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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

Pure Mathematics 4

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA14 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	2	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	27	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	53	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	79	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	105	
6 Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	131	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	157	

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1

CHAPTER 1 • KEY SKILLS

Proof

Specification P4: Proof by contradiction • 10 questions • 73 marks • about 88 minutes

1 The method

Assume the statement is false. Use correct algebra to reach something impossible. Finish: “this is a contradiction, so the statement is true”.

2 Writing the assumption

“For all $x \dots$ ” becomes “there is some x for which it is false”.
 “There are no integers. . .” becomes “assume there are integers. . .”.

3 Parity and divisibility

Even = $2k$, odd = $2k + 1$.
 $(2k + 1)^2 = 4k^2 + 4k + 1$ is odd, so if n^2 is even, n is even. Not a multiple of 3: $3k + 1$ or $3k + 2$.

4 Irrational numbers

Write $\sqrt{6} = \frac{a}{b}$ where a and b are integers with *no common factors*. Show a and b share a factor.

5 Integer equations

Factorise, e.g. $x^2 - y^2 = (x - y)(x + y)$, then use factor pairs or odd and even.
 2^a is even but 3^b is odd.

6 Inequalities

Assume it fails for some x . Multiply only by something you know is positive, then complete the square: $(x - 2)^2 < 0$ is impossible.

7 No real solutions

Assume the curves meet and eliminate y . A quadratic in x^2 with a negative discriminant has no real solutions.

8 Calculus and ranges

A stationary point means $\frac{dy}{dx} = 0$. Use $e^{kx} > 0$, $-1 \leq \cos x \leq 1$ and $\sin^2 x \leq 1$

9 Infinitely many primes

Assume p is the largest prime. $p! + 1$ is not divisible by $2, 3, \dots, p$, so it has a prime factor bigger than p .

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	6	10	10	8	9	10
1 The method	•	•	•	•	•	•	•	•	•	•
2 Writing the assumption	•	•	•	•			•	•	•	•
3 Parity and divisibility	•		•		•	•		•		
4 Irrational numbers	•				•	•				•
5 Integer equations			•		•					
6 Inequalities				•			•			
7 No real solutions		•							•	
8 Calculus and ranges		•					•			
9 Infinitely many primes								•		
Interleaved: deduction, exhaustion and counter-examples (P2)			•	•		•		•		•
Interleaved: logarithms and change of base (P2)					•					
Interleaved: quadratics, discriminant, completing the square (P1)				•					•	
Interleaved: differentiation, e and trigonometric identities (P3)		•					•			

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Start with the assumption in words (“Assume there is . . . for some x ”, never “for all x ”), and include “no common factors” for an irrational number. Use algebra, not a calculator or a sketch, and end with the contradiction and a conclusion.



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1. A student writes the following attempt to prove that $\sqrt{6}$ is irrational.

Line 1 Assume that $\sqrt{6} = \frac{a}{b}$, where a and b are integers.

Line 2 Then $a^2 = 6b^2$, so a^2 is even and therefore a is even.

Line 3 Let $a = 2m$. Then $4m^2 = 6b^2$, so $2m^2 = 3b^2$

Line 4 Therefore b is even.

Line 5 So a and b are both even. This is a contradiction, so $\sqrt{6}$ is irrational.

- (a) State what is missing from Line 1 and explain why the proof needs it. (2)
- (b) Explain why “ a^2 is even and therefore a is even” in Line 2 is true. (1)
- (c) Line 4 does not follow from Line 3 as it is written. Write out the missing steps. (2)



Question 1 continued

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(Total for Question 1 is 5 marks)



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2. The curve C has equation

$$y = e^{3x} + 6x - 2\sin 3x \quad x \in \mathbb{R}$$

Use proof by contradiction to prove that C has no stationary points.

(5)

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Question 2 continued

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(Total for Question 2 is 5 marks)



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3. (a) Prove that, for all integers x and y , the numbers $x + y$ and $x - y$ are either both even or both odd.

(2)

- (b) Hence use proof by contradiction to prove that there are no positive integers x and y such that

$$x^2 - y^2 = 18$$

(3)

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Question 3 continued

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(Total for Question 3 is 5 marks)



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4. (a) Prove by contradiction that, for all real numbers x with $x > 1$,

$$\frac{x^2}{x-1} \geq 4$$

(4)

- (b) Show that the result in part (a) is not true for all real numbers x with $x \neq 1$

(1)

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Question 4 continued

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(Total for Question 4 is 5 marks)



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5. (a) Use proof by contradiction to prove that $\log_2 3$ is irrational.

(4)

(b) Hence prove that $\log_8 9$ is irrational.

(2)

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Question 5 continued

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(Total for Question 5 is 6 marks)



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- (b)** Hence use proof by contradiction to prove that $\sqrt[3]{3}$ is irrational. **(5)**

A student tries to use the same method to prove that $\sqrt[3]{8}$ is irrational, starting with $\sqrt[3]{8} = \frac{a}{b}$, where a and b are integers with no common factors, and deducing that a is even.

- (c) Explain why the method does not lead to a contradiction for $\sqrt[3]{8}$ (2)

Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)



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7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Show that

$$\sec^2 x + \operatorname{cosec}^2 x \equiv 4 \operatorname{cosec}^2 2x \quad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(3)

(b) Hence use proof by contradiction to prove that

$$\sec^2 x + \operatorname{cosec}^2 x \geq 4$$

for all values of x for which it is defined.

(3)

(c) Solve, for $0 < x < \pi$,

$$\sec^2 x + \operatorname{cosec}^2 x = \frac{16}{3}$$

giving your answers as exact multiples of π .

(4)



Question 7 continued

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Question 7 continued

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(Total for Question 7 is 10 marks)



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8. In this question n is an integer with $n \geq 2$, and $n! = 1 \times 2 \times 3 \times \dots \times n$

(a) Prove that none of the integers $2, 3, \dots, n$ is a factor of $n! + 1$

(2)

(b) Hence use proof by contradiction to prove that there are infinitely many prime numbers.

(4)

A student says “ $n! + 1$ is always a prime number”.

(c) Show that the student is wrong.

(2)

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Question 8 continued

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(Total for Question 8 is 8 marks)



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9. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

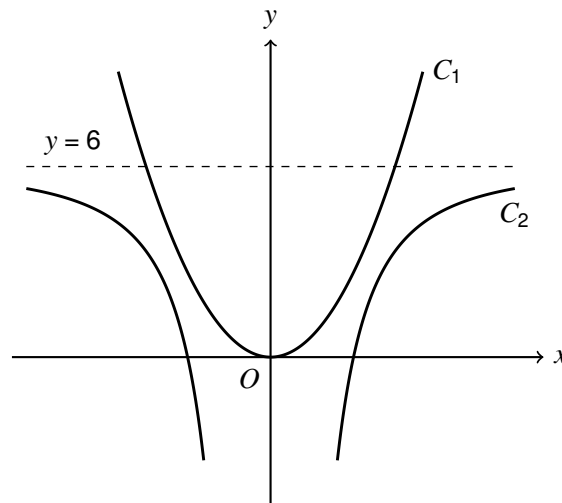


Figure 1

Figure 1 shows a sketch of the curves C_1 and C_2 with equations

$$C_1: y = 2x^2 \qquad C_2: y = 6 - \frac{8}{x^2} \quad x \neq 0$$

The curves appear not to meet.

- (a) Use algebra and proof by contradiction to prove that C_1 and C_2 do not meet.

(4)

The equation of C_2 is changed to $y = k - \frac{8}{x^2}$, $x \neq 0$, where k is a constant.

- (b) Find the set of values of k for which C_1 and the new curve meet.

(3)

- (c) In the case $k = 10$, find the coordinates of all the points where C_1 and the new curve meet.

(2)



Question 9 continued

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(Total for Question 9 is 9 marks)



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10. (a) Prove by contradiction that the sum of a rational number and an irrational number is irrational.

(3)

- (b) Use proof by contradiction to prove that $\sqrt{2} + \sqrt{3}$ is irrational.

You may assume that $\sqrt{6}$ is irrational.

(4)

- (c) Give an example to show that the sum of two irrational numbers can be rational.

(1)

- (d) Hence, or otherwise, prove that $\sqrt{3} - \sqrt{2}$ is irrational.

(2)

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TOTAL FOR CHAPTER 1 IS 73 MARKS

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CHAPTER 2 • KEY SKILLS

2

Partial Fractions

Specification P4: Algebra and functions • 10 questions • 78 marks • about 94 minutes

1 The identity

$$\frac{5x+1}{(x-1)(x+2)} \equiv \frac{A}{x-1} + \frac{B}{x+2}$$

Multiply by the whole denominator:

$$5x+1 \equiv A(x+2) + B(x-1)$$

2 Finding constants

Substitute the value of x that makes each factor zero. Compare coefficients (e.g. of x^2) for any constant left over.

