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A-LEVEL MATHEMATICS • FURTHER PURE 2

Polar Coordinates

Worked Solutions

Pearson Edexcel International A Level (WFM02) • Calculator allowed: show every step

Full worked solutions for the free Polar Coordinates worksheet, A-Level and International A-Level Maths

A **Examples** & Your turn page 2

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Tap a section to jump straight to it

Full worked solutions for every question on the Further Pure 2 Polar Coordinates worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet | Exam questions by topic | Further Pure 2 exam questions | More worksheets

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A Your turn answers

Compare each line of your working with the solution.

CORE

A1 $r^2 = 6r\sin\theta$: $\boxed{x^2+y^2=6y}$, a circle centre $(0,3)$, radius 3

A2 $r^2\left(\cos^2\theta-\sin^2\theta\right)=9$: $\boxed{r^2\cos 2\theta = 9}$

A3 $r=0$ at $\theta=0$ and $\theta=\dfrac{\pi}{3}$; greatest $r=3$ at $\boxed{\theta=\dfrac{\pi}{6}}$

STRETCH

A4 $\dfrac{1}{2}\int_0^{2\pi}\left(16+8\sin\theta+\sin^2\theta\right)\mathrm{d}\theta=\dfrac{1}{2}(32\pi+\pi)$: $\boxed{\dfrac{33\pi}{2}}$

A5 $\dfrac{1}{2}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}16\cos^2 2\theta\,\mathrm{d}\theta$: $\boxed{2\pi}$

A6 $y=6\sin^2\theta\cos\theta$; $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=6\sin\theta\left(2\cos^2\theta-\sin^2\theta\right)=0$, $\tan^2\theta=2$: $\boxed{\theta=\arctan\sqrt{2},\ r=2\sqrt{2}}$

A7 $\dfrac{1}{2}\int_0^{\frac{\pi}{2}}\left(16-(2+2\sin\theta)^2\right)\mathrm{d}\theta=\dfrac{1}{2}[10\theta+8\cos\theta+\sin 2\theta]_0^{\frac{\pi}{2}}$: $\boxed{\dfrac{5\pi}{2}-4}$

B Fluency solutions

One mark for each correct part. The notes in italics explain why each sequence was chosen.

CORE

B1 **(a)** $\boxed{x^2+y^2=25}$ **(b)** $\boxed{y=x,\ x>0}$ **(c)** $\boxed{x=3}$ **(d)** $r^4=8r^2\sin\theta\cos\theta$: $\boxed{(x^2+y^2)^2=8xy}$

Multiply by r or r^2 to create x, y and x^2+y^2

B2 **(a)** $\boxed{r=4}$ **(b)** $\boxed{r=5\sec\theta}$ **(c)** $\boxed{r=6\sin\theta}$ **(d)** $\boxed{r^2\cos 2\theta=4}$

Substitute $x=r\cos\theta$, $y=r\sin\theta$

B3 **(a)** $\boxed{5, 3, 1}$ **(b)** $\boxed{5 \text{ and } 1}$ **(c)** $\boxed{r \geqslant 1, \text{ never } 0}$ **(d)** $\boxed{\text{the initial line}}$

A few values of r show the shape.

B4 **(a)** $\boxed{9\pi}$ **(b)** $\frac{1}{2}\int_0^{\frac{\pi}{4}} \sec^2\theta \, d\theta$: $\boxed{\frac{1}{2}}$ **(c)** $\boxed{\frac{3\pi}{2}}$ **(d)** $\boxed{\pi}$

(b) is a triangle and (d) a circle of radius 1: the formula agrees with geometry.

B5 **(a)** $\boxed{|\theta| \leqslant \frac{\pi}{6}, \frac{\pi}{2} \leqslant \theta \leqslant \frac{5\pi}{6}, -\frac{5\pi}{6} \leqslant \theta \leqslant -\frac{\pi}{2}}$ **(b)** $\boxed{3}$ **(c)** $\boxed{\frac{\pi}{3}}$

Only $r \geqslant 0$ is drawn.

