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Circles & Circle Theorems

Worked Solutions

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • No calculator

Full worked solutions for the free Circles and Circle Theorems worksheet, GCSE and iGCSE Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Circles & Circle Theorems worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. In the exam, method marks are given for correct working even when the final answer is wrong. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **P** = process mark, **A** = accuracy mark, **B** = independent mark, **C** = communication mark.

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A**Your turn answers**

Compare each line of your working with the solution. Every angle needs a reason.

FOUNDATION

A1 ST is a **tangent**; the shaded region is a **segment**

A2 $OA = OB$ (radii), so angle $OBA = 28^\circ$ (base angles of an isosceles triangle) $x = 180 - 56 = \mathbf{124}$

A3 $360 \div 8 = \mathbf{45}^\circ$ at the centre base angle $= (180 - 45) \div 2 = \mathbf{67.5}^\circ$

HIGHER

A4 The angle at the centre is twice the angle at the circumference: $x = 130 \div 2 = \mathbf{65}$

A5 The angle in a semicircle is 90° (angle ACB). Angles in a triangle: $x = 180 - 90 - 61 = \mathbf{29}$

A6 Opposite angles of a cyclic quadrilateral add up to 180° : $x = 180 - 68 = \mathbf{112}$

A7 Tangents meet radii at 90° . Angles in quadrilateral $OAPB$ add up to 360° : $x = 360 - 90 - 90 - 142 = \mathbf{38}$

B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

FOUNDATION

B1 (a) **chord** (b) **tangent** (c) **radius** (d) **diameter**

B2 (a) $(180 - 40) \div 2 = \mathbf{70}^\circ$ (b) $\mathbf{45}^\circ$ (c) $180 - 50 = \mathbf{130}^\circ$ (d) $180 - 128 = \mathbf{52}^\circ$

Two radii always make an isosceles triangle.

B3 (a) $360 \div 5 = \mathbf{72}^\circ$ (b) $(180 - 72) \div 2 = \mathbf{54}^\circ$ (c) $\mathbf{36}^\circ$ (d) $(180 - 36) \div 2 = \mathbf{72}^\circ$



B4 (a) 14 cm (b) 11.5 cm (c) $6 + 6 + 8 = 20$ cm (d) the base angles are $(180 - 60) \div 2 = 60^\circ$, so the triangle is equilateral and $AB = 9$ cm

B5 (a) true (b) false: it touches the circle at one point (c) true

B6 (a) 100° (b) $180 - 2x$ (c) $3x = 180 - 2x$: 36 (d) $180 - 2x = x + 30$: 50

HIGHER

B7 (a) angle at the centre is twice the angle at the circumference: 50 (b) angles in the same segment are equal: 70
(c) angle in a semicircle is 90° : $180 - 90 - 35 = 55$ (d) $136 \div 2 = 68$

B8 (a) 108° (b) 85° (c) $4x + 20 = 180$: 40 (d) the exterior angle equals the interior opposite angle: 80°

B9 (a) 90° (tangent meets radius) (b) $180 - 90 - 30 = 60^\circ$ (c) 7 cm (tangents from a point are equal)
(d) $360 - 90 - 90 - 70 = 110^\circ$

B10 (a) alternate segment theorem: 55 (b) 60 (c) 62° (d) angles on a straight line: $180 - 48 - 71 = 61^\circ$

C**Reasoning solutions**

Your wording can be different. Check that your reason makes the same point.

FOUNDATION

C1 A radius is not a chord (it does not join two points on the circle). The longest chord is a diameter.

C2 A sector is between two radii and an arc. The region between a chord and an arc is a segment.

C3 OA and OB are both radii of the same circle, so they are equal. A triangle with two equal sides is isosceles.

C4 $OA = OB$ (radii), so angle $OBA = 35^\circ$ (base angles of an isosceles triangle). Angle $AOB = 180 - 35 - 35 = 110^\circ$ (angles in a triangle)



HIGHER

- C5** The angle at the centre is twice the angle at the circumference, not the other way round. For example, an angle of 40° at the circumference gives 80° at the centre.
- C6** A diameter makes a straight angle, 180° , at the centre. The angle at the circumference is half of this: 90°
- C7** Angle $BAD = 140 \div 2 = 70^\circ$ (the angle at the centre is twice the angle at the circumference)
 $x = 180 - 70 = \boxed{110}$ (opposite angles of a cyclic quadrilateral add up to 180°)
- C8** (a) Angle $BOD = 2a$ (the angle at the centre is twice the angle at the circumference)
(b) Angle $BCD = c$ stands on the other arc, so the reflex angle $BOD = 2c$. The angles at O make a full turn:
 $2a + 2c = 360$, so $a + c = 180^\circ$