3 Factorise first

Use the factor theorem on a cubic, or $x^2 - 4 = (x-2)(x+2)$.

$(x^2 - 4)(x+2) = (x-2)(x+2)^2$ has a *repeated* factor.

4 Repeated factors

$(x-a)^2$ needs two fractions:

$$\frac{A}{x-b} + \frac{B}{x-a} + \frac{C}{(x-a)^2}$$

5 Improper fractions

Degree of numerator $\geq$ degree of denominator: divide first, or start the identity with A (or $Ax+B$).

6 Unknown coefficients

If some constants are given, compare coefficients: the x^2 terms give $A+B+C=1$ when the numerator is $x^2 + \dots$

7 Differentiating

Differentiate term by term:

$$k(ax+b)^{-1} \rightarrow -ka(ax+b)^{-2} \text{ and}$$

$$k(ax+b)^{-2} \rightarrow -2ka(ax+b)^{-3}$$

8 Series

Split each term, write out the first and last few terms of the sum, and cancel:

$$\frac{1}{r} - \frac{1}{r+1} \text{ telescopes.}$$

9 Graphs

$A + \frac{B}{x-a} + \dots$ has asymptotes $x=a$

and $y=A$. With $u = \frac{1}{x-a}$ a repeated factor gives a quadratic in u .

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	6	5	5	5	10	10	10	11	11
1 The identity	•			•			•		•	
2 Finding constants	•	•	•	•	•	•	•	•	•	•
3 Factorise first	•				•	•				
4 Repeated factors		•	•		•					•
5 Improper fractions			•			•		•		•
6 Unknown coefficients		•	•	•						
7 Differentiating		•				•			•	
8 Series							•			
9 Graphs								•		•
Interleaved: factor theorem and sigma notation (P2)	•						•			
Interleaved: inequalities and completing the square (P1)								•		•
Interleaved: differentiation, e and ln, number of solutions (P3)		•				•			•	•
Interleaved: proof by contradiction (Chapter 1)								•		

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write the identity with the whole denominator, and give the final answer as the full expression, not just A , B and C . Watch the signs: write $-\frac{3}{x+3}$, not $+\frac{-3}{x+3}$. Substituting values of x is quicker than simultaneous equations.



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1.

$$f(x) = \frac{4x^2 + 3x - 25}{x^3 - 2x^2 - 5x + 6}$$

- (a) Show that $(x - 1)$ is a factor of $x^3 - 2x^2 - 5x + 6$, and hence factorise $x^3 - 2x^2 - 5x + 6$ completely.

(2)

- (b) Hence express $f(x)$ in partial fractions.

(3)

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2.

$$f(t) = \frac{15t^2 - 10t + 7}{(t+1)(3t-1)^2} \quad t \neq -1, t \neq \frac{1}{3}$$

(a) Given that

$$f(t) \equiv \frac{A}{t+1} + \frac{B}{3t-1} + \frac{C}{(3t-1)^2}$$

find the values of the constants A , B and C .

(4)

(b) Hence find the value of $f'(0)$

(2)

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3.

$$g(x) = \frac{2x^3 - 2x^2 + x + 3}{x(x-1)^2} \quad x \neq 0, x \neq 1$$

(a) Show that $g(x) = 2 + \frac{2x^2 - x + 3}{x(x-1)^2}$ (2)

(b) Hence express $g(x)$ in partial fractions. (3)

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Question 3 continued

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4. Given that

$$\frac{x^2 + px + q}{(x+1)(x-2)(x+4)} \equiv \frac{2}{x+1} + \frac{B}{x-2} + \frac{3}{x+4}$$

where p , q and B are constants, find the value of B , the value of p and the value of q .

(5)

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Question 4 continued

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5.

$$h(x) = \frac{3x^2 + x + 2}{(x^2 - 4)(x + 2)} \quad x \neq \pm 2$$

Express $h(x)$ in partial fractions.

(5)

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$$g(x) = \frac{2x^3 + 7x^2 + 3x + 9}{x(x+3)} \quad x > 0$$

$$g(x) \equiv Ax + B + \frac{C}{x} + \frac{D}{x+3}$$

(2)

(3)

(1)

Question 6 continued

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Question 6 continued

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(3)

$$\sum_{r=1}^n \frac{2}{r(r+1)(r+2)} = \frac{1}{2} - \frac{1}{(n+1)(n+2)}$$

(4)

(2)

(d) Explain why $\sum_{r=1}^n \frac{2}{r(r+1)(r+2)} < \frac{1}{2}$ for every positive integer n .

(1)

Question 7 continued

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8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

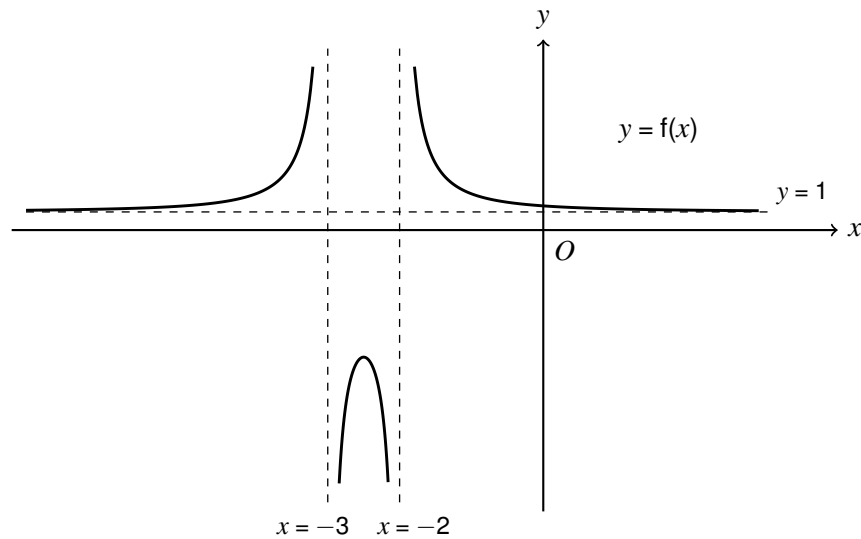


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \frac{x^2 + 5x + 8}{(x+2)(x+3)} \quad x \neq -2, x \neq -3$$

The curve has asymptotes $x = -3$, $x = -2$ and $y = 1$

(a) Express $f(x)$ in the form $A + \frac{B}{x+2} + \frac{C}{x+3}$, where A , B and C are constants.

(4)

(b) Use proof by contradiction to prove that there is no real value of x for which $f(x) = 1$

(3)

(c) Using algebra, find the set of values of x for which $f(x) > 3$, giving your answer in exact form.

(3)



Question 8 continued

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9. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{6e^x}{(e^x + 1)(e^x + 3)} \quad x \in \mathbb{R}$$

- (a) Express $\frac{6u}{(u+1)(u+3)}$ in partial fractions.

(3)

- (b) Hence, or otherwise, show that

$$f'(x) = \frac{3e^x}{(e^x + 1)^2} - \frac{9e^x}{(e^x + 3)^2}$$

(3)

- (c) Hence find the exact x coordinate of the stationary point of the curve $y = f(x)$, giving your answer in the form $a \ln b$, where a and b are rational.

(3)

- (d) Find the exact maximum value of $f(x)$, giving your answer in the form $p + q\sqrt{3}$, where p and q are integers.

(2)



Question 9 continued

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Question 9 continued

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(Total for Question 9 is 11 marks)



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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

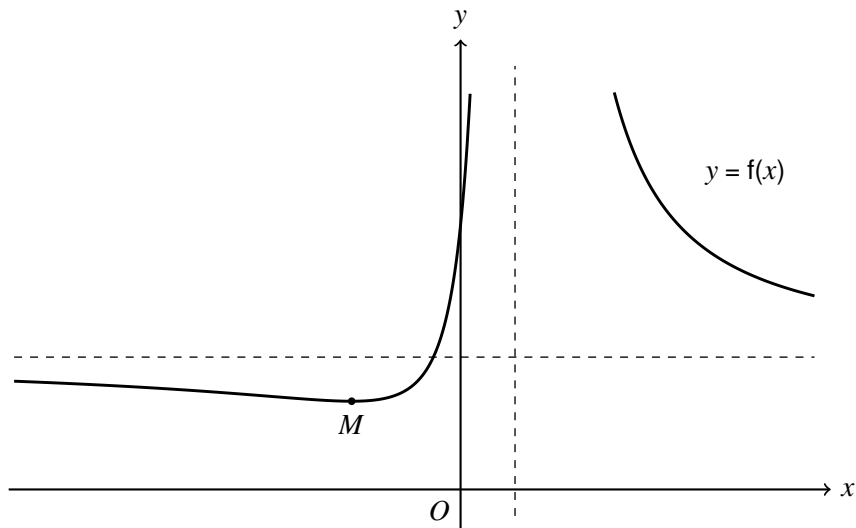


Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$, where

$$f(x) = \frac{x^2 + 2}{(x - 1)^2} \quad x \neq 1$$

The curve has asymptotes $x = 1$ and $y = 1$, and a minimum point M .