B6 **(a)** $\boxed{2\cos\theta(1+2\sin\theta)}$ **(b)** $\boxed{\left(4, \frac{\pi}{2}\right), \left(1, -\frac{\pi}{6}\right), \left(1, -\frac{5\pi}{6}\right)}$

(c) $\boxed{-2(2\sin\theta - 1)(\sin\theta + 1)}$ **(d)** $\boxed{\left(3, \frac{\pi}{6}\right), \left(3, \frac{5\pi}{6}\right)}$

Parallel: $\frac{dy}{d\theta} = 0$*; perpendicular:* $\frac{dx}{d\theta} = 0$

STRETCH

B7 **(a)** $4\cos\theta = 2$: $\boxed{\theta = \pm\frac{\pi}{3}}$ **(b)** $\int_0^{\frac{\pi}{3}} \left(16\cos^2\theta - 4\right) d\theta$: $\boxed{\frac{4\pi}{3} + 2\sqrt{3}}$ **(c)** $\boxed{\frac{8\pi}{3} - 2\sqrt{3}}$ **(d)** sum 4π

Split at the intersections.

B8 **(a)** $2\sec\theta = 4$: $\boxed{\theta = \pm\frac{\pi}{3}}$ **(b)** $\boxed{\frac{16\pi}{3}}$ **(c)** $\frac{1}{2} \times 2 \times 4\sqrt{3}$: $\boxed{4\sqrt{3}}$ **(d)** $\boxed{\frac{16\pi}{3} - 4\sqrt{3}}$

A sector minus a triangle gives a segment.

B9 **(a)** $\boxed{A\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right), B(0, 5)}$ **(b)** $9 + 25 - 30\cos\frac{\pi}{3}$: $\boxed{\sqrt{19}}$ **(c)** $\frac{27}{4} + \frac{49}{4} = 19$ **(d)** $\boxed{\frac{15\sqrt{3}}{4}}$

The angle between OA and OB is the difference of the angles.

B10 **(a)** differentiate $y = f(\theta)\sin\theta$ by the product rule **(b)** $\boxed{f'(\theta)\cos\theta - f(\theta)\sin\theta}$

(c) $y = \sin 2\theta$: $\boxed{\left(\sqrt{2}, \frac{\pi}{4}\right)}$ **(d)** $x = 2\cos^2\theta$: $\boxed{(2, 0)}$

The general results apply to every curve.

C Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 The area is $\frac{1}{2}\int r^2\,\mathrm{d}\theta$: $\boxed{9\pi}$

C2 $(a+b)^2$ has a middle term: $\boxed{r^2 = 4 + 4\sin\theta + \sin^2\theta}$

C3 0 to 2π covers the whole curve; one loop is $-\frac{\pi}{4} \leqslant \theta \leqslant \frac{\pi}{4}$: $\boxed{\frac{9\pi}{8}}$

C4 Points with $r < 0$ are not drawn: $\boxed{0 \leqslant \theta \leqslant \frac{\pi}{4}, \frac{3\pi}{4} \leqslant \theta \leqslant \frac{5\pi}{4}, \frac{7\pi}{4} \leqslant \theta < 2\pi}$

STRETCH

C5 $\frac{\mathrm{d}r}{\mathrm{d}\theta} = 0$ gives the greatest or least distance from O, not a horizontal tangent: $\boxed{\frac{\mathrm{d}y}{\mathrm{d}\theta} = 0 \text{ with } y = r\sin\theta}$

C6 Both curves also pass through the pole (at different θ): $\boxed{O \text{ and } \left(\sqrt{2}, \frac{\pi}{4}\right)}$

C7 He has found the sector OPQ, which includes the triangle OPQ; subtract it: $\frac{4\pi}{3} + 2\sqrt{3} - 3\sqrt{3}$: $\boxed{\frac{4\pi}{3} - \sqrt{3}}$

C8 **(a)** $\frac{1}{2}\int_0^{\alpha} a^2\,\mathrm{d}\theta = \frac{1}{2}a^2\alpha$, as required

