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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 1

Further Pure 1

Chapter 2: Roots of Quadratic Equations

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM01/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM01 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Complex Numbers** Arithmetic, conjugates, Argand diagrams, modulus and argument, roots of polynomial equations	10	80	separate file	
2	**Roots of Quadratic Equations** Sums and products of roots, symmetric functions, equations with transformed roots	10	80	2	
3	**Numerical Solutions of Equations** Interval bisection, linear interpolation, Newton–Raphson	10	80	separate file	
4	**Coordinate Systems** Parametric equations, parabolas, rectangular hyperbolas, tangents and normals	10	80	separate file	
5	**Matrices** Matrix arithmetic, determinants, inverses of 2×2 matrices	10	80	separate file	
6	**Transformations Using Matrices** Linear transformations, combined transformations, area scale factor, inverses	10	80	separate file	
7	**Series** Standard sums of r, r^2 and r^3, sigma notation	10	80	separate file	
8	**Proof** Mathematical induction for sums, divisibility, matrices and sequences	10	80	separate file	

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CHAPTER 2 • KEY SKILLS

2 Roots of Quadratic Equations

Specification FP1: Roots of quadratic equations • 10 questions • 80 marks • about 96 minutes

1 Sum and product

For $ax^2+bx+c=0$ with roots α, β: $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$. Watch the sign of the sum.

2 Sum of squares

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

3 Sum of cubes

$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$

4 Reciprocals

$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$ and $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$

5 Difference of roots

$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$; then $\alpha^3-\beta^3=(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)$

6 Forming equations

New roots with sum S and product P: $x^2-Sx+P=0$. Multiply to integer coefficients and keep "= 0".

7 Without solving

Work only with $\alpha+\beta$ and $\alpha\beta$: finding the roots themselves earns no marks.

8 Unknown coefficients

Turn a condition on the roots (one is twice the other, they differ by 2) into equations using the sum and product.

9 Complex roots

Real coefficients give conjugate roots, so $\alpha\beta=\alpha\alpha^*=|\alpha|^2$. Symmetric sums can be negative.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Sum and product	•	•	•	•	•	•	•	•	•	•
2 Sum of squares	•	•		•		•		•	•	•
3 Sum of cubes	•					•				
4 Reciprocals		•					•			
5 Difference of roots		•						•		•
6 Forming equations			•	•		•	•	•	•	•
7 Without solving	•	•	•	•		•	•	•		
8 Unknown coefficients					•			•	•	•
9 Complex roots						•			•	
Interleaved: complex numbers (Chapter 1)						•			•	
Interleaved: algebraic fractions and surds (P1)		•		•	•		•			•
Interleaved: discriminant and quadratics (P1)					•	•			•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write down $\alpha+\beta$ and $\alpha\beta$ first. Rewrite every expression in terms of them before substituting, and finish a new equation with integer coefficients and "= 0".

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1. The roots of the equation $2x^2 - 5x + 4 = 0$ are α and β. Without solving the equation,

(a) write down the values of $\alpha + \beta$ and $\alpha\beta$,

(1)

(b) find the value of $\alpha^2 + \beta^2$,

(2)

(c) find the value of $\alpha^3 + \beta^3$.

(3)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. The roots of the equation $x^2 + 3x - 7 = 0$ are α and β, where $\alpha > \beta$. Without solving the equation, find the exact value of

(a) $\frac{1}{\alpha} + \frac{1}{\beta}$

(2)

(b) $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$

(2)

(c) $\alpha - \beta$

(2)

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Question 2 continued

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3. Let α and β denote the two solutions of $x^2 - 4x + 2 = 0$. A second quadratic has solutions $2\alpha + 1$ and $2\beta + 1$. Do not solve either equation.

(a) Calculate $(2\alpha + 1) + (2\beta + 1)$.

(2)

(b) Calculate $(2\alpha + 1)(2\beta + 1)$.

(2)

(c) Hence write the second quadratic in the form $x^2 + px + q = 0$, stating the integers p and q.

(2)

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Question 3 continued

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4. α and β are the roots of $3x^2 + 2x + 5 = 0$

(a) Show, without finding α and β, that $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = -\frac{26}{15}$

(3)

(b) Deduce an equation with integer coefficients whose roots are the ratios $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$

(3)

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Question 4 continued

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5. The equation $x^2 - kx + 18 = 0$, where k is a constant, has one root which is twice the other.

(a) Find the possible values of k.

(4)

(b) For the positive value of k, state the roots of the equation.

(1)

(c) Find the values of k for which the equation has equal roots.

(1)

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Question 5 continued

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6. The roots of the equation $x^2 + 2x + 6 = 0$ are α and β. Without solving the equation,

(a) write down the values of $\alpha + \beta$ and $\alpha\beta$,

(1)

(b) show that $\alpha^2 + \beta^2 = -8$,

(2)

(c) find the value of $\alpha^3 + \beta^3$,

(2)

(d) find a quadratic equation, with integer coefficients, whose roots are α^3 and β^3

(3)

(e) Explain how $\alpha^2 + \beta^2$ can be negative, and state the roots α and β.

(2)

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Question 6 continued

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7. Two numbers α and β satisfy $2\alpha^2 - 3\alpha + 4 = 0$ and $2\beta^2 - 3\beta + 4 = 0$, with $\alpha \neq \beta$. Two new numbers are defined by

$$u = \alpha + \frac{1}{\beta} \quad \text{and} \quad v = \beta + \frac{1}{\alpha}$$

(a) State the values of $\alpha + \beta$ and $\alpha\beta$.

(1)

(b) Show that $u + v = \frac{9}{4}$

(3)

(c) Find the value of uv.

(3)

(d) Hence obtain an equation of the form $ax^2 + bx + c = 0$, with integers a, b and c, whose solutions are u and v.

(3)

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Question 7 continued

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8. For a constant c, the numbers α and β, with $\alpha > \beta$, satisfy $x^2 - 5x + c = 0$, and the sum of their squares is 13

(a) Determine c.

(3)

(b) Write down an equation, in the form $x^2 + px + q = 0$, satisfied by both α^2 and β^2

(3)

(c) Using only $\alpha + \beta$ and $\alpha\beta$, calculate $\alpha^3 - \beta^3$.

(4)

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Question 8 continued

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9. The equation $x^2 - 4x + k = 0$, where k is a real constant, has non-real roots α and β. Given that $|\alpha| = 5$

(a) explain why $\alpha\beta = |\alpha|^2$, and hence find the value of k,

(3)

(b) find α and β in the form $p \pm qi$,

(2)

(c) find a quadratic equation, with integer coefficients, whose roots are α^2 and β^2,

(3)

(d) verify that the product of the roots in part (c) is equal to $\left|\alpha^2\right|^2$.

(2)

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10. The roots of the equation $x^2 - (k+2)x + 2k + 1 = 0$, where k is a real constant, are α and β.

(a) Show that $\alpha^2 + \beta^2 = k^2 + 2$

(3)

(b) Hence state the least possible value of $\alpha^2 + \beta^2$.

(1)

(c) Given that the roots differ by 2, find the exact possible values of k.

(3)

(d) Find the values of k for which the equation has equal roots.

(2)

(e) For $k = 4$, find a quadratic equation whose roots are $\alpha + \beta$ and $\alpha\beta$.

(1)

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Question 10 continued

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TOTAL FOR CHAPTER 2 IS 80 MARKS

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