



eClassroom

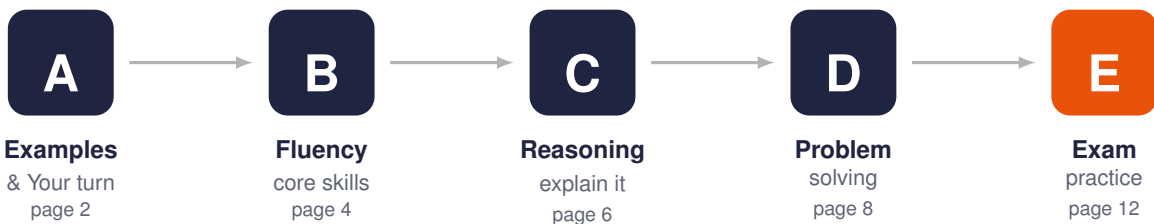
A-LEVEL MATHEMATICS • PURE 1

Trigonometric Ratios

Worksheet

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Free Trigonometric Ratios worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: use the sine rule and the cosine rule to find sides and angles • find the area of a triangle with $\frac{1}{2}ab\sin C$ • solve problems that need more than one triangle • sketch and use the graphs of $\sin x$, $\cos x$ and $\tan x$ • *Stretch:* handle the ambiguous case, work with transformed trigonometric graphs, count solutions and prove results.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH The ambiguous case, an area that gives two angles, quadrilaterals and bearings, and transformed trigonometric graphs.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

In triangle ABC , $AC = 6$ cm, $AB = 9$ cm and angle $BAC = 50^\circ$. Find BC to 3 significant figures.

Two sides and the angle between them: the cosine rule.

$$BC^2 = 6^2 + 9^2 - 2 \times 6 \times 9 \cos 50^\circ = 47.58 \dots$$

$$BC = 6.90 \text{ cm}$$

Your turn A1

In triangle PQR , $PQ = 7$ cm, $PR = 11$ cm and angle $QPR = 115^\circ$. Find QR to 3 significant figures.

Worked example A2

In triangle ABC , angle $A = 48^\circ$, angle $B = 71^\circ$ and $BC = 12$ cm. Find AC .

BC is opposite A and AC is opposite B : pair each side with its opposite angle.

$$\frac{AC}{\sin 71^\circ} = \frac{12}{\sin 48^\circ}$$

$$AC = 15.3 \text{ cm}$$

Your turn A2

In triangle ABC , angle $C = 36^\circ$, angle $A = 102^\circ$ and $AB = 8$ cm. Find BC .

Worked example A3

Find the area of a triangle with sides 8 cm and 11 cm and an angle of 40° between them.

The angle must be the one between the two sides.

$$\frac{1}{2} \times 8 \times 11 \times \sin 40^\circ$$

$$28.3 \text{ cm}^2$$

Your turn A3

Find the area of a triangle with sides 5.4 cm and 7.2 cm and an angle of 128° between them.



STRETCH

Worked example A4: the largest angle

A triangle has sides 5 cm, 7 cm and 10 cm. Find its largest angle.

The largest angle is opposite the longest side. A negative cosine means obtuse.

$$\cos \theta = \frac{25 + 49 - 100}{2 \times 5 \times 7} = -\frac{26}{70}$$

$$\theta = 111.8^\circ$$

Your turn A4

A triangle has sides 4 cm, 9 cm and 11 cm. Find its largest angle.

Worked example A5: the ambiguous case

In triangle ABC , $BC = 7$ cm, $AB = 9$ cm and angle $A = 40^\circ$. Find the two possible values of angle C .

$$\sin C = \frac{9 \sin 40^\circ}{7} = 0.826 \dots$$

$\sin C = \sin(180^\circ - C)$. Both fit, because $40^\circ + 124.3^\circ < 180^\circ$

$$C = 55.7^\circ \text{ or } 124.3^\circ$$

Your turn A5

In triangle ABC , $AC = 10$ cm, $AB = 13$ cm and angle $B = 35^\circ$. Find the two possible values of angle C .

Worked example A6: a translated graph

For $y = \cos(x - 60^\circ)$, $0 \leq x \leq 360^\circ$, find the y intercept, the x intercepts and the maximum point.