(b) $\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4a^2\cos^2\theta\,\mathrm{d}\theta = a^2\left[\theta + \frac{1}{2}\sin 2\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \pi a^2$; the curve is the circle $x^2 + y^2 = 2ax$ of radius a

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D Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $\frac{1}{2}\int_0^{\frac{\pi}{2}}\left(4+4\sin\theta+\frac{1}{2}-\frac{1}{2}\cos 2\theta\right)\mathrm{d}\theta=\frac{1}{2}\left[\frac{9}{2}\theta-4\cos\theta-\frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{2}}=\frac{1}{2}\left(\frac{9\pi}{4}+4\right)$: $\boxed{\frac{9\pi}{8}+2}$

D2 **(a)** $4\sin\theta=2\sqrt{2}$: $\boxed{\theta=\frac{\pi}{4},\frac{3\pi}{4}}$

(b) $\frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}\left(16\sin^2\theta-8\right)\mathrm{d}\theta=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}-4\cos 2\theta\,\mathrm{d}\theta=\left[-2\sin 2\theta\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$: $\boxed{4}$

D3 $r\geqslant 0$ for $-\frac{\pi}{6}\leqslant\theta\leqslant\frac{\pi}{6}$; $\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}9\cos^2 3\theta\,\mathrm{d}\theta=\frac{9}{4}\left[\theta+\frac{1}{6}\sin 6\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$: $\boxed{\frac{3\pi}{4}}$

D4 $x=8\sin\theta\cos^2\theta$; $\frac{\mathrm{d}x}{\mathrm{d}\theta}=8\cos\theta\left(\cos^2\theta-2\sin^2\theta\right)=0$, $\tan^2\theta=\frac{1}{2}$; $r=\frac{8\tan\theta}{1+\tan^2\theta}$: $\boxed{\left(\frac{8\sqrt{2}}{3},\arctan\frac{1}{\sqrt{2}}\right)}$

STRETCH

D5 **(a)** $y=4\cos^2\theta\sin\theta$; $\frac{\mathrm{d}y}{\mathrm{d}\theta}=4\cos\theta\left(\cos^2\theta-2\sin^2\theta\right)=0$ gives $\tan^2\theta=\frac{1}{2}$, as required; $\cos^2\theta=\frac{2}{3}$: $\boxed{r=\frac{8}{3}}$

(b) $\frac{1}{2}\int_0^{2\pi}16\cos^4\theta\,\mathrm{d}\theta=8\times\frac{3\pi}{4}$: $\boxed{6\pi}$

D6 **(a)** $\tan\theta=\sqrt{3}$: $\boxed{P\left(\sqrt{3},\frac{\pi}{3}\right)}$

(b) $\frac{1}{2}\int_0^{\frac{\pi}{3}}4\sin^2\theta\,\mathrm{d}\theta+\frac{1}{2}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}12\cos^2\theta\,\mathrm{d}\theta=\left(\frac{\pi}{3}-\frac{\sqrt{3}}{4}\right)+\left(\frac{\pi}{2}-\frac{3\sqrt{3}}{4}\right)$: $\boxed{\frac{5\pi}{6}-\sqrt{3}}$

D7 **(a)** $4\cos\theta=3\sec\theta$, $\cos^2\theta=\frac{3}{4}$: $\boxed{\left(2\sqrt{3},\pm\frac{\pi}{6}\right)}$

(b) sector $\frac{1}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}16\cos^2\theta\,\mathrm{d}\theta=\frac{4\pi}{3}+2\sqrt{3}$; triangle $\frac{1}{2}\times 3\times 2\sqrt{3}=3\sqrt{3}$: $\boxed{\frac{4\pi}{3}-\sqrt{3}}$

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D8 **(a)** $\frac{1}{2}\int_0^{2\pi}\left(9+12\sin\theta+4\sin^2\theta\right)\mathrm{d}\theta=\frac{1}{2}(18\pi+4\pi)$: $\boxed{11\pi}$