- (a) Express $f(x)$ in the form $A + \frac{B}{x-1} + \frac{C}{(x-1)^2}$, where A , B and C are constants. (4)
- (b) Using $u = \frac{1}{x-1}$, or otherwise, find the exact coordinates of M . (3)
- (c) Show that the equation $f(x) = 1$ has exactly one solution, and find it. (2)
- (d) Find the set of values of the constant k for which the equation $f(x) = k$ has exactly two solutions. (2)



Question 10 continued

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Question 10 continued

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(Total for Question 10 is 11 marks)

TOTAL FOR CHAPTER 2 IS 78 MARKS

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CHAPTER 3 • KEY SKILLS

3

Coordinate Geometry in the (x, y) Plane

Specification P4: Parametric equations • 10 questions • 76 marks • about 91 minutes

1 Points from t

Substitute t to find a point. For a condition such as $y = 0$, solve for t first, then find x .

2 Eliminating t

Make t the subject of one equation and substitute into the other, e.g.
 $x(t+1) = 2t \Rightarrow t = \frac{x}{2-x}$. The answer has no t in it.

3 Pythagorean identities

$\sin^2 t + \cos^2 t = 1$, $1 + \tan^2 t = \sec^2 t$.
 $x = a + r \cos t$, $y = b + r \sin t$ is a circle.

4 Double angles and addition

$\sin 2t = 2 \sin t \cos t$, $\cos 2t = 1 - 2 \sin^2 t$.
 Expand $\cos\left(t + \frac{\pi}{6}\right)$ first, then eliminate t .

5 Exponentials and logs

$x = \ln(t+1) \Rightarrow t = e^x - 1$. Write the answer in the form asked for.

6 Domain and range

Domain of f = values of $x(t)$; range = values of $y(t)$. Check turning points inside the interval, not only the ends.

7 Curve sketching

Use key values of t . Two values of t giving the same point means the curve crosses itself.

8 Meeting a line

Substitute $x(t)$ and $y(t)$ into the line to get an equation in t . Reject values of t outside the domain.

9 Modelling

Say what t , x and y mean. Give answers in context with units and state a limitation of the model.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	5	11	9	10	10	11
1 Points from t			•		•	•		•	•	•
2 Eliminating t	•				•	•				
3 Pythagorean identities		•		•			•	•	•	•
4 Double angles and addition			•	•					•	•
5 Exponentials and logs					•					
6 Domain and range	•	•			•		•		•	
7 Curve sketching			•			•	•			•
8 Meeting a line	•					•	•			
9 Modelling								•		
Interleaved: trigonometric identities and R forms (P2, P3)		•	•	•			•	•	•	•
Interleaved: circles, quadratics and factorising (P1, P2)						•	•			
Interleaved: e, ln and algebraic division (P3)					•				•	
Interleaved: proof by contradiction (Chapter 1)									•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. A Cartesian equation must contain only x and y . Work in radians, keep the interval for t in mind for every domain, range and intersection, and find any constant the question asks for (e.g. state $k = 3$).



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1. The curve C has parametric equations

$$x = \frac{2t}{t+1} \quad y = \frac{t-3}{t+1} \quad t > -1$$

- (a) Show that all points on C lie on a straight line, and find the equation of this line in the form $y = mx + c$ (3)
- (b) Determine whether C meets the line with equation $x + y = 13$, giving a reason for your answer. (2)

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2. The curve C has parametric equations

$$x = 2 + \tan t \quad y = 4 \sec^2 t - 3 \quad -\frac{\pi}{4} \leq t \leq \frac{\pi}{3}$$

The curve C has equation $y = f(x)$

- (a) Find $f(x)$ in the form $a(x+b)^2 + c$, where a , b and c are integers.

(2)

- (b) Find the domain and the range of f .

(3)

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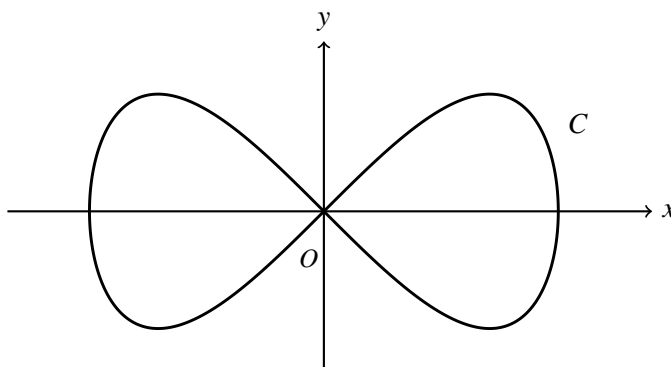


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 2 \sin t \quad y = \sin 2t \quad 0 \leq t < 2\pi$$

(a) Show that a Cartesian equation of C is $4y^2 = x^2(4 - x^2)$ (3)

(b) Find the coordinates of the points where C meets the x -axis. (2)



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Question 3 continued

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4. The curve C has parametric equations

$$x = 2 \sin t \quad y = 2 \cos \left(t + \frac{\pi}{6} \right) \quad 0 \leq t < 2\pi$$

Show that all points on C satisfy the equation

$$x^2 + xy + y^2 = k$$

where k is a constant to be found.

(5)

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5. The curve C has parametric equations

$$x = \ln(t + 1) \quad y = t^2 - 2t \quad t > -1$$

- (a) Show that a Cartesian equation of C is $y = e^{2x} + pe^x + q$, where p and q are integers to be found. (2)
- (b) Find the exact coordinates of the points where C meets the x -axis. (2)
- (c) State the range of values of y on C . (1)

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6. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

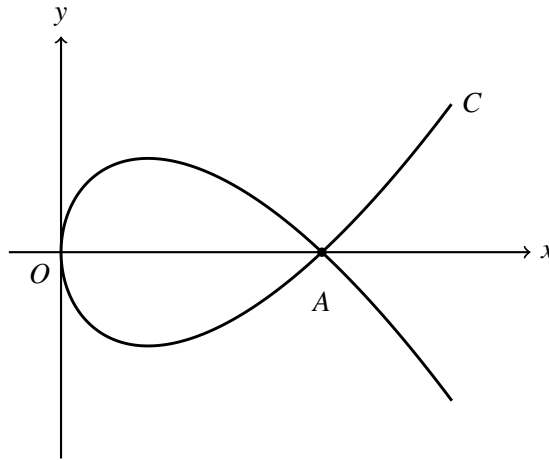


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = t^2 \quad y = t^3 - 3t \quad t \in \mathbb{R}$$

The curve meets the x -axis at the origin O and at the point A , where C crosses itself.

- (a) Find the coordinates of A and the two values of t at A . (3)

- (b) Show that a Cartesian equation of C is $y^2 = x(x - 3)^2$ (3)

The line with equation $y = 2x$ meets C at O and at two other points.

- (c) Find the coordinates of these two other points. (3)

- (d) Find the value of the constant m for which the line $y = mx$ meets C at exactly two distinct points, giving a reason for your answer. (2)



Question 6 continued

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Question 6 continued

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Solutions relying on calculator technology are not acceptable.

The curve C has parametric equations

$$x = 4 \cos t - 1 \quad y = 4 \sin t + 3 \quad 0 \leq t \leq \pi$$

- (a)** Show that C is part of a circle, stating the centre and the radius of the circle. **(3)**

- (b)** Sketch C , showing the coordinates of its end points and of its highest point. **(2)**

The line with equation $y = x + k$, where k is a constant, meets C at two distinct points.

- (c) Find the set of possible values of k . (4)

Question 7 continued

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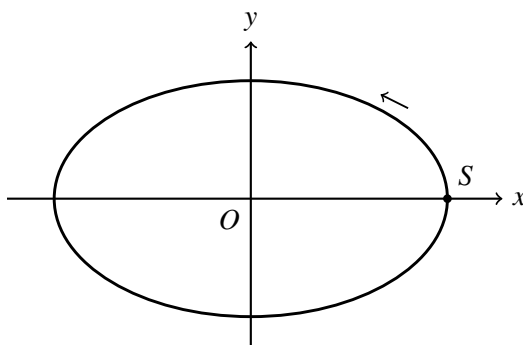
**Figure 3**

Figure 3 shows a model of a running track. The runner starts at the point S and runs anticlockwise. In the model, the position of the runner t minutes after the start is given by

$$x = 50 \cos t \quad y = 30 \sin t \quad 0 \leq t < 2\pi$$

where x and y are in metres and O is the centre of the track.