The graph of $\cos x$ moves 60° to the right.

$$\cos(-60^\circ) = \frac{1}{2}; \quad x - 60^\circ = 90^\circ \text{ or } 270^\circ$$

$$(0, \frac{1}{2}); \quad 150^\circ, 330^\circ; \quad \max(60^\circ, 1)$$

Your turn A6

For $y = \sin(x + 45^\circ)$, $0 \leq x \leq 360^\circ$, find the y intercept, the x intercepts and the maximum point.

Worked example A7: a stretched graph

For $y = 3 \sin 2x$, $0 \leq x \leq 360^\circ$, state the period, and the number of solutions of $3 \sin 2x = 2$

$\sin 2x$ completes a cycle every 180° , so there are two full waves.

The line $y = 2$ crosses each wave twice.

$$\text{period } 180^\circ; \quad 4 \text{ solutions}$$

Your turn A7

For $y = 2 \cos 3x$, $0 \leq x \leq 360^\circ$, state the period, and the number of solutions of $2 \cos 3x = 1$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method*

- B1** In triangle ABC , $AB = 5$ cm and $AC = 8$ cm. Find BC , to 3 significant figures where necessary, when angle BAC is
- (a) 60° (b) 120° (c) 90° (d) 30° [4 marks]
- B2** In triangle ABC , angle $A = 40^\circ$ and angle $B = 60^\circ$. Find, to 3 significant figures,
- (a) AC , given that $BC = 10$ cm
- (b) AB , given that $BC = 10$ cm
- (c) BC , given instead that $AC = 10$ cm
- (d) angle A in a different triangle with $BC = 7$ cm, $AC = 10$ cm and angle $B = 40^\circ$, given that angle A is acute [4 marks]
- B3** A triangle has sides of 6 cm and 10 cm. Find the exact area of the triangle when the angle between these sides is
- (a) 30° (b) 150° (c) 90° (d) 45° [4 marks]
- B4** Find the largest angle of the triangle with sides
- (a) 3, 5, 7 (b) 5, 7, 8 (the angle opposite 7) (c) 3, 4, 5 (d) 4, 5, 6 [4 marks]



CORE

*continued: the graphs***B5** Find the solutions of each equation for $0 \leq x \leq 360^\circ$

(a) $\sin x = \frac{1}{2}$

(b) $\cos x = \frac{1}{2}$

(c) $\tan x = 1$

[3 marks]

B6 State the number of solutions of each equation for $0 \leq x \leq 360^\circ$

(a) $\sin x = 0.7$

(b) $\sin x = 1$

(c) $\cos x = -1.2$

(d) $\tan x = 5$

[4 marks]

STRETCH

B7 In triangle ABC , angle $A = 30^\circ$ and $AC = 6$ cm. State how many triangles are possible when BC is

(a) 4 cm

(b) 3 cm

(c) 2 cm

(d) 8 cm

[4 marks]

B8 Write down the maximum value and the period of

(a) $y = 3 \sin x$

(b) $y = \sin 3x$

(c) $y = \sin(x + 30^\circ) + 2$

(d) $y = -2 \cos \frac{x}{2}$

[4 marks]

B9 State the number of solutions of each equation for $0 \leq x \leq 360^\circ$

(a) $\sin 2x = 0.5$

(b) $\sin \frac{x}{2} = 0.5$

(c) $\cos(x - 30^\circ) = 1$

(d) $\tan 3x = 1$

[4 marks]

B10 A triangle has sides 4 cm and 6 cm. Give exact answers.**(a)** Find its area when the angle between these sides is 60° **(b)** Find the third side when the angle between these sides is 60° **(c)** Find its area when the angle between these sides is 135° **(d)** Find the third side when the angle between these sides is 135° , in the form $\sqrt{a + b\sqrt{2}}$

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** In triangle ABC , $AB = 6$ cm, $BC = 8$ cm and angle $BAC = 50^\circ$. Sam writes: $\text{area} = \frac{1}{2} \times 6 \times 8 \times \sin 50^\circ$
Explain why Sam's method is wrong. [2 marks]
- C2** Priya solves $\sin x = \frac{1}{2}$ for $0 \leq x \leq 360^\circ$ and writes $x = 30^\circ$ only.
Use the graph of $y = \sin x$ to explain what she has missed, and give the full solution. [2 marks]
- C3** Show that the triangle with sides 7 cm, 8 cm and 13 cm has an angle of 120° , and find its exact area. [3 marks]
- C4** Kai says: "The graph of $y = \sin 2x$ has period 720° , because the 2 stretches it."
Explain Kai's mistake, and state the number of solutions of $\sin 2x = 0.3$ for $0 \leq x \leq 360^\circ$ [3 marks]



