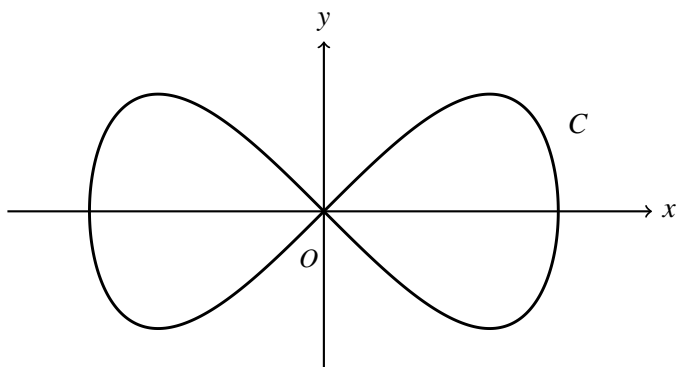


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 2 \sin t \quad y = \sin 2t \quad 0 \leq t < 2\pi$$

(a) Show that a Cartesian equation of C is $4y^2 = x^2(4 - x^2)$ (3)

(b) Find the coordinates of the points where C meets the x -axis. (2)



[illegible]

4. The curve C has parametric equations

$$x = 2 \sin t \quad y = 2 \cos \left(t + \frac{\pi}{6} \right) \quad 0 \leq t < 2\pi$$

Show that all points on C satisfy the equation

$$x^2 + xy + y^2 = k$$

where k is a constant to be found.

(5)

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5. The curve C has parametric equations

$$x = \ln(t + 1) \quad y = t^2 - 2t \quad t > -1$$

- (a) Show that a Cartesian equation of C is $y = e^{2x} + pe^x + q$, where p and q are integers to be found. (2)
- (b) Find the exact coordinates of the points where C meets the x -axis. (2)
- (c) State the range of values of y on C . (1)

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(Total for Question 5 is 5 marks)



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6. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

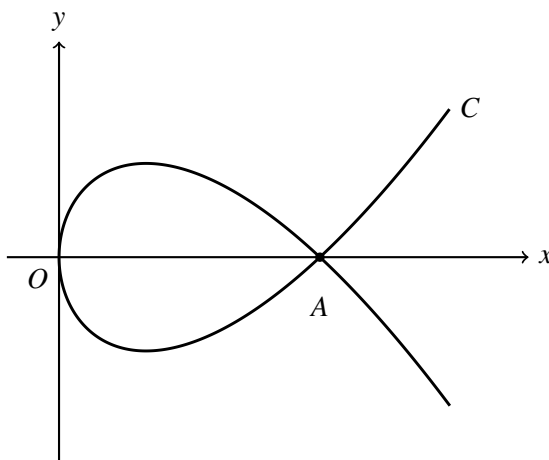


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = t^2 \quad y = t^3 - 3t \quad t \in \mathbb{R}$$

The curve meets the x -axis at the origin O and at the point A , where C crosses itself.

- (a) Find the coordinates of A and the two values of t at A . (3)

- (b) Show that a Cartesian equation of C is $y^2 = x(x - 3)^2$ (3)

The line with equation $y = 2x$ meets C at O and at two other points.

- (c) Find the coordinates of these two other points. (3)

- (d) Find the value of the constant m for which the line $y = mx$ meets C at exactly two distinct points, giving a reason for your answer. (2)



Question 6 continued

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Question 6 continued

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(Total for Question 6 is 11 marks)



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7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has parametric equations

$$x = 4 \cos t - 1 \quad y = 4 \sin t + 3 \quad 0 \leq t \leq \pi$$

(a) Show that C is part of a circle, stating the centre and the radius of the circle.

(3)

(b) Sketch C , showing the coordinates of its end points and of its highest point.

(2)

The line with equation $y = x + k$, where k is a constant, meets C at two distinct points.

(c) Find the set of possible values of k .

(4)

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Question 7 continued

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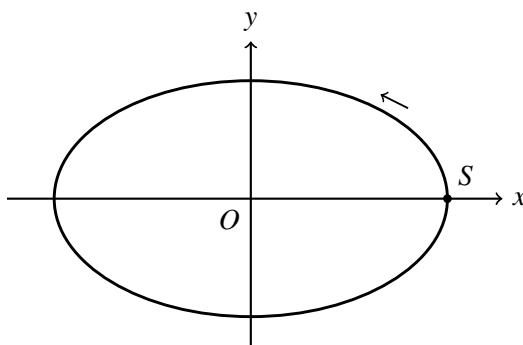


Figure 3

Figure 3 shows a model of a running track. The runner starts at the point S and runs anticlockwise. In the model, the position of the runner t minutes after the start is given by

$$x = 50 \cos t \quad y = 30 \sin t \quad 0 \leq t < 2\pi$$

where x and y are in metres and O is the centre of the track.

- (a) Find a Cartesian equation of the track. (2)
- (b) Show that the distance, d metres, of the runner from O satisfies $d^2 = 900 + 1600 \cos^2 t$ (2)
- (c) Find the four values of t at which the runner is exactly 40 m from O , giving your answers to 3 significant figures. (3)
- (d) Find, to the nearest second, the total time for which the runner is more than 40 m from O . (2)
- (e) Give one limitation of the model. (1)



Question 8 continued

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Solutions relying on calculator technology are not acceptable.

The curve C has parametric equations

$$x = \tan t \quad y = \cos 2t \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

- (a) Show that a Cartesian equation of C is $y = \frac{1-x^2}{1+x^2}$ (3)
- (b) Write y in the form $A + \frac{B}{1+x^2}$, where A and B are integers. (2)
- (c) Use proof by contradiction to prove that there is no point on C with $y \leq -1$ (3)
- (d) Find the exact coordinates of the points on C where $y = \frac{1}{2}$ (2)

Question 9 continued

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Question 9 continued

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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

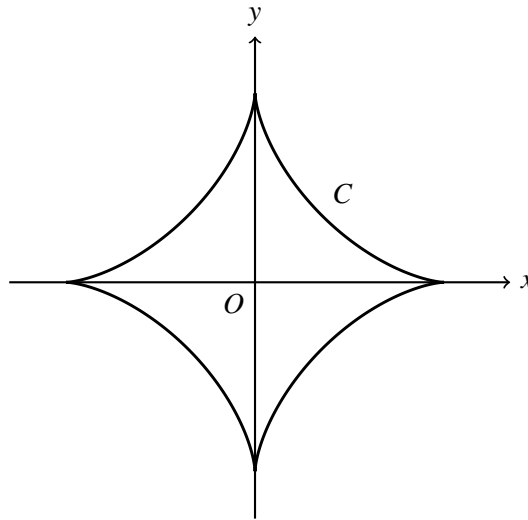


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = 2 \cos^3 t \quad y = 2 \sin^3 t \quad 0 \leq t < 2\pi$$

- (a) Show that all points on C satisfy the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = k$, stating the exact value of the constant k . (3)
- (b) Show that $x^2 + y^2 = 4 - 3 \sin^2 2t$ (4)
- (c) Hence find the least distance from O to a point on C , and the exact coordinates of the point in the first quadrant where it occurs. (3)
- (d) Write down the greatest distance from O to a point on C , and the coordinates of the points where it occurs. (1)



Question 10 continued

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(Total for Question 10 is 11 marks)**TOTAL FOR CHAPTER 3 IS 76 MARKS****Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk