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INTERNATIONAL A LEVEL MATHEMATICS • STATISTICS 2

Statistics 2

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WST02/01) • Calculator and Statistical Tables allowed

Multi-step exam questions at the difficulty of the real WST02 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Binomial Distributions** Conditions, probabilities and tables, mean and variance, unknown n and p	10	80	2	
2	**Poisson Distributions** Probabilities, modelling conditions, scaling the rate, adding Poisson variables	10	80	28	
3	**Approximations** Poisson and normal approximations, continuity corrections	10	80	54	
4	**Continuous Random Variables** Probability density and cumulative distribution functions, mean, variance, median, mode	10	81	80	
5	**Continuous Uniform Distribution** Rectangular distribution, mean and variance, modelling	10	80	106	
6	**Sampling and Sampling Distributions** Populations and samples, statistics, sampling distributions	10	80	132	
7	**Hypothesis Testing** Critical regions, significance levels, binomial and Poisson tests	10	80	158	

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CHAPTER 1 • KEY SKILLS

1 Binomial Distributions

Specification S2: Binomial distribution • 10 questions • 80 marks • about 96 minutes

1 The model

A fixed number of independent trials, each a success or failure, with constant probability p. Write $X \sim \mathrm{B}(n, p)$, with both parameters.

2 The formula

$\mathrm{P}(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$. Use it for a single value, or when n or p is not in the tables.

3 Using the tables

Tables give $\mathrm{P}(X \leqslant x)$.

$\mathrm{P}(X \geqslant 5) = 1 - \mathrm{P}(X \leqslant 4)$;

$\mathrm{P}(2 \leqslant X < 6) = \mathrm{P}(X \leqslant 5) - \mathrm{P}(X \leqslant 1)$

4 When $p > 0.5$

Count failures instead: if $X \sim \mathrm{B}(20, 0.9)$ then $Y = 20 - X \sim \mathrm{B}(20, 0.1)$, and $\mathrm{P}(X \geqslant 18) = \mathrm{P}(Y \leqslant 2)$

5 Mean and variance

$\mathrm{E}(X) = np$ and $\mathrm{Var}(X) = np(1-p)$.

Square a standard deviation first; $\frac{\mathrm{Var}(X)}{\mathrm{E}(X)} = 1 - p$

6 Conditional probability

$\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$, e.g.

$\mathrm{P}(X = 3 \mid X \leqslant 4) = \dfrac{\mathrm{P}(X = 3)}{\mathrm{P}(X \leqslant 4)}$

7 Two-stage models

First find the probability of one "success" (a round won, a bad week), then use a new binomial for how many of these happen.

8 Finding n

$\mathrm{P}(\text{at least one}) = 1 - (1-p)^n$. Solve $(1-p)^n < k$ with logarithms, then round n up.

9 Accuracy

Table values to 4 decimal places; other answers to 3 significant figures. Show the probability statement before the value.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 The model	•	•			•	•	•			
2 The formula	•	•		•		•		•	•	•
3 Using the tables	•		•	•		•	•	•	•	•
4 When $p > 0.5$		•								
5 Mean and variance			•			•	•	•		•
6 Conditional probability				•			•	•		
7 Two-stage models						•	•		•	
8 Finding n					•					
9 Accuracy	•	•	•	•		•	•	•	•	•
Interleaved: probability, tree diagrams, expectation (S1)				•		•	•		•	
Interleaved: logarithms and inequalities (P1, P2)					•					•
Interleaved: algebra and solving equations (P1)			•					•	•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WST02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Define the random variable and its distribution in full, e.g. "X = number of seeds that grow, $X \sim \mathrm{B}(12, 0.3)$". Write the probability statement, then its rewriting for the tables, then the value.

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1. A spinner lands on red with probability 0.3 each time it is spun. The spinner is spun 12 times, and X is the number of times it lands on red.

(a) Write down the distribution of X, giving any parameters.

(1)

(b) Find P($X = 4$)

(2)

(c) Find P($X \geqslant 5$)

(2)

(d) Find P($2 \leqslant X < 6$)

(1)

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Question 1 continued

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2. The probability that a battery of a particular type lasts longer than its guaranteed life is 0.9. Batteries are sold in packs of 20, and the batteries in a pack may be regarded as independent.

(a) Find the probability that at least 18 batteries in a pack last longer than their guaranteed life.

(3)

(b) Find the probability that all 20 batteries in a pack last longer than their guaranteed life.

(1)

(c) Find the probability that at most 16 batteries in a pack last longer than their guaranteed life.

(2)

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Question 2 continued

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3. The random variable $X \sim \text{B}(n, p)$. Given that $\text{E}(X) = 6$ and $\text{Var}(X) = 4.2$

(a) Find the value of n and the value of p.

(3)

(b) Find $\text{P}(X < 4)$

(1)

(c) Find the probability that X is more than one standard deviation above its mean.

(2)

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Question 3 continued

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4. The random variable $X \sim \text{B}(10, 0.4)$

(a) Find $\text{P}(X \leqslant 4)$

(1)

(b) Find $\text{P}(X = 3)$

(2)

(c) Find $\text{P}(X = 3 \mid X \leqslant 4)$

(3)

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Question 4 continued

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5. A machine produces components, and 5% of its components are defective, independently of each other. A random sample of n components is taken.