STRETCH

- C5** In triangle ABC , $BC = 8$ cm, $AC = 11$ cm and angle $A = 38^\circ$. Lee uses the sine rule, gets angle $B = 57.8^\circ$, and says the triangle is fixed.
Show that two different triangles fit this information, and give both values of angle B . [3 marks]
- C6** In triangle ABC , the perpendicular from A to BC has length h . Using $BC = a$ and $AC = b$, show that $h = b \sin C$, and hence prove that the area of the triangle is $\frac{1}{2}ab \sin C$ [3 marks]
- C7** Use the cosine rule to explain why there is no triangle with sides 3 cm, 4 cm and 8 cm, and state the fact about triangles that this shows. [3 marks]
- C8** Two sides of a triangle are 6 cm and 10 cm, and the angle between them is θ
- (a) Explain why the area of the triangle is greatest when $\theta = 90^\circ$, and state the greatest area. (2)
- (b) Given that the area is $15\sqrt{3} \text{ cm}^2$, find the two possible values of θ (2)
- (c) For the obtuse value of θ , find the exact length of the third side. (1)
- [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

- D1** A company logo is in the shape of the triangle ABC shown in Figure 1. In the logo, $AB = 7$ cm, $AC = 5$ cm and angle $BAC = 60^\circ$

Using exact values for the trigonometric ratios,

- (a) show that the length of the side BC is $\sqrt{39}$ cm, (2)
- (b) find the exact area of the logo, giving your answer in the form $p\sqrt{3}$, where p is a rational number. (1)

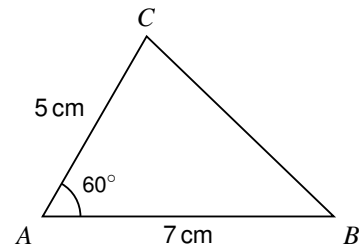


Diagram NOT accurately drawn

Figure 1

[3 marks]

- D2** Figure 2 shows the design for a triangular metal bracket PQR . In the design, $PQ = 9$ cm, $QR = 6$ cm and angle $QPR = 35^\circ$. The angle PRQ is obtuse, as shown in Figure 2.

- (a) Find the size of the obtuse angle PRQ , giving your answer in degrees to 2 decimal places. (2)
- (b) Hence find the length of the side PR , giving your answer in cm to 3 significant figures. (2)

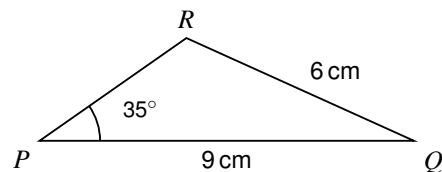


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Figure 2

[4 marks]



D3 Figure 3 shows a sketch of the graph of $y = \tan x$ for $0 \leq x \leq 360^\circ$. The graph has two vertical asymptotes, which are shown as dashed lines in Figure 3.

(a) Write down the equations of the two asymptotes shown in Figure 3.

(1)

(b) State the number of solutions of the equation $\tan x = 2$ for $0 \leq x \leq 360^\circ$

(1)

Given that one solution of $\tan x = 2$ is $x = 63.4^\circ$ to 1 decimal place,

(c) use Figure 3 to find the other solution in this interval, explaining how the shape of the graph gives your answer.

(1)

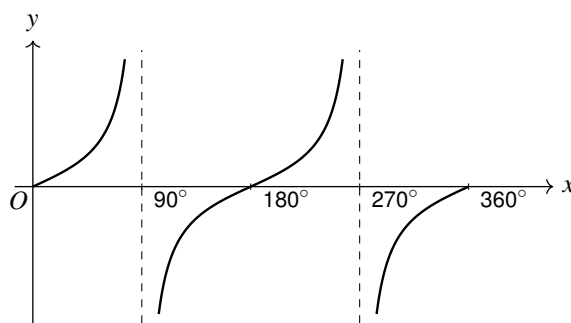


Figure 3

[3 marks]

D4 Figure 4 shows a triangular plot of land ABC . The sides AB and AC have lengths 8 m and 6 m, and the angle between them, angle BAC , is obtuse.

The area of the plot is $12\sqrt{3} \text{ m}^2$

(a) Show that $\sin(\text{angle } BAC) = \frac{\sqrt{3}}{2}$

(1)

(b) Hence find the exact length of the third side BC , giving your answer as a simplified surd.