- (a) Find a Cartesian equation of the track. (2)
- (b) Show that the distance, d metres, of the runner from O satisfies $d^2 = 900 + 1600 \cos^2 t$ (2)
- (c) Find the four values of t at which the runner is exactly 40 m from O , giving your answers to 3 significant figures. (3)
- (d) Find, to the nearest second, the total time for which the runner is more than 40 m from O . (2)
- (e) Give one limitation of the model. (1)



Question 8 continued

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Question 8 continued

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Solutions relying on calculator technology are not acceptable.

The curve C has parametric equations

$$x = \tan t \quad y = \cos 2t \quad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

- (a) Show that a Cartesian equation of C is $y = \frac{1-x^2}{1+x^2}$ (3)
- (b) Write y in the form $A + \frac{B}{1+x^2}$, where A and B are integers. (2)
- (c) Use proof by contradiction to prove that there is no point on C with $y \leq -1$ (3)
- (d) Find the exact coordinates of the points on C where $y = \frac{1}{2}$ (2)

Question 9 continued

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Question 9 continued

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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

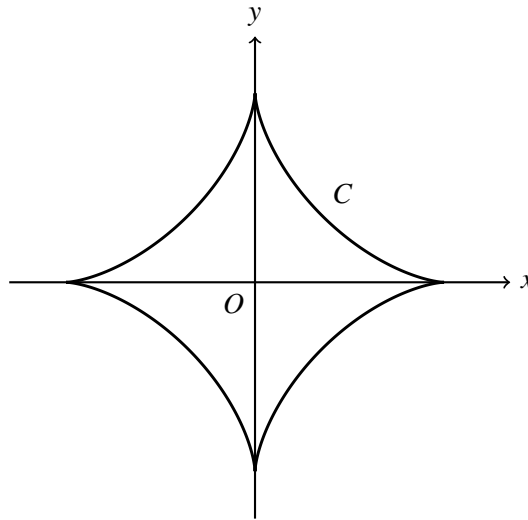


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = 2\cos^3 t \quad y = 2\sin^3 t \quad 0 \leq t < 2\pi$$

- (a) Show that all points on C satisfy the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = k$, stating the exact value of the constant k . (3)
- (b) Show that $x^2 + y^2 = 4 - 3\sin^2 2t$ (4)
- (c) Hence find the least distance from O to a point on C , and the exact coordinates of the point in the first quadrant where it occurs. (3)
- (d) Write down the greatest distance from O to a point on C , and the coordinates of the points where it occurs. (1)



Question 10 continued

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Question 10 continued

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TOTAL FOR CHAPTER 3 IS 76 MARKS

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4

CHAPTER 4 • KEY SKILLS

Binomial Expansion

Specification P4: Sequences and series • 10 questions • 81 marks • about 97 minutes

1 The expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots \text{ for any rational } n, \text{ valid for } |x| < 1$$

2 Replacing x

In $(1-4x)^{-3}$ use $(-4x)$ in every term, with brackets: $(-4x)^2 = 16x^2$ and $(-4x)^3 = -64x^3$

3 Taking out a^n

$$(9+4x)^{-\frac{1}{2}} = 9^{-\frac{1}{2}} \left(1 + \frac{4}{9}x\right)^{-\frac{1}{2}} = \frac{1}{3} \left(1 + \frac{4}{9}x\right)^{-\frac{1}{2}}$$

Not $9 \left(1 + \frac{4}{9}x\right)^{-\frac{1}{2}}$

4 Validity

$(a+bx)^n$ is valid for $|x| < \left|\frac{a}{b}\right|$. Use a strict inequality, no $\pm$. For two series, take the narrower interval.

5 Unknown constants

Write each coefficient in terms of the unknowns and compare, e.g. $nk = -2$ and $\frac{n(n-1)}{2}k^2 = 6$

6 Products

$(a+bx)(1+px+qx^2)$: collect each power of x carefully. Only keep the terms asked for.

7 Partial fractions first

Split, then expand each part:

$$\frac{3}{x-2} = -\frac{3}{2} \left(1 - \frac{x}{2}\right)^{-1}$$

8 Approximations

Choose x inside the valid interval, find what the original expression equals (it may be a reciprocal), then substitute into the series.

9 Accuracy

Percentage error

$$= \frac{|\text{approx} - \text{exact}|}{\text{exact}} \times 100. \text{ The smaller } |x|, \text{ the better the approximation.}$$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	6	6	6	6	11	11	10	10	10
1 The expansion	•	•	•	•	•	•	•	•	•	•
2 Replacing x	•	•		•	•	•				•
3 Taking out a^n	•			•	•			•	•	
4 Validity	•		•	•		•	•	•	•	•
5 Unknown constants		•	•					•		
6 Products		•				•				
7 Partial fractions first				•			•			
8 Approximations					•	•		•	•	
9 Accuracy						•			•	•
Interleaved: partial fractions (Chapter 2)				•			•			
Interleaved: surds, indices and simultaneous equations (P1)			•		•	•		•		
Interleaved: circles: tangent and radius (P2)									•	
Interleaved: proof by contradiction, completing the square (Chapter 1, P1)										•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Show the unsimplified binomial coefficients, then simplify each term. Take out a^n , not a . When approximating, check what the original expression is equal to at your value of x , and give a fraction when a rational answer is asked for.



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1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\frac{1}{\sqrt{9+4x}}$$

giving each coefficient as a fraction in its simplest form.

(4)

- (b) State the range of values of x for which this expansion is valid.

(1)

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2. (a) Find the binomial expansion of $(1 - 6x)^{\frac{1}{3}}$ in ascending powers of x , up to and including the term in x^2 , simplifying each term. (3)

Given that

$$(a + bx)(1 - 6x)^{\frac{1}{3}} = 3 - 7x + cx^2 + \dots$$

where a , b and c are constants,

- (b) find the values of a , b and c . (3)

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3. In the binomial expansion of $(1 + kx)^n$, where k and n are constants and n is not a positive integer, the coefficient of x is -2 and the coefficient of x^2 is 6

(a) Find the value of k and the value of n .

(4)

(b) Find the coefficient of x^3

(1)

(c) State the range of values of x for which the expansion is valid.

(1)

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(Total for Question 3 is 6 marks)



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4.

$$f(x) = \frac{x+7}{(1+x)(x-2)}$$

- (a) Express $f(x)$ in partial fractions. (2)
- (b) Hence find the series expansion of $f(x)$, in ascending powers of x , up to and including the term in x^2 . Give each coefficient as a simplified fraction. (3)
- (c) State the range of values of x for which your expansion is valid. (1)

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5. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of $(4 - x)^{-\frac{1}{2}}$, giving each coefficient in its simplest form. (4)
- (b) By substituting $x = 1$ into your expansion, find an approximation to $\sqrt{3}$, giving your answer as a fraction. (2)

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 11 marks)



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7.

$$f(x) = \frac{6x^2 + 3}{(1+x)(1-2x)^2} \quad |x| < \frac{1}{2}$$

- (a) Find the values of the constants A , B and C such that

$$f(x) \equiv \frac{A}{1+x} + \frac{B}{1-2x} + \frac{C}{(1-2x)^2}$$

(4)

- (b) Hence find the series expansion of $f(x)$, in ascending powers of x , up to and including the term in x^2

(4)

- (c) Explain why the expansion is valid only for $|x| < \frac{1}{2}$

(1)

- (d) Show that the coefficient of x^n in the expansion of $f(x)$ is $(-1)^n + (3n+2)2^n$

(2)



Question 7 continued

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Question 7 continued

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8. The binomial expansion of

$$(2 - kx)^{-3} \quad |kx| < 2$$

where k is a positive constant, may be written as $A + Bx + Cx^2 + Dx^3 + \dots$

- (a) Find the value of A .

(1)

Given that $C = 2B$,

- (b) show that $k^2 - 2k = 0$

(3)

- (c) Hence find the value of k and the value of D .

(3)

- (d) State the range of values of x for which the expansion is valid.

(1)

- (e) Use the expansion, with a suitable value of x , to find an approximation to $\frac{1}{1.9^3}$, giving your answer to 4 decimal places.