(a) Show that the probability that the sample contains at least one defective component is greater than 0.9 when $0.95^n < 0.1$

(2)

(b) Find the smallest value of n for which the probability that the sample contains at least one defective component is greater than 0.9

(3)

(c) State one assumption needed for the binomial model to be valid here.

(1)

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Question 5 continued

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6. In a game, a player throws a ball at a target 5 times. On each throw the probability of hitting the target is 0.4, independently of all other throws. The player wins the round if the target is hit at least 3 times.

(a) Show that the probability of winning a round is 0.3174

(3)

A player plays 8 rounds.

(b) Find the probability that the player wins exactly 3 rounds.

(3)

(c) Find the probability that the player's first win is in the fourth round.

(2)

(d) Find the mean and the variance of the number of rounds won in 8 rounds.

(2)

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Question 6 continued

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7. A factory has two machines, A and B. Machine A makes 60% of the factory's items and 5% of these are defective. Machine B makes the rest of the items and 17.5% of these are defective.

(a) Show that the probability that a randomly chosen item is defective is 0.1

(2)

A random sample of 20 items is taken from the factory's production.

(b) Find the probability that the sample contains more than 3 defective items.

(2)

(c) Given that an item is defective, find the probability that it was made by machine A.

(2)

A batch of 50 items is checked.

(d) Find the mean and the standard deviation of the number of defective items in the batch.

(2)

(e) Find the probability that the number of defective items in the batch is no more than its mean.

(2)

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Question 7 continued

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8. The random variable $X \sim \text{B}(7, p)$, where $0 < p < 1$. Given that $\text{P}(X = 2) = 2\,\text{P}(X = 1)$

(a) Show that $p = 0.4$

(4)

(b) Find $\text{P}(X \geqslant 4)$

(2)

(c) Find $\text{P}(X = 5 \mid X \geqslant 4)$

(2)

(d) Find $\text{E}(X)$ and $\text{Var}(X)$.

(2)

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Question 8 continued

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9. At a fair, a player pays 50p to play a game. A spinner, which lands on red with probability 0.3, is spun 5 times. If it lands on red at least 3 times, the player receives a prize of £k; otherwise the player receives nothing.

(a) Find the probability that a player wins the prize.

(2)

(b) Find the value of k for which the game is fair, so that the expected amount a player receives equals the cost of playing.

(2)

The organiser sets $k = 3$

(c) Find the organiser's expected profit from 200 games.

(3)

(d) Find the probability that the organiser makes a loss over 10 games, and comment on your answer in the light of part (c).

(3)

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Question 9 continued

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Question 9 continued

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10. The random variable $Y \sim \text{B}(n, p)$ has mean 6 and variance 3.6

(a) Find the value of n and the value of p.

(3)

(b) Show that $\dfrac{\text{P}(Y = r+1)}{\text{P}(Y = r)} = \dfrac{2(15-r)}{3(r+1)}$ and hence show that the mode of Y is 6

(4)

(c) Find $\text{P}(Y = 6)$

(1)

(d) Find $\text{P}(|Y-6| \leqslant 2)$

(2)

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Question 10 continued

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TOTAL FOR CHAPTER 1 IS 80 MARKS

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CHAPTER 2 • KEY SKILLS

2 Poisson Distributions

Specification S2: Poisson distribution • 10 questions • 80 marks • about 96 minutes

1 The model

Events in an interval of time or space, at a mean rate λ: $X \sim \text{Po}(\lambda)$, with the parameter stated.

2 The formula

$\text{P}(X = r) = \dfrac{\text{e}^{-\lambda}\lambda^r}{r!}$; in particular $\text{P}(X = 0) = \text{e}^{-\lambda}$

3 Using the tables

Tables give $\text{P}(X \leqslant x)$: $\text{P}(X > 5) = 1 - \text{P}(X \leqslant 5)$ and $\text{P}(X = 3) = \text{P}(X \leqslant 3) - \text{P}(X \leqslant 2)$

4 Scaling the rate

7 per 10 minutes is 3.5 per 5 minutes and 0.7 per minute. Scale λ in proportion to the length of the interval.

5 Conditions in context

The events occur singly, independently, randomly, at a constant rate. Name the context (calls, customers, defects).

6 Mean and variance

$\text{E}(X) = \text{Var}(X) = \lambda$. If a sample has mean close to variance, a Poisson model is plausible.

7 Adding

If $X \sim \text{Po}(\lambda)$ and $Y \sim \text{Po}(\mu)$ are independent, then $X + Y \sim \text{Po}(\lambda + \mu)$

8 Inside other models

A Poisson probability can become the p of a binomial ("busy minutes"), or part of a conditional probability.

9 Finding λ or t

$\text{e}^{-\lambda t} = k$ gives $t = -\dfrac{\ln k}{\lambda}$. Equations between probabilities cancel $\text{e}^{-\lambda}$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 The model	•					•	•	•		•
2 The formula	•	•		•	•		•	•	•	
3 Using the tables	•	•	•	•		•	•	•	•	•
4 Scaling the rate		•		•	•	•	•	•		
5 Conditions in context	•					•				•
6 Mean and variance			•							
7 Adding				•				•	•	
8 Inside other models						•	•	•		•
9 Finding λ or t					•			•	•	
Interleaved: binomial distribution (Chapter 1)						•				
Interleaved: conditional probability and expectation (S1)			•	•			•	•		•
Interleaved: exponentials and logarithms (P2)					•			•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WST02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write the distribution with its parameter for the interval in the question, e.g. "$Y \sim$ Po(3.5) for calls in 5 minutes". State conditions in the context of the question, with "rate", not "probability".