(2)

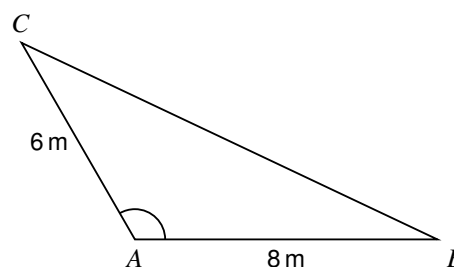


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Figure 4

[3 marks]



STRETCH

- D5** Figure 5 shows the plan of a flower bed $ABCD$ in the shape of a quadrilateral. A straight path of stepping stones is laid along the diagonal AC .

In the plan, $AB = 5$ m, $BC = 8$ m, $CD = 9$ m, $DA = 10$ m and angle $ABC = 120^\circ$

(a) Find the length of the path AC , giving your answer in metres to 3 significant figures.

(1)

(b) Find the size of angle ADC , giving your answer in degrees to 1 decimal place.

(2)

(c) Find the total area of the flower bed, giving your answer in m^2 to 3 significant figures.

(1)

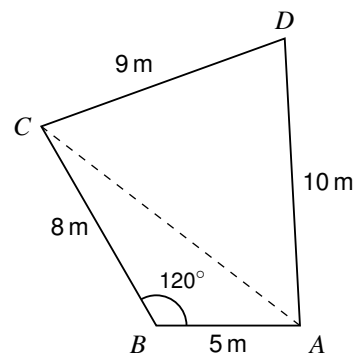


Diagram NOT accurately drawn

Figure 5

[4 marks]

- D6** A boat sails 12 km from the harbour A on a bearing of 060° to the point B . It then sails 8 km on a bearing of 170° to the point C , as shown in Figure 6.

(a) Show that angle $ABC = 70^\circ$, and find the distance AC , giving your answer in km to 3 significant figures.

(2)

The boat then returns directly to the harbour.

(b) Find the bearing of C from A , giving your answer to the nearest 0.1°

(2)

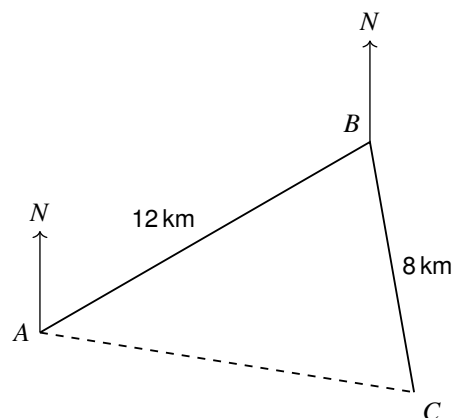


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Figure 6

[4 marks]



D7 Figure 7 shows the curve with equation $y = 3 \cos(x + 30^\circ)$ for $0 \leq x \leq 360^\circ$. The curve crosses the y -axis at the point P and has a minimum point at the point Q , as shown in Figure 7.

(a) Write down the exact coordinates of P .

(1)

(b) Write down the coordinates of Q .

(1)

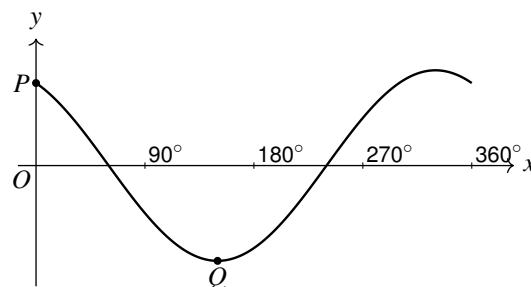


Figure 7

(c) State the number of solutions of $3 \cos(x + 30^\circ) = 2$ for $0 \leq x \leq 360^\circ$, giving a reason.

(1)

The curve with equation $y = 3 \cos(x + 30^\circ) + k$, where k is a constant, has a maximum value of 7

(d) State the value of k , and the minimum value of this curve.

(1)

[4 marks]

D8 Figure 8 shows the curve with equation $y = A \sin bx$, $0 \leq x \leq 360^\circ$, where A and b are positive constants. The curve has a maximum point at $(45^\circ, 3)$, as shown in Figure 8, and it completes exactly two full cycles in the interval $0 \leq x \leq 360^\circ$

(a) Write down the value of A and the value of b .