(2)



Question 8 continued

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Question 8 continued

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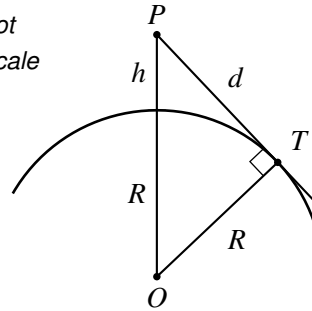
Diagram not
drawn to scale**Figure 1**

Figure 1 shows the Earth, modelled as a sphere with centre O and radius R km. An observer at P is h km above the surface. The line PT touches the surface at T , so the distance to the horizon is $d = PT$ km.

(a) Show that $d = \sqrt{2Rh} \left(1 + \frac{h}{2R} \right)^{\frac{1}{2}}$ (2)

(b) Hence show that, for small values of h ,

$$d \approx \sqrt{2Rh} \left(1 + \frac{h}{4R} - \frac{h^2}{32R^2} \right)$$
(3)

(c) State, in terms of R , the values of h for which the expansion is valid. (1)

A simpler model uses $d \approx \sqrt{2Rh}$. Take $R = 6400$

(d) Find the percentage error in the simpler model for an observer at a height of 400 km, giving your answer to 2 significant figures. (3)

(e) Explain why the simpler model always underestimates d . (1)



Question 9 continued

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Question 9 continued

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10. (a) Find the binomial expansion of $(1 + 2x)^{\frac{3}{2}}$ in ascending powers of x , up to and including the term in x^3 , simplifying each term. (4)
- (b) State the range of values of x for which this expansion is valid. (1)

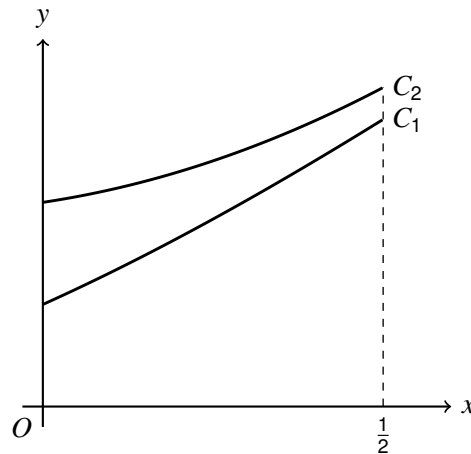


Figure 2

Figure 2 shows the curves C_1 and C_2 , for $0 \leq x \leq \frac{1}{2}$, with equations

$$C_1: y = 1 + 3x + \frac{3}{2}x^2 - \frac{1}{2}x^3 \quad C_2: y = 2 + x + \frac{5}{2}x^2$$

The curve C_1 is the approximation to $y = (1 + 2x)^{\frac{3}{2}}$ given by part (a).

- (c) Use algebra and proof by contradiction to prove that C_1 and C_2 do not meet for $0 < x < \frac{1}{2}$. (4)
- (d) Explain why part (c) does not prove that the curve with equation $y = (1 + 2x)^{\frac{3}{2}}$ does not meet C_2 for $0 < x < \frac{1}{2}$. (1)



Question 10 continued

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 4 IS 81 MARKS

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5

CHAPTER 5 • KEY SKILLS

Differentiation

Specification P4: Differentiation • 10 questions • 81 marks • about 97 minutes

1 Parametric

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. Use the value of t at the point, not x .

2 Tangents and normals

Normal gradient = $-\frac{1}{m}$. Give the form asked for: $ax + by + c = 0$ needs integers and “= 0”.

3 Implicit

$y^2 \rightarrow 2y \frac{dy}{dx}$; $xy \rightarrow y + x \frac{dy}{dx}$ (product rule); a constant $\rightarrow 0$. Collect the $\frac{dy}{dx}$ terms.

4 Exponentials a^x

$\frac{d}{dx}(a^x) = a^x \ln a$, e.g. $2^x \rightarrow 2^x \ln 2$ (since $2^x = e^{x \ln 2}$)

5 Taking logarithms

For $y = x^{\cos x}$: $\ln y = \cos x \ln x$, then $\frac{1}{y} \frac{dy}{dx} = \dots$ and multiply by y .

6 Stationary and vertical

$\frac{dy}{dx} = 0$: numerator = 0, then use the curve's equation. Vertical tangent: denominator = 0.

7 Connected rates

$\frac{dS}{dt} = \frac{dS}{dr} \times \frac{dr}{dt}$. Quote the formula you use first.

8 Forming equations

Net rate = rate in – rate out.
“Proportional to” needs a constant k .
Similar triangles link r and h .

9 Interpreting

A negative rate means decreasing. A level stops changing when the rate is 0. Give units.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	5	7	6	10	10	10	10	11
1 Parametric		•				•			•	
2 Tangents and normals	•	•			•	•			•	
3 Implicit	•			•	•		•			•
4 Exponentials a^x					•					
5 Taking logarithms										•
6 Stationary and vertical				•			•			•
7 Connected rates			•					•		
8 Forming equations								•		
9 Interpreting			•					•		•
Interleaved: trigonometric identities, double angles (P3)		•							•	
Interleaved: e, ln and numerical methods (P3)					•					•
Interleaved: quadratics, discriminant, coordinate geometry (P1)	•			•		•	•			
Interleaved: proof and parametric curves (Chapters 1, 3)						•	•		•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. In implicit work every y term gets a $\frac{dy}{dx}$; do not write “ $\frac{dy}{dx} =$ ” at the start of the line. Check tangent or normal, and the form of the answer. For rates, write the chain rule in full before substituting.



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1. The curve C has equation

$$2x^2 + xy - y^3 = 9$$

The point $P(2, 1)$ lies on C .

Find an equation of the normal to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(6)

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Question 1 continued

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2. The curve C has parametric equations

$$x = 4 \sin^2 t \quad y = 3 \tan t \quad 0 < t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t , giving your answer in its simplest form.

(3)

(b) Find an equation of the tangent to C at the point where $t = \frac{\pi}{6}$, giving your answer in the form $y = mx + c$, where m and c are exact constants.

(3)

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Question 2 continued

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3. A spherical balloon is being inflated. At time t seconds the radius of the balloon is r cm, its volume is V cm³ and its surface area is S cm², where

$$V = \frac{4}{3}\pi r^3 \quad S = 4\pi r^2$$

The volume of the balloon increases at a constant rate of $50 \text{ cm}^3 \text{ s}^{-1}$.

Find the rate at which the surface area of the balloon is increasing at the instant when $r = 5$

(5)

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Question 3 continued

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(Total for Question 3 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

4. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**The curve C has equation

$$x^3 - 3xy + y^2 = k$$

where k is a constant.

- (a) Find $\frac{dy}{dx}$ in terms of x and y .

(4)The point $P(p, 4)$, where $p > 0$, is a stationary point of C .

- (b) Find the value of p and the value of k .

(3)

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Question 4 continued

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(Total for Question 4 is 7 marks)



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5. The curve C has equation

$$2^x + xy = y^2 \quad y > 0$$

- (a) Show that $\frac{dy}{dx} = \frac{2^x \ln 2 + y}{2y - x}$

(4)

The point $P(0, 1)$ lies on C . The tangent to C at P meets the x -axis at the point Q .

- (b) Find the exact x coordinate of Q .

(2)

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Question 5 continued

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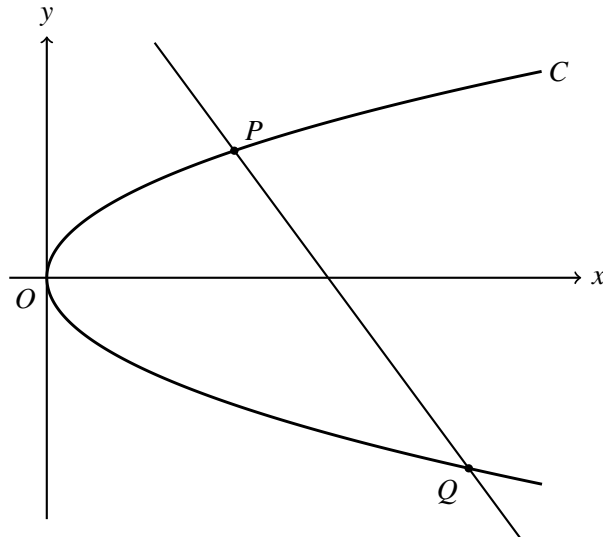


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = t^2 \quad y = 2t \quad t \in \mathbb{R}$$

The point P on C has parameter $t = 2$. The normal to C at P meets C again at the point Q .