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1. A shop sells pairs of running shoes at a mean rate of 4 pairs per hour. The number of pairs sold in an hour is X.

(a) Write down a suitable distribution to model X.

(1)

(b) State one condition, in context, needed for this distribution to be a suitable model.

(1)

(c) Find P(X = 6)

(2)

(d) Find the probability that more than 5 pairs are sold in an hour.

(2)

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Question 1 continued

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2. Telephone calls arrive at a help desk at a mean rate of 7 every 10 minutes.

(a) Find the probability that exactly 3 calls arrive in a 5-minute period.

(2)

(b) Find the probability that at least 10 calls arrive in a 10-minute period.

(2)

(c) Find the probability that no calls arrive in a 1-minute period.

(2)

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Question 2 continued

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3. The numbers of flaws in 100 one-metre lengths of fabric were recorded.

Number of flaws, x	0	1	2	3	4	5
Frequency, f	22	33	25	13	5	2

(a) Find the mean and the variance of the number of flaws per metre.

(3)

(b) Explain how your answers to part (a) support a Poisson model for the number of flaws per metre.

(1)

(c) Using a Poisson distribution with mean 1.5, find the probability that a one-metre length has at least 3 flaws, and compare it with the data.

(2)

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Question 3 continued

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4. Emails arrive in the inbox of Ana at a mean rate of 2.5 per hour and in the inbox of Ben at a mean rate of 1.5 per hour. The numbers of emails arriving in the two inboxes are independent Poisson variables.

(a) Find the probability that exactly 4 emails arrive altogether in a one-hour period.

(2)

(b) Find the probability that no more than 5 emails arrive altogether in a two-hour period.

(2)

(c) Find the probability that, in a one-hour period, exactly 2 emails arrive for Ana and exactly 2 for Ben.

(2)

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5. Faults occur in a computer network at a mean rate of 0.2 per hour, and the number of faults in any time interval has a Poisson distribution.

(a) Find the longest time, in minutes, for which the probability of no faults is at least 0.9

(3)

The random variable $X \sim \text{Po}(\lambda)$ has $\text{P}(X = 0) = 0.0498$

(b) Find the value of λ to the nearest whole number.

(2)

(c) Hence find $\text{P}(X = 2)$

(1)

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6. Cars pass a checkpoint at a mean rate of 2.5 per minute. A minute is called "busy" if at least 5 cars pass the checkpoint in that minute.

(a) Show that the probability that a minute is busy is 0.1088

(2)

(b) Find the probability that at least 2 of 10 separate one-minute periods are busy.

(4)

(c) Find the probability that at least 12 cars pass the checkpoint in a 4-minute period.

(2)

(d) State, in context, one condition needed for the Poisson model, and one assumption needed for the model in part (b).

(2)

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Question 6 continued

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7. The number of goals scored in a football match is modelled by a Poisson distribution with mean 3

(a) Find the probability that more than 3 goals are scored in a match.

(2)

(b) Given that at least one goal is scored in a match, find the probability that exactly 2 goals are scored.

(3)

(c) Find the probability that no more than 6 goals are scored altogether in two matches.

(2)

(d) Show that 2 and 3 are equally likely numbers of goals in a match, and explain why these are the modes.

(2)

(e) Suggest one reason why the Poisson model may not be suitable for goals scored in a match.

(1)

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Question 7 continued

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Question 7 continued

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8. A cable has two types of defect, which occur independently. Type A defects occur at a mean rate of 0.6 per 10 m and type B defects at a mean rate of 0.4 per 10 m, each following a Poisson distribution.

(a) Find the probability that a 10 m length of cable has no defects of either type.

(1)

(b) Find the probability that a 50 m length of cable has at least 3 defects.

(2)

(c) Find the length of cable, in metres, for which the probability of no defects is 0.5

(3)

A 10 m length of cable has exactly 2 defects.

(d) Find the probability that both defects are in the first 5 m of this length.

(4)

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Question 8 continued

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(Total for Question 8 is 10 marks)

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9. The random variable $X \sim \text{Po}(\lambda)$, where $\lambda > 0$, and $\text{P}(X = 2) = 3\,\text{P}(X = 4)$

(a) Show that $\lambda = 2$

(3)

(b) Find $\text{P}(X \geqslant 4)$

(2)

The random variable $Y \sim \text{Po}(\mu)$ is independent of X, and $\text{P}(X + Y = 0) = \text{e}^{-5}$

(c) Find the value of μ.

(3)

(d) Find $\text{P}(X + Y \leqslant 3)$

(2)

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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 10 marks)

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10. A shop hires out boats by the day and has 5 boats. The daily demand for boats, X, is modelled by a Poisson distribution with mean 3.5

(a) Find the probability that, on a given day, the demand cannot all be met.