(2)

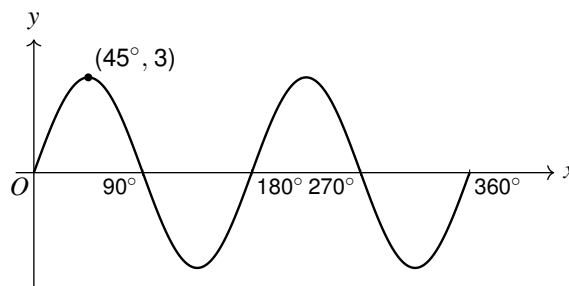


Figure 8

(b) State the number of solutions of $A \sin bx = 1.5$ for $0 \leq x \leq 360^\circ$

(1)

(c) Given that $\sin 30^\circ = \frac{1}{2}$, find all the solutions of the equation $A \sin bx = 1.5$ in the interval $0 \leq x \leq 360^\circ$, showing how each solution arises from the graph.

(2)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA11. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** Figure 9 shows the plan of a field ABC , which is in the shape of a triangle.

Given that

- $AB = 120$ m
- angle $ABC = 38^\circ$
- angle $BAC = 72^\circ$

- (a) find the length of the side AC , giving your answer in metres to 1 decimal place.

(2)

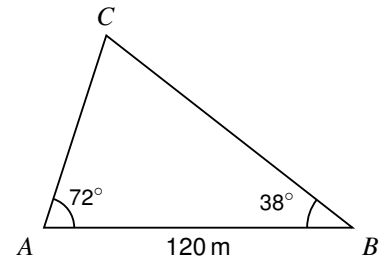


Figure 9

.....
A farmer plants the whole field with grass seed. One bag of seed covers 250 m^2

- (b) Find the area of the field, in m^2 to 3 significant figures, and hence the number of bags of seed the farmer must buy.

.....
(3)

(Total for Question E1 is 5 marks)



E2 Figure 10 shows part of the curve C with equation

$$y = 4 \sin(x - 30^\circ) \quad 0 \leq x \leq 360^\circ$$

The point P is the maximum point of C , the point Q is where C crosses the x -axis between P and the minimum point, and the point R is where C crosses the y -axis.

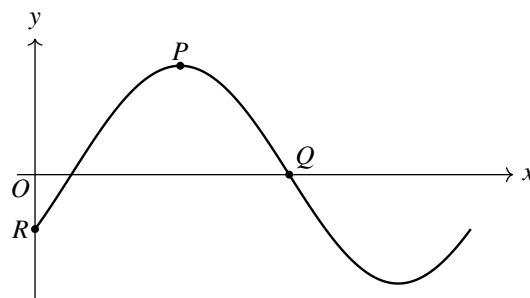


Figure 10

(a) Write down the coordinates of P and the coordinates of Q .

.....
(2)

(b) Find the coordinates of R .

.....
(1)

The curve C is translated to the curve with equation $y = 4 \sin(x - 30^\circ) + k$, where k is a constant. The minimum point of the translated curve lies on the x -axis.

(c) State the value of k .

.....
(1)

(d) State the number of solutions of $4 \sin(x - 30^\circ) = -3$ for $0 \leq x \leq 360^\circ$, giving a reason for your answer.

.....
(2)
(Total for Question E2 is 6 marks)



STRETCH

E3 Figure 11 shows the triangle ABC , which is the cross-section of a wedge used to hold a door open.

In the triangle, $AB = x$ cm, $AC = (x + 2)$ cm, $BC = 2\sqrt{7}$ cm and angle $BAC = 60^\circ$, where x is a positive constant.

- (a) Use the cosine rule to show that $x^2 + 2x - 24 = 0$
(3)

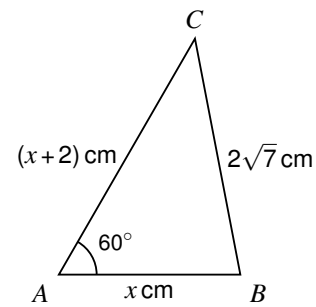


Diagram NOT accurately drawn

Figure 11

- (b) Hence find the value of x .

.....
(1)

- (c) Find the exact area of triangle ABC .

.....
(2)

- (d) Find the exact shortest distance from the point A to the side BC of the triangle, giving your answer in the form $\frac{p\sqrt{21}}{q}$, where p and q are integers.

.....
(2)

(Total for Question E3 is 8 marks)

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