(b) $y=(3+2\sin\theta)\sin\theta$; $\frac{\mathrm{d}y}{\mathrm{d}\theta}=\cos\theta(3+4\sin\theta)=0$: $\boxed{\left(5,\frac{\pi}{2}\right),\left(1,\frac{3\pi}{2}\right),\left(\frac{3}{2},3.99\right),\left(\frac{3}{2},5.44\right)}$

E Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE *Exam questions for everyone*

E1 **(a)** $\boxed{r=\frac{3}{2}}$ *B1*

(b) $\frac{1}{2}\int_0^{\frac{\pi}{6}}9\cos^2 2\theta\,\mathrm{d}\theta=\frac{9}{4}\int_0^{\frac{\pi}{6}}(1+\cos 4\theta)\,\mathrm{d}\theta$ *M1*

$=\frac{9}{4}\left[\theta+\frac{1}{4}\sin 4\theta\right]_0^{\frac{\pi}{6}}$ *A1*

$=\frac{9}{4}\left(\frac{\pi}{6}+\frac{\sqrt{3}}{8}\right)=\frac{3\pi}{8}+\frac{9\sqrt{3}}{32}$, as required *A1**

(c) $\frac{9}{4}\left[\theta+\frac{1}{4}\sin 4\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}=\frac{9}{4}\left(\frac{\pi}{4}-\frac{\pi}{6}-\frac{\sqrt{3}}{8}\right)$ *M1*

$\boxed{\frac{3\pi}{16}-\frac{9\sqrt{3}}{32}}$ *A1*

E2 **(a)** $y=3(1-\cos\theta)\sin\theta$ *M1*

$\frac{\mathrm{d}y}{\mathrm{d}\theta}=3(\cos\theta-\cos 2\theta)=-3(2\cos\theta+1)(\cos\theta-1)=0$ *M1*

$\cos\theta=-\frac{1}{2}$: $\boxed{P\left(\frac{9}{2},\frac{2\pi}{3}\right)}$ *A1*

(b) $\frac{1}{2}\int_0^{\frac{2\pi}{3}}9(1-\cos\theta)^2\,\mathrm{d}\theta=\frac{9}{2}\int_0^{\frac{2\pi}{3}}\left(\frac{3}{2}-2\cos\theta+\frac{1}{2}\cos 2\theta\right)\mathrm{d}\theta$ *M1 A1*

$=\frac{9}{2}\left[\frac{3}{2}\theta-2\sin\theta+\frac{1}{4}\sin 2\theta\right]_0^{\frac{2\pi}{3}}=\frac{9}{2}\left(\pi-\sqrt{3}-\frac{\sqrt{3}}{8}\right)$ *M1*

$\boxed{\frac{9\pi}{2}-\frac{81\sqrt{3}}{16}}$ *A1*

STRETCH

E3 **(a)** $2(1+\cos\theta)=3,\ \cos\theta=\frac{1}{2}$ *M1*

$\boxed{A\left(3,\frac{\pi}{3}\right),B\left(3,-\frac{\pi}{3}\right)}$ *A1*

(b) $2\times\frac{1}{2}\int_0^{\frac{\pi}{3}}\left(4(1+\cos\theta)^2-9\right)\mathrm{d}\theta$ *M1*

$=\int_0^{\frac{\pi}{3}}(-3+8\cos\theta+2\cos2\theta)\,\mathrm{d}\theta$ *M1 A1*

$=[-3\theta+8\sin\theta+\sin2\theta]_0^{\frac{\pi}{3}}=-\pi+4\sqrt{3}+\frac{\sqrt{3}}{2}$ *M1*

$=\frac{9\sqrt{3}}{2}-\pi$, as required *A1**

(c) Area of C_1: $\frac{1}{2}\int_0^{2\pi}4(1+\cos\theta)^2\,\mathrm{d}\theta=6\pi$ *M1*

$6\pi-\left(\frac{9\sqrt{3}}{2}-\pi\right)$: $\boxed{7\pi-\frac{9\sqrt{3}}{2}}$ *A1*

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