(2)

(b) Find the expected number of boats hired out on a day.

(4)

(c) Find the smallest number of boats the shop needs so that the probability that the demand cannot all be met is less than 0.05

(2)

(d) Given that the demand on a particular day is more than 3, find the probability that it cannot all be met.

(2)

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 2 IS 80 MARKS

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CHAPTER 3 • KEY SKILLS

3 Approximations

Specification S2: Approximations • 10 questions • 80 marks • about 96 minutes

1 Poisson for binomial

n large and p small (close to 0):

$\mathrm{B}(n, p) \approx \mathrm{Po}(np)$

2 When p is close to 1

Count the other outcome first: 98% germinate, so failures

$\sim \mathrm{B}(250, 0.02) \approx \mathrm{Po}(5)$

3 Normal for binomial

n large and p close to 0.5:

$\mathrm{B}(n, p) \approx \mathrm{N}\big(np,\ np(1-p)\big)$

4 Normal for Poisson

λ large: $\mathrm{Po}(\lambda) \approx \mathrm{N}(\lambda, \lambda)$, so the standard deviation is $\sqrt{\lambda}$

5 Continuity correction

$\mathrm{P}(X \geqslant 70) \rightarrow \mathrm{P}(Y > 69.5)$;

$\mathrm{P}(X < 70) \rightarrow \mathrm{P}(Y < 69.5)$;

$\mathrm{P}(X = 40) \rightarrow \mathrm{P}(39.5 < Y < 40.5)$

6 Standardising

$z = \dfrac{x - \mu}{\sigma}$ to 2 decimal places, then the tables. Use the standard deviation, not the variance.

7 Choosing and justifying

Quote the condition with the numbers: "$n = 150$ is large and $p = 0.02$ is small".

8 Working backwards

Use the percentage points: $\mathrm{P}(Z > 2.3263) = 0.01$, then undo the continuity correction and round up to a whole number.

9 Comparing with exact

An approximation is good when it is close to the exact probability; percentage error

$= \dfrac{|\text{approx} - \text{exact}|}{\text{exact}} \times 100$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Poisson for binomial	•			•				•		•
2 When p is close to 1				•						
3 Normal for binomial		•			•		•	•		•
4 Normal for Poisson			•			•			•	
5 Continuity correction		•	•		•	•	•	•	•	•
6 Standardising		•	•		•	•	•	•	•	•
7 Choosing and justifying	•	•		•		•		•		•
8 Working backwards							•		•	
9 Comparing with exact					•		•			•
Interleaved: binomial and Poisson distributions (Chapters 1, 2)	•	•	•	•	•	•	•	•	•	•
Interleaved: the normal distribution and percentage points (S1)		•	•		•	•	•	•	•	•
Interleaved: inequalities (P1)							•		•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WST02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write the original distribution, then the approximating distribution with its parameters. Show the continuity correction and the z-value. When p is close to 1, count the other outcome before approximating.

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1. On average, 2% of the eggs from a farm are cracked. A box contains 150 eggs, and X is the number of cracked eggs in the box.

(a) Using a suitable approximation, find $P(X \leqslant 2)$

(3)

(b) Give a reason why your approximation is suitable.

(1)

(c) Using the same approximation, find $P(X = 4)$

(2)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. In a large town, 40% of adults support a new bypass. A random sample of 150 adults is taken, and X is the number in the sample who support the bypass.

(a) Using a suitable approximation, find the probability that at least 70 adults in the sample support the bypass.

(4)

(b) Explain why a normal approximation is suitable here.

(1)

(c) Explain why a continuity correction is needed.

(1)

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Question 2 continued

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3. Cars arrive at a car park at a mean rate of 40 per hour. The number of cars arriving in an hour is modelled by a Poisson distribution.

(a) Using a suitable approximation, find the probability that more than 50 cars arrive in an hour.

(4)

(b) Using the same approximation, find the probability that exactly 40 cars arrive in an hour.

(2)

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Question 3 continued

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4. On average 98% of the seeds of a certain plant germinate. A packet contains 250 seeds.

(a) Explain why a normal approximation to the number of seeds that germinate is not suitable.

(1)

(b) Using a suitable approximation, find the probability that more than 245 seeds in the packet germinate.

(4)

(c) Find the probability that at least 240 seeds in the packet germinate.

(1)

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Question 4 continued

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(Total for Question 4 is 6 marks)

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5. The random variable $X \sim \text{B}(30, 0.5)$

(a) Find the exact value of $\text{P}(X \leqslant 12)$

(1)

(b) Use a normal approximation to estimate $\text{P}(X \leqslant 12)$

(3)

(c) Find the percentage error in your estimate, and comment on the accuracy of the approximation.

(2)

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Question 5 continued

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(Total for Question 5 is 6 marks)

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6. Customers arrive at a ticket office at a mean rate of 1.4 per minute, and the number arriving in any interval follows a Poisson distribution.

(a) Find the probability that at least 3 customers arrive in a one-minute period.

(2)

(b) Find the probability that, in 5 separate one-minute periods, exactly one has no customers arriving.

(3)

(c) Using a suitable approximation, find the probability that at least 50 customers arrive in a 30-minute period.

(4)

(d) Explain why the approximation in part (c) is suitable.