D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

FOUNDATION

- D1** Triangle OAB is isosceles: angle $OBA = (180 - 80) \div 2 = 50^\circ$. Triangle OBC is isosceles: angle $OBC = (180 - 120) \div 2 = 30^\circ$
 $x = 50 + 30 = \boxed{80}$
- D2** (a) $360 \div 5 = \boxed{72}^\circ$ (b) $\boxed{54}^\circ$ (c) $54 + 54 = \boxed{108}^\circ$ (d) angle $BCA = (180 - 108) \div 2 = 36^\circ$, so $ACD = 108 - 36 = \boxed{72}^\circ$
- D3** Angle $AOC = 180 - 60 = 120^\circ$ (angles on a straight line). Triangle AOC is isosceles (radii): $x = (180 - 120) \div 2 = \boxed{30}$
- D4** Each hour is $360 \div 12 = 30^\circ$, so the angle is $4 \times 30 = \boxed{120}^\circ$. The triangle is isosceles (radii): base angles $\boxed{30}^\circ$ each



HIGHER

- D5** Angle $OAP = 90^\circ$ (tangent meets radius), so angle $OAB = 90 - 64 = 26^\circ$. Triangle OAB is isosceles: angle $OBA = 26^\circ$
 $x = 180 - 26 - 26 = 128$
- D6** $x = 85$: the exterior angle of a cyclic quadrilateral equals the interior opposite angle
 $y = 180 - 85 = 95$: angles on a straight line (or opposite angles of a cyclic quadrilateral)
- D7** The angle at the centre is twice the angle at the circumference: $3x = 2(x + 10)$, so $x = 20$ Angle $AOB = 3 \times 20 = 60^\circ$
- D8** (a) $OA = OB$ (radii), OP is common, and angles $OAP = OBP = 90^\circ$ (tangent meets radius). So the triangles are congruent (RHS).
 (b) Corresponding sides of congruent triangles are equal, so $PA = PB$. Corresponding angles are equal, so angle $APO =$ angle BPO : OP bisects angle APB

E

Exam practice solutions

Marks are shown on the right of each line.

FOUNDATION

Foundation and Higher crossover

- E1** (a) chord B1
 (b) tangent B1
 (c) sector B1
- E2** $OA = OB$, so the triangle is isosceles M1
 $(180 - 120) \div 2$ M1
 $x = 30$ A1
- E3** $360 \div 9 = 40^\circ$ M1 A1
 Base angle $(180 - 40) \div 2 = 70^\circ$ B1
- E4** Angle $OBA = 25^\circ$ and angle $OBC = 35^\circ$ (isosceles triangles, radii) M1 M1
 $x = 25 + 35 = 60$ A1



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- E5** The equal angles are the base angles, OAB and OBA , not angle AOB C1
 $180 - 38 - 38$ M1
 $= 104^\circ$ A1

HIGHER

- E6** Angle $AOB = 2 \times 40 = 80^\circ$ (angle at the centre is twice the angle at the circumference) B1
 Triangle OAB is isosceles (radii): $(180 - 80) \div 2$ M1
 $x = 50$ A1
- E7** Opposite angles of a cyclic quadrilateral add up to 180° C1
 $180 - 105$ M1
 $x = 75$ A1
- E8** Angle $ACB = 55^\circ$ (alternate segment theorem) B1 C1
 Angle $ABC = 55^\circ$ (alternate segment theorem, using the tangent on the other side, or isosceles triangle) B1
 $x = 180 - 55 - 55 = 70$ (angles in a triangle) A1
- E9** Let angle $ACO = p$ and angle $BCO = q$. $OA = OC$ (radii), so angle $OAC = p$; $OB = OC$, so angle $OBC = q$ M1
 The exterior angle of triangle OAC at O : angle $AOD = p + p = 2p$. Similarly angle $BOD = 2q$ M1 M1
 Angle $AOB = 2p + 2q = 2(p + q) = 2 \times \text{angle } ACB$ A1
 Every step uses equal radii and the exterior angle of a triangle C1