- (a) Find $\frac{dy}{dx}$ in terms of t . (2)
- (b) Show that the normal to C at P has equation $y = 12 - 2x$ (3)
- (c) Use algebra to find the coordinates of Q . (3)
- (d) Find the area of triangle OPQ , where O is the origin. (2)



Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)



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7. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

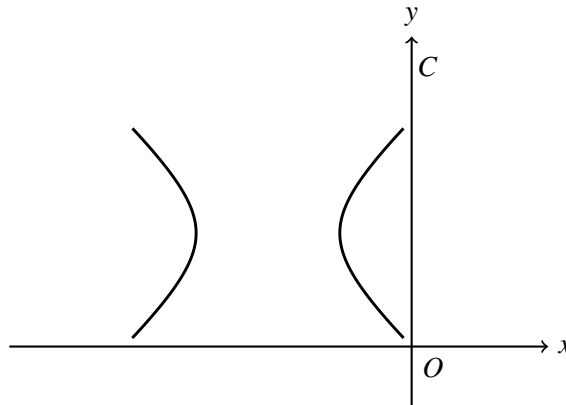
**Figure 2**

Figure 2 shows a sketch of the curve C with equation

$$y^2 - 6y = 3x^2 + 12x$$

The curve is not defined for values of x in the interval $p < x < q$

- (a) Find $\frac{dy}{dx}$ in terms of x and y . (3)
- (b) Using your answer to part (a), or otherwise, find the value of p and the value of q . (4)
- (c) Use proof by contradiction to prove that C has no stationary points. (3)



Question 7 continued

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Question 7 continued

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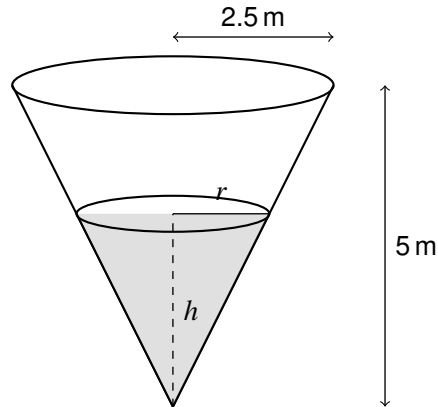
**Figure 3**

Figure 3 shows a container in the shape of an inverted right circular cone, with height 5 m and top radius 2.5 m. At time t minutes, the depth of water in the container is h metres, the radius of the water surface is r metres and the volume of water is $V \text{ m}^3$

- (a) Show that $V = \frac{\pi h^3}{12}$ (2)

Water flows into the container at a constant rate of $2\pi \text{ m}^3$ per minute and leaks out at a rate of $\frac{1}{2}\pi h \text{ m}^3$ per minute.

- (b) Show that $\frac{dh}{dt} = \frac{8-2h}{h^2}$ (4)

- (c) Find the rate at which the depth of water is increasing when $h = 2$ (1)

- (d) Find the depth at which, according to the model, the water level would stop rising. (2)

- (e) Explain whether, according to the model, the container will overflow. (1)



Question 8 continued

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9. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

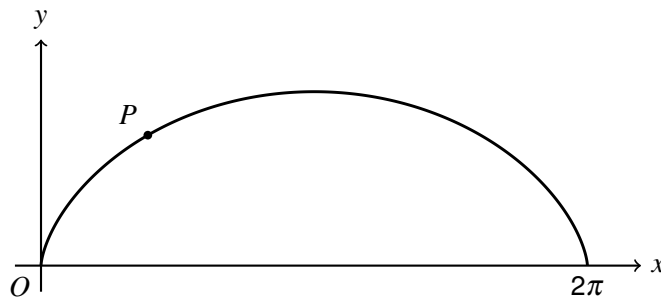


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = t - \sin t \quad y = 1 - \cos t \quad 0 \leq t \leq 2\pi$$

- (a) Show that $\frac{dy}{dx} = \frac{\sin t}{1 - \cos t}$, $t \neq 0, 2\pi$ (2)

- (b) Hence show that $\frac{dy}{dx} = \cot\left(\frac{t}{2}\right)$ (2)

The point P on C has parameter $t = \frac{2\pi}{3}$

- (c) Find an equation of the tangent to C at P , giving your answer in the form $y = mx + c$, where m and c are exact. (3)
- (d) Show that, for $0 < t < 2\pi$, the normal to C at the point with parameter t passes through the point $(t, 0)$ (3)



Question 9 continued

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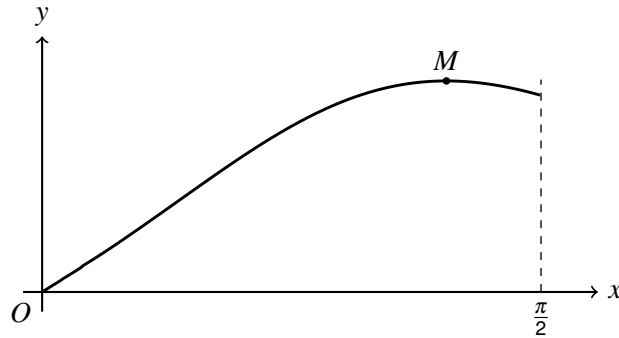


Figure 5

Figure 5 shows a sketch of the curve C with equation

$$y = x^{\cos x} \quad 0 < x < \frac{\pi}{2}$$

The curve has a maximum point M .

- (a) By first taking natural logarithms, show that $\frac{dy}{dx} = x^{\cos x} \left(\frac{\cos x}{x} - \sin x \ln x \right)$ (4)

- (b) Hence show that the x coordinate of M is a solution of the equation $x \tan x \ln x = 1$ (2)

The x coordinate of M is α .

- (c) Show that α lies between 1.2 and 1.3 (2)

- (d) Show that $\alpha = 1.273$ correct to 3 decimal places. (2)

- (e) Hence find the y coordinate of M , giving your answer to 3 significant figures. (1)



Question 10 continued

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(Total for Question 10 is 11 marks)**TOTAL FOR CHAPTER 5 IS 81 MARKS****Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

6

CHAPTER 6 • KEY SKILLS

Integration

Specification P4: Integration • 10 questions • 86 marks • about 103 minutes

1 Integration by parts

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx. \text{ Let } u = x \text{ (or } x^2), \text{ but always } u = \ln x.$$

2 Substitution

Find $\frac{du}{dx}$, replace dx (not just rename it du), change the limits, and leave no x in the integral.

3 Partial fractions

Split first.

$$\int \frac{k}{ax+b} dx = \frac{k}{a} \ln|ax+b| + c: \text{ do not lose the } \frac{1}{a}.$$

4 Trigonometric integrals

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x);$$

$$\int \sec^2 x dx = \tan x;$$

$$\int \tan x dx = \ln|\sec x|$$

5 Parametric areas

$$\text{Area} = \int y \frac{dx}{dt} dt, \text{ with the limits as values of } t.$$

6 Volumes of revolution

$$V = \pi \int y^2 dx: \text{ square the whole of } y.$$

$$\text{Parametric: } \pi \int y^2 \frac{dx}{dt} dt$$

7 Separable equations

Separate, integrate both sides and add $+c$ at once. Use the condition to find c , then rearrange.

8 Modelling

As $t \rightarrow \infty$, $e^{-kt} \rightarrow 0$ gives the long-term value. Find times exactly with $\ln$.

9 Exact answers

Combine logarithms: $\ln 5 - \ln \frac{1}{3} = \ln 15$.
Give the form asked for, e.g. $a + b \ln 2$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	6	8	5	6	11	10	12	12	11
1 Integration by parts	•						•		•	•
2 Substitution		•				•				
3 Partial fractions			•					•		
4 Trigonometric integrals	•				•	•				•
5 Parametric areas						•				
6 Volumes of revolution				•		•			•	
7 Separable equations					•			•		•
8 Modelling								•	•	
9 Exact answers	•	•	•	•			•	•		•
Interleaved: trigonometric identities and inverse functions (P3)					•	•				•
Interleaved: partial fractions and parametric curves (Chapters 2, 3)			•			•		•		
Interleaved: e, ln and numerical methods (P3)							•	•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Quote the formula first ($V = \pi \int y^2 dx$, the parts formula). Include $+c$ as soon as you integrate a differential equation. With a substitution, change dx and the limits. Write the answer in the exact form asked for.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Use integration by parts to find the exact value of

$$\int_0^{\frac{\pi}{4}} x \sec^2 x \, dx$$

giving your answer in the form $a\pi + b \ln 2$, where a and b are rational constants.

(5)

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Question 1 continued

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(Total for Question 1 is 5 marks)



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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Use the substitution $u = \sqrt{x}$ to show that

$$\int_4^9 \frac{1}{\sqrt{x}-1} dx = a + b \ln 2$$

where a and b are integers to be found.