(1)

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Question 6 continued

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Question 6 continued

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7. A test has 50 multiple-choice questions, each with 4 possible answers, only one of which is correct. A candidate guesses the answer to every question at random. X is the number of correct answers.

(a) Find the mean and the variance of X.

(2)

(b) Using a normal approximation, find P($X \geqslant 20$)

(4)

The pass mark is k. The examiners want the probability that a candidate who guesses every answer passes to be less than 1%.

(c) Using a normal approximation, find the smallest possible value of k.

(4)

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Question 7 continued

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Question 7 continued

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8. On average, 1.5% of the bolts made by a machine are faulty. The bolts are packed in boxes of 400, and a box is rejected if it contains more than 8 faulty bolts.

(a) Using a suitable approximation, show that the probability that a box is rejected is 0.1528

(3)

A sample of 60 boxes is inspected, and R is the number of boxes rejected.

(b) Using a normal approximation, find the probability that at least 12 boxes are rejected.

(5)

(c) Comment on the suitability of the normal approximation in part (b).

(2)

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Question 8 continued

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Question 8 continued

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9. A small company receives emails at a mean rate of 25 per working day. The number of emails received in any period is modelled by a Poisson distribution.

(a) Using a suitable approximation, find the probability that no more than 20 emails are received on a working day.

(3)

The company's system can deal with k emails in a day. It wants the probability of receiving more than k emails in a day to be less than 0.05

(b) Using a normal approximation, find the smallest possible value of k.

(4)

(c) Find the probability that more than 140 emails are received in a 5-day working week.

(3)

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Question 9 continued

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Question 9 continued

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10. The random variable $X \sim \text{B}(n, p)$ has mean 6 and variance 5.88

(a) Find the value of n and the value of p.

(3)

(b) Use a Poisson approximation to estimate $\text{P}(X \leqslant 4)$

(2)

(c) Use a normal approximation to estimate $\text{P}(X \leqslant 4)$

(3)

The exact value of $\text{P}(X \leqslant 4)$ is 0.2824

(d) State, with a reason, which approximation is more suitable here.

(2)

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Question 10 continued

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TOTAL FOR CHAPTER 3 IS 80 MARKS

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CHAPTER 4 • KEY SKILLS

4 Continuous Random Variables

Specification S2: Continuous random variables • 10 questions • 81 marks • about 97 minutes

1 Probability density

$f(x) \geqslant 0$ and the total area is 1: $\int f(x)\,dx = 1$. Use this to find an unknown constant.

2 Probabilities

$P(a < X < b) = \int_a^b f(x)\,dx$ (an area).

For a continuous variable $P(X = a) = 0$, so $<$ and $\leqslant$ give the same answer.

3 The cdf

$F(x) = P(X \leqslant x)$. Integrate each piece from its start, adding F at the join. Write $F(x) = 0$ and $F(x) = 1$ outside the range.

4 pdf from cdf

$f(x) = F'(x)$: differentiate, do not integrate. Beyond the range, F = 1

5 Mean and variance

$E(X) = \int xf(x)\,dx$,

$Var(X) = E(X^2) - [E(X)]^2$.

$E(aX + b) = aE(X) + b$,

$Var(aX + b) = a^2 Var(X)$

6 Median and quartiles

Solve $F(m) = 0.5$ (or 0.25, 0.75) using the piece that contains the answer; reject roots outside its range.

7 Mode

The value where f is greatest: solve $f'(x) = 0$, not $f(x) = 0$, or use an end point.

8 Skewness

Mean $<$ median $<$ mode: negative skew. Mode $<$ median $<$ mean: positive skew. Symmetric: all equal.

9 Locating a value

Evaluate F either side, e.g. $F(3.6) < 0.5 < F(3.7)$, and conclude "so the median lies between 3.6 and 3.7".

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	11
1 Probability density	•				•	•		•		•
2 Probabilities	•	•		•	•			•	•	•
3 The cdf		•				•			•	
4 pdf from cdf				•					•	
5 Mean and variance			•			•	•	•	•	•
6 Median and quartiles		•		•	•	•		•		
7 Mode							•			
8 Skewness							•			
9 Locating a value							•			
Interleaved: integration and differentiation of polynomials (P1, P2)	•	•	•	•	•	•	•	•	•	•
Interleaved: expectation algebra and quadratics (S1, P1)			•	•	•	•			•	•
Interleaved: binomial distribution (Chapter 1)										•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WST02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. When asked to "use algebraic integration", show the integral, the integrated expression and the substituted limits. Define a cdf for every value of x. Use symmetry when the graph shows it.

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1. The continuous random variable X has probability density function

$$f(x) = \begin{cases} k\left(x^2+1\right) & 0 \leqslant x \leqslant 3 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{12}$

(2)

(b) Use algebraic integration to find $P(1 < X < 2)$

(3)

(c) Write down $P(X = 2)$

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. The time, X minutes, that a customer waits to be served at a bank is modelled by the probability density function

$$f(x) = \begin{cases} \frac{x}{8} & 0 \leqslant x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the cumulative distribution function $F(x)$ for all values of x.

(3)

(b) Find the median waiting time, giving your answer to 3 significant figures.

(2)

(c) Find the probability that a customer waits more than 3 minutes.

(1)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3. The depth of snow, X metres, at a ski station on a random day in January is modelled by a random variable with probability density function

$$f(x) = \begin{cases} \frac{3}{8}x^2 & 0 \leqslant x \leqslant 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Use algebraic integration to find $E(X)$.

(2)

(b) Show that $\text{Var}(X) = 0.15$

(3)

(c) Find $E(2X+3)$ and $\text{Var}(2X+3)$.

(1)

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Question 3 continued

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(Total for Question 3 is 6 marks)

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4. The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 1 \\ \dfrac{x^2 + 2x - 3}{12} & 1 \leqslant x \leqslant 3 \\ 1 & x > 3 \end{cases}$$

(a) Find the probability density function of X.

(2)

(b) Find $P(1.5 \leqslant X \leqslant 4)$

(2)

(c) Find the median of X, giving your answer to 3 significant figures.

(2)

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Question 4 continued

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5. The continuous random variable X has probability density function

$$f(x) = \begin{cases} \frac{x}{4} & 0 \leqslant x \leqslant 2 \\ \frac{4-x}{4} & 2 < x \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Sketch the graph of $f(x)$ and show that the total area under it is 1

(2)

(b) Find $P(1 < X < 3)$

(2)

(c) Find the upper quartile of X, giving your answer in the form $a - \sqrt{b}$

(2)

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Question 5 continued

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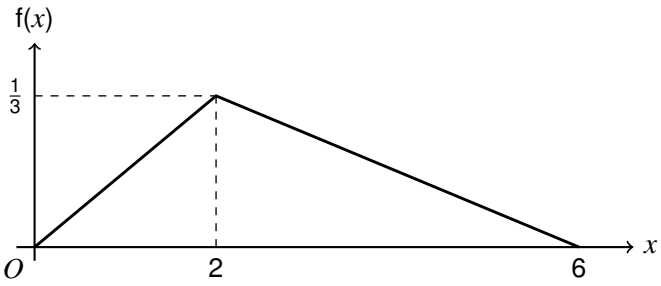

Figure 1

The continuous random variable X has probability density function

$$\text{f}(x) = \begin{cases} \dfrac{x}{6} & 0 \leqslant x \leqslant 2 \\ \dfrac{6-x}{12} & 2 < x \leqslant 6 \\ 0 & \text{otherwise} \end{cases}$$

as shown in Figure 1.

(a) Show that $\text{f}(x)$ is a valid probability density function.

(2)

(b) Use algebraic integration to find $\text{E}(X)$.

(3)

(c) Show that, for $2 < x \leqslant 6$, $\text{F}(x) = \dfrac{12x - x^2 - 12}{24}$

(3)

(d) Find the median of X, giving your answer in the form $a - b\sqrt{3}$

(2)

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)

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7. The continuous random variable X has probability density function

$$f(x) = \begin{cases} \dfrac{x^2(6-x)}{108} & 0 \leqslant x \leqslant 6 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the mode of X.

(3)

(b) Find $E(X)$.

(3)

(c) Show that the median of X lies between 3.6 and 3.7

(2)

(d) Describe the skewness of the distribution, giving a reason.

(2)

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Question 7 continued

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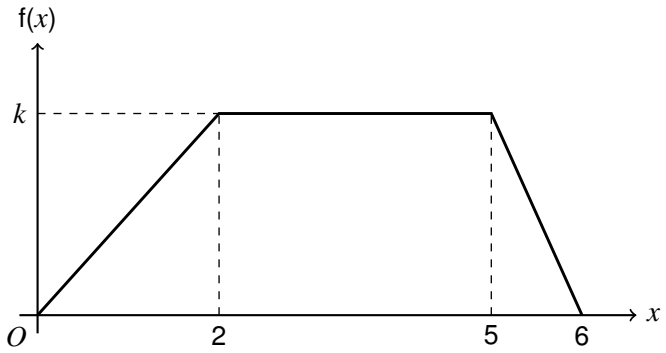

Figure 2

The continuous random variable X has the probability density function shown in Figure 2. The graph consists of three straight line segments: from (0, 0) to (2, k), from (2, k) to (5, k) and from (5, k) to (6, 0).

(a) Show that $k = \frac{2}{9}$

(2)

(b) Find P($1 < X < 5.5$)

(2)

(c) Find the median of X.

(3)

(d) Find E(X).

(3)

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9. The continuous random variable X has cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 0 \\ ax^2 + bx & 0 \leqslant x \leqslant 2 \\ 1 & x > 2 \end{cases}$$

where a and b are constants. Given that $E(X) = \frac{7}{6}$

(a) Show that $4a + 2b = 1$ and $16a + 6b = \frac{7}{2}$

(4)

(b) Find the value of a and the value of b.

(2)

(c) Find $\text{Var}(X)$.

(3)

(d) Find $P\left(X > E(X)\right)$

(1)

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Question 9 continued

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Question 9 continued

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10. The lifetime, T hours, of a rechargeable battery on a single charge is modelled by the probability density function

$$f(t) = \begin{cases} \frac{3}{32}\left(4t - t^2\right) & 0 \leqslant t \leqslant 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $P(T > 3) = \frac{5}{32}$

(3)

Ten fully charged batteries are used, independently.

(b) Find the probability that at least 2 of them last more than 3 hours.