(6)

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Question 2 continued

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(Total for Question 2 is 6 marks)



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3. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

$$f(x) = \frac{5}{(2x-1)(x+2)} \quad x > \frac{1}{2}$$

- (a) Express $f(x)$ in partial fractions.

(3)

- (b) Given that

$$\int_k^{2k} f(x) \, dx = \ln \frac{14}{9} \quad k > \frac{1}{2}$$

use algebraic integration to find the value of k .

(5)



Question 3 continued

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(Total for Question 3 is 8 marks)



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4. In this question you must show all stages of your working.
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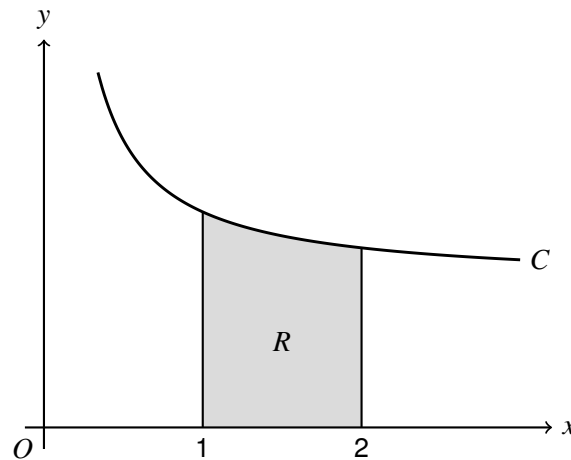


Figure 1

Figure 1 shows part of the curve C with equation $y = 2 + \frac{1}{x}$, $x > 0$

The region R , shown shaded, is bounded by C , the lines $x = 1$ and $x = 2$ and the x -axis. The region R is rotated through 2π radians about the x -axis.

Use algebraic integration to find the exact volume of the solid generated, giving your answer in the form $\pi(p + q \ln 2)$, where p and q are rational.

(5)



Question 4 continued

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(Total for Question 4 is 5 marks)



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5. Find the particular solution of the differential equation

$$\frac{dy}{dx} = (2x + 1) \cos^2 y \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

for which $y = \frac{\pi}{4}$ when $x = 0$, giving your answer in the form $y = f(x)$

(6)

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Question 5 continued

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(Total for Question 5 is 6 marks)



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6. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

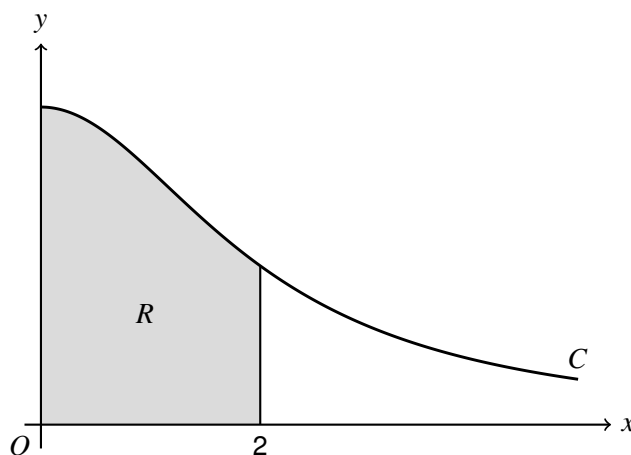


Figure 2

Figure 2 shows part of the curve C with parametric equations

$$x = 2 \tan t \quad y = 4 \cos^2 t \quad 0 \leq t < \frac{\pi}{2}$$

The region R , shown shaded, is bounded by C , the coordinate axes and the line $x = 2$

- (a) Show that the area of R is given by $\int_0^{\frac{\pi}{4}} k \, dt$, where k is a constant to be found, and hence find the exact area of R . (4)

- (b) Show that a Cartesian equation of C is $y = \frac{16}{4+x^2}$. (2)

The region R is rotated through 2π radians about the x -axis.

- (c) Use algebraic integration to find the exact volume of the solid generated, giving your answer in the form $\pi(a\pi + b)$, where a and b are integers. (5)



Question 6 continued

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Question 6 continued

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(Total for Question 6 is 11 marks)



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7. (a) Find $\int e^{-x} \sin x dx$

Diagram not drawn to scale

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = e^{-x} \sin x$, $x \geq 0$

The region R_1 is bounded by the curve and the x -axis for $0 \leq x \leq \pi$. The region R_2 is bounded by the curve and the x -axis for $\pi \leq x \leq 2\pi$

(b) Find the exact area of R_1 **(2)**

(c) Show that the area of R_1 is e^π times the area of R_2 (3)

Question 7 continued

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Question 7 continued

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(Total for Question 7 is 10 marks)



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8. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**

The number of fish in a lake, P thousands, t years after the start of a study, is modelled by the differential equation

$$\frac{dP}{dt} = \frac{P(10 - P)}{20} \quad 0 < P < 10$$

At the start of the study there were 2000 fish in the lake.

- (a) Express $\frac{1}{P(10 - P)}$ in partial fractions. (3)
- (b) Solve the differential equation to show that $P = \frac{10}{1 + 4e^{-\frac{1}{2}t}}$ (6)
- (c) Find, according to the model, the number of fish in the lake in the long term. (1)
- (d) Find the exact time, in years, at which there are 8000 fish in the lake, giving your answer in the form $a \ln 2$ (2)



Question 8 continued

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Question 8 continued

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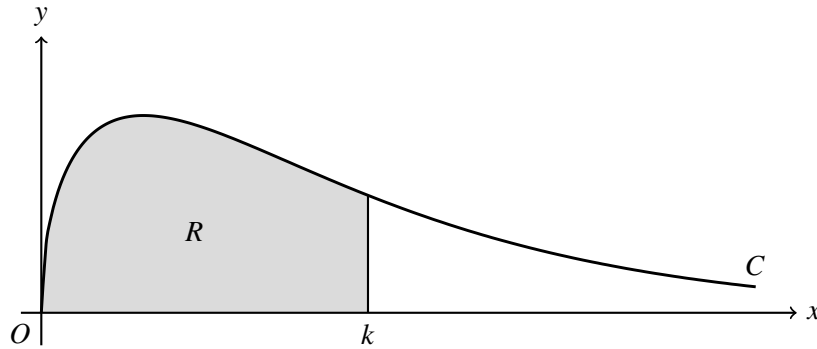


Figure 4

Figure 4 shows part of the curve C with equation $y = \sqrt{x}e^{-\frac{1}{2}x}$, $x \geq 0$

The region R , shown shaded, is bounded by C , the x -axis and the line $x = k$, where $k > 0$. The region R is rotated through 2π radians about the x -axis to form a solid of volume V .

(a) Show that $V = \pi(1 - (k+1)e^{-k})$ (5)

(b) State the limit of V as $k \rightarrow \infty$ (1)

The value of k for which V is 90% of this limit is β .

(c) Show that β satisfies the equation $\beta = \ln(10\beta + 10)$ (2)

(d) Using the iteration formula $\beta_{n+1} = \ln(10\beta_n + 10)$ with $\beta_1 = 4$, find the values of β_2 and β_3 , giving your answers to 4 decimal places. (2)

(e) Show that $\beta = 3.890$ correct to 3 decimal places. (2)



Question 9 continued

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Question 9 continued

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(Total for Question 9 is 12 marks)



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10.

(a) Find $\int x \cos 2x \, dx$

(3)

(b) Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{x \cos^2 x}{y} \quad y > 0$$

to show that $y^2 = \frac{1}{2}x^2 + \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x + \frac{3}{4}$

(6)

(c) Find the exact value of y when $x = \frac{\pi}{2}$

(2)

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 11 marks)

TOTAL FOR CHAPTER 6 IS 86 MARKS

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7

CHAPTER 7 • KEY SKILLS

Vectors

Specification P4: Vectors • 10 questions • 82 marks • about 98 minutes

1 Vectors and distance

$\vec{AB} = \mathbf{b} - \mathbf{a}$ (the end minus the start).
 $|\mathbf{x}\mathbf{i} + \mathbf{y}\mathbf{j} + \mathbf{z}\mathbf{k}| = \sqrt{x^2 + y^2 + z^2}$. A unit vector is $\frac{\mathbf{a}}{|\mathbf{a}|}$

2 Equation of a line

$\mathbf{r} = \mathbf{a} + \lambda \mathbf{d}$: a point on the line plus a multiple of the direction. Always write “ $\mathbf{r} =$ ”.

3 Points on a line

Set $\mathbf{a} + \lambda \mathbf{d}$ equal to the point: one component gives λ , the others must agree.

4 Intersecting lines

Equate the lines, solve two components for λ and μ , then use the third to check (or to find a constant).