(3)

(c) Write down $E(T)$, giving a reason, and find $Var(T)$.

(3)

The cost of running a device on one charge is C pence, where $C = 5T + 2$

(d) Find the mean and the standard deviation of C.

(2)

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Question 10 continued

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Question 10 continued

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TOTAL FOR CHAPTER 4 IS 81 MARKS

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CHAPTER 5 • KEY SKILLS

5 Continuous Uniform Distribution

Specification S2: Continuous uniform distribution • 10 questions • 80 marks • about 96 minutes

1 The pdf

$X \sim \mathrm{U}[a,b]$: $\mathrm{f}(x) = \dfrac{1}{b-a}$ for $a \leqslant x \leqslant b$, 0 otherwise. The graph is a rectangle of area 1.

2 Probabilities

$\mathrm{P}(c < X < d) = \dfrac{d-c}{b-a}$, after trimming c and d to lie in $[a,b]$

3 The cdf

$\mathrm{F}(x) = 0$ for $x < a$, $\dfrac{x-a}{b-a}$ for $a \leqslant x \leqslant b$, and 1 for $x > b$

4 Mean and variance

$\mathrm{E}(X) = \dfrac{a+b}{2}$, $\mathrm{Var}(X) = \dfrac{(b-a)^2}{12}$, and $\mathrm{E}\left(X^2\right) = \mathrm{Var}(X) + [\mathrm{E}(X)]^2$

5 Finding a and b

From the mean, $a+b$; from the variance, $(b-a)^2$, so square root to get $b-a$. Then solve two linear equations.

6 Linear functions

$\mathrm{E}(4X+3) = 4\mathrm{E}(X)+3$ and $\mathrm{Var}(4X+3) = 16\mathrm{Var}(X)$

7 Modelling

Rounding errors to the nearest unit: $\mathrm{U}[-0.5, 0.5]$. A point chosen at random on a length: uniform on that length.

8 Inside other models

A uniform probability can be the p of a binomial, which may itself need an approximation.

9 Conditional probability

$\mathrm{P}(W > 9 \mid W > 5) = \dfrac{\mathrm{P}(W > 9)}{\mathrm{P}(W > 5)}$: lengths in proportion.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 The pdf	•									
2 Probabilities	•	•	•		•	•	•	•	•	•
3 The cdf				•						
4 Mean and variance	•		•	•	•		•	•		•
5 Finding a and b		•								•
6 Linear functions					•					
7 Modelling			•			•	•	•	•	
8 Inside other models			•			•			•	
9 Conditional probability							•		•	
Interleaved: binomial distribution and approximations (Chapters 1, 3)			•			•			•	
Interleaved: continuous random variables (Chapter 4)				•				•		•
Interleaved: inequalities and algebra (P1)		•					•	•		•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WST02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Draw the rectangle and mark the interval: most probabilities are a length divided by $b-a$. Decide carefully what is uniform, and on which interval, before calculating.

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1. The continuous random variable X is uniformly distributed over the interval $[2, 10]$.

(a) Write down the probability density function of X, and sketch its graph.

(2)

(b) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

(2)

(c) Find $\mathrm{P}(X > 7.5)$

(1)

(d) Write down $\mathrm{P}(X = 5)$

(1)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2. The random variable $Y \sim \text{U}[a, b]$, where $a < b$. Given that $\text{E}(Y) = 20$ and $\text{Var}(Y) = 48$

(a) Find the value of a and the value of b.

(4)

(b) Find $\text{P}(Y < 14)$

(1)

(c) Find $\text{P}\left(|Y - 20| > 9\right)$

(1)

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Question 2 continued

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3. The lengths of wooden planks are measured to the nearest centimetre. The rounding error, E cm, is the difference between the recorded length and the true length.

(a) Suggest a suitable model for the distribution of E.

(1)

(b) Find the probability that the rounding error is less than 0.1 cm in size.

(1)

(c) Find the variance of E.

(1)

Twenty planks are measured, with their rounding errors independent.

(d) Find the probability that more than 5 of them have a rounding error less than 0.1 cm in size.

(3)

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Question 3 continued

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4. The random variable $X \sim \text{U}[-3, 5]$

(a) Find the cumulative distribution function of X for all values of x.

(3)

(b) Find the value of q such that $\text{P}(X < q) = 0.3$

(1)

(c) Find $\text{E}\left(X^2\right)$.

(2)

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Question 4 continued

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5. The random variable $X \sim \text{U}[1, 7]$, and $Y = 4X + 3$

(a) Find $\text{E}(X)$ and $\text{Var}(X)$.

(2)

(b) Find $\text{E}(Y)$ and $\text{Var}(Y)$.

(2)

(c) Find $\text{P}(Y > 20)$

(2)

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Question 5 continued

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6. The length, L mm, of a bolt made by a machine is modelled as $L \sim \text{U}[19.6, 20.6]$. A bolt can be used only if its length is between 19.8 mm and 20.4 mm.

(a) Show that the probability that a bolt can be used is 0.6

(2)

(b) A box contains 10 bolts. Find the probability that at least 8 of the bolts in the box can be used.

(3)

(c) Using a suitable approximation, find the probability that more than 125 of a random sample of 200 bolts can be used.