5 Parallel or skew

Parallel: $\mathbf{d}_1 = k\mathbf{d}_2$. Skew: not parallel and no point of intersection. State both, with reasons.

6 Scalar product and angles

$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$, with both vectors pointing out of the angle. A negative value means an obtuse angle.

7 Perpendiculars

$\mathbf{a} \cdot \mathbf{b} = 0$. The foot X from P : $\vec{PX} \cdot \mathbf{d} = 0$ (not $\vec{OX} \cdot \mathbf{d}$). The reflection is $\mathbf{p}' = 2\mathbf{x} - \mathbf{p}$

8 Areas

Triangle: $\frac{1}{2}|\mathbf{a}||\mathbf{b}|\sin \theta$; parallelogram: $|\mathbf{a}||\mathbf{b}|\sin \theta$. Exact $\sin \theta = \sqrt{1 - \cos^2 \theta}$

9 Distances along a line

$|\lambda \mathbf{d}| = |\lambda||\mathbf{d}|$: a distance along a line gives two points, one each side. Draw a diagram.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	5	6	5	6	10	10	11	12	11
1 Vectors and distance	•	•			•	•	•		•	•
2 Equation of a line		•					•			
3 Points on a line		•						•		•
4 Intersecting lines			•	•				•	•	
5 Parallel or skew				•						
6 Scalar product and angles	•					•		•	•	•
7 Perpendiculars					•		•		•	•
8 Areas						•	•	•	•	•
9 Distances along a line								•		
Interleaved: quadratics and completing the square (P1)					•					•
Interleaved: exact trigonometry, Pythagoras, circle theorems (P1, P2)	•					•		•	•	•
Interleaved: proof and minimising with calculus (Chapters 1, 5)				•	•					

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Draw a diagram. Use direction vectors (not position vectors) for angles, and vectors pointing out of the angle. Finish every argument: “so the lines do not intersect”, “not parallel, so skew”. Give coordinates when coordinates are asked for.



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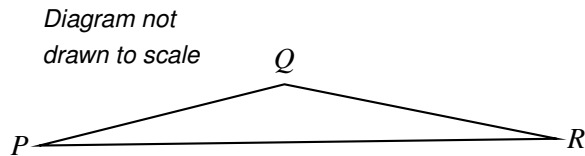


Figure 1

Figure 1 shows a triangle PQR . Relative to a fixed origin O ,

$$\overrightarrow{OP} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} \quad \overrightarrow{OQ} = 4\mathbf{i} + 3\mathbf{j} \quad \overrightarrow{OR} = 6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

- (a) Find $\overrightarrow{QP}$ and $\overrightarrow{QR}$ (2)
- (b) Find the size of angle PQR , giving your answer in degrees to one decimal place. (3)
- (c) State, with a reason, which angle of triangle PQR is the largest. (1)



Question 1 continued

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(Total for Question 1 is 6 marks)



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2. Relative to a fixed origin O , the point A has coordinates $(2, -1, 3)$ and the point B has coordinates $(4, 3, -1)$. The line l passes through A and B .

(a) Find a vector equation of l .

(2)

The point $C(p, 9, q)$ lies on l .

(b) Find the value of p and the value of q .

(2)

(c) Find a unit vector parallel to l .

(1)

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Question 2 continued

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(Total for Question 2 is 5 marks)



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3. Relative to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k}) \quad l_2: \mathbf{r} = (3\mathbf{i} + q\mathbf{j} + 8\mathbf{k}) + \mu(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$$

where λ and μ are scalar parameters and q is a constant. Given that l_1 and l_2 intersect,

- (a) find the value of q ,

(4)

- (b) find the coordinates of the point of intersection.

(2)

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Question 3 continued

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(Total for Question 3 is 6 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

4. Relative to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad l_2: \mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

where λ and μ are scalar parameters.

Prove that l_1 and l_2 are skew.

(5)

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Question 4 continued

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(Total for Question 4 is 5 marks)



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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Relative to a fixed origin O , the line l has equation

$$\mathbf{r} = (-4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$$

The point P lies on l .

(a) Show that $OP^2 = 9\lambda^2 - 36\lambda + 56$

(2)

(b) Hence, by completing the square, find the coordinates of the point on l that is closest to O , and the shortest distance from O to l , giving the distance in the form $a\sqrt{b}$

(4)

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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

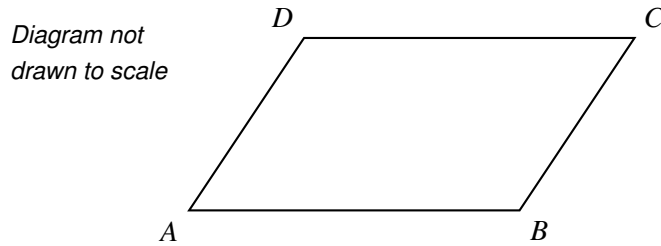


Figure 2

Figure 2 shows a parallelogram $ABCD$. Relative to a fixed origin O , the points A , B and C have coordinates $(1, 2, 0)$, $(3, 1, 2)$ and $(4, 3, 5)$ respectively.

- (a) Find the coordinates of D . (2)
- (b) Show that $\cos \angle ABC = -\frac{\sqrt{14}}{7}$ (3)
- (c) Find the exact area of the parallelogram, giving your answer in the form $p\sqrt{q}$, where p and q are integers. (3)
- (d) Hence find the exact shortest distance from A to the line through B and C . (2)



Question 6 continued

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Question 6 continued

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Question 7 continued

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8. Relative to a fixed origin O , the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} -3 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad l_2: \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

where λ and μ are scalar parameters. The lines meet at the point P .

- (a) Find the coordinates of P .

(3)

The acute angle between l_1 and l_2 is θ .

- (b) Show that $\cos \theta = \frac{4}{9}$

(3)

The point Q lies on l_1 where $\lambda = 5$. The point R lies on l_2 such that $PR = PQ$.

- (c) Find the coordinates of the two possible positions of R .

(3)

- (d) Find the exact area of triangle QPR , explaining why it is the same for both positions of R .

(2)



Question 8 continued

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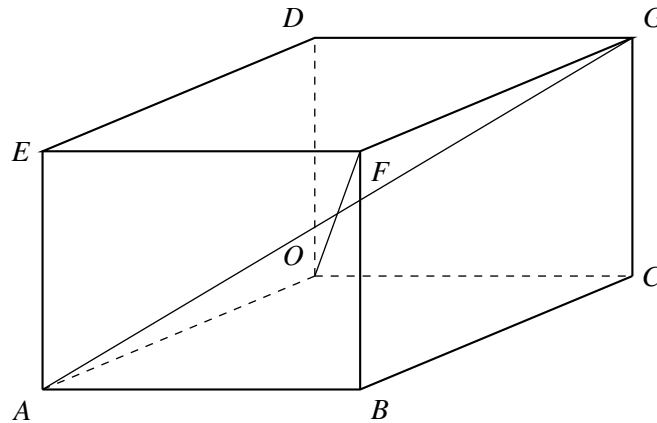


Figure 3

Figure 3 shows a cuboid $OABCDEFG$. Relative to the origin O , the vertices A , C and D have position vectors $6\mathbf{i}$, $4\mathbf{j}$ and $3\mathbf{k}$ respectively.

- (a) Write down $\overrightarrow{OF}$ and $\overrightarrow{AG}$ (2)
- (b) Find the acute angle between the lines OF and AG , giving your answer in degrees to one decimal place. (3)
- (c) Show that the lines OF and AG intersect, and find the coordinates of the point of intersection. (3)
- (d) Show that angle AOG is a right angle, and hence find the area of triangle OAG (2)
- (e) Hence find the exact shortest distance from O to the line AG , giving your answer in the form $k\sqrt{61}$ (2)



Question 9 continued

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10. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**Relative to a fixed origin O , the line l has equation

$$\mathbf{r} = (\mathbf{j} + 2\mathbf{k}) + \lambda(\mathbf{i} + \mathbf{j})$$

The points A and B have coordinates $(5, 1, 3)$ and $(1, 3, 3)$ respectively.

- (a) Explain why neither A nor B lies on l .

(1)The point C lies on l and angle $ACB = 90^\circ$

- (b) Show that the value of λ at C satisfies $\lambda^2 - 4\lambda + 3 = 0$

(4)

- (c) Hence find the coordinates of the two possible positions of C .

(2)

- (d) Explain, with reference to the sphere with diameter AB , why there cannot be more than two points on l at which angle $ACB = 90^\circ$

(2)

- (e) For the position of C with the smaller value of λ , find the area of triangle ACB .

(2)

Question 10 continued

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TOTAL FOR CHAPTER 7 IS 82 MARKS

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