(4)

(d) State one assumption you have made in parts (b) and (c).

(1)

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Question 6 continued

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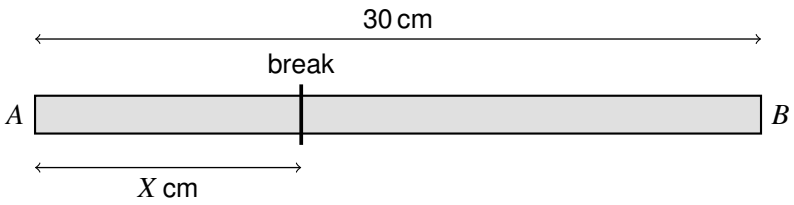

Figure 1

A rod AB of length 30 cm is broken into two pieces at a point chosen at random along its length. The distance of the break from A is X cm, as shown in Figure 1.

(a) Find the probability that the longer piece is more than twice the length of the shorter piece.

(3)

(b) Explain why the length of the shorter piece is uniformly distributed on [0, 15], and find its mean.

(2)

(c) Find the standard deviation of the length of the shorter piece.

(2)

(d) Given that the break is within 15 cm of A, find the probability that the shorter piece is longer than 6 cm.

(2)

(e) State one assumption made in this model.

(1)

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Question 7 continued

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8. A square tile is cut so that its side length, L cm, is modelled by $L \sim \text{U}[2, 6]$. The area of the tile is A cm^2.

(a) Find E(L) and Var(L).

(2)

(b) Find E(A).

(3)

(c) Find P($A > 20$)

(2)

(d) Find the median of A, and use it with your answer to part (b) to comment on the shape of the distribution of A.

(3)

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Question 8 continued

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9. Buses leave a bus stop every 12 minutes. Jamal arrives at the stop at a random time and waits for the next bus. His waiting time, W minutes, is modelled by $W \sim \text{U}[0, 12]$

(a) Find $\text{P}(W > 8)$

(1)

(b) On one day Jamal has already waited 5 minutes. Find the probability that he must wait at least another 4 minutes.

(3)

(c) Jamal uses the bus on 5 days. Find the probability that he waits more than 8 minutes on exactly 2 of these days.

(3)

(d) Find the expected total time Jamal waits over the 5 days.

(2)

(e) Give one reason why the uniform model may not be realistic.

(1)

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Question 9 continued

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10. The random variable $X \sim \text{U}[-a, 3a]$, where a is a positive constant. Given that $\text{P}(X > 1) = 0.6$

(a) Show that $a = \frac{5}{3}$

(3)

(b) Find $\text{E}(X)$ and $\text{Var}(X)$.

(2)

(c) Find $\text{P}\left(|X| < 1\right)$

(2)

(d) Use integration to find $\text{E}\left(|X|\right)$.

(3)

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Question 10 continued

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TOTAL FOR CHAPTER 5 IS 80 MARKS

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CHAPTER 6 • KEY SKILLS

6 Sampling and Sampling Distributions

Specification S2: Sampling distributions • 10 questions • 80 marks • about 96 minutes

1 Population and census

The population is the whole set of items. A census surveys every member; a sample surveys some of them.

2 Unit and frame

A sampling unit is one member of the population; the sampling frame is the list of sampling units used to choose the sample.

3 Census or sample

Census: accurate, but slow, costly, and impossible if testing destroys items. Sample: quicker and cheaper, but may not represent the population.

4 A statistic

A function of the sample values only. If it contains an unknown parameter such as μ, σ or p, it is not a statistic.

5 Sampling distribution

All possible values of the statistic, with their probabilities. Check the probabilities add to 1.

6 Listing samples

List every sample in a table. A sample such as $(1, 2)$ can occur in 2 orders; $(1, 2, 5)$ from mixed bags needs care with which bag each came from.

7 Different statistics

Mean, median, maximum, range and total each have their own distribution; some take only a few values.

8 Binomial samples

The number of successes in a sample of n is $\mathrm{B}(n, p)$; the sample proportion $\frac{Y}{n}$ has mean p and variance $\frac{p(1-p)}{n}$

9 Expectation of a statistic

Find E from its sampling distribution and compare with the population parameter, e.g. $\mathrm{E}(\bar{X}) = \mu$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Population and census	•							•		
2 Unit and frame	•							•		
3 Census or sample	•							•		
4 A statistic		•		•	•			•	•	
5 Sampling distribution		•	•	•	•	•	•		•	•
6 Listing samples			•	•		•	•		•	•
7 Different statistics				•		•			•	•
8 Binomial samples					•			•		
9 Expectation of a statistic		•		•	•		•		•	•
Interleaved: expectation and variance (S1)		•		•	•		•		•	•
Interleaved: binomial and approximations (Chapters 1, 3)					•			•		
Interleaved: conditional probability and algebra (S1, P1)									•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WST02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Draw up a table of every possible sample with its probability before finding any statistic. Definitions must refer to the context of the question.

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1. A head teacher wants to find out the views of the 1200 students in her school about a new timetable. She has a list of all the students.

(a) Describe the population.

(1)

(b) Describe a suitable sampling frame.

(1)

(c) State the sampling units.

(1)

(d) Give one advantage of taking a census and one advantage of taking a sample.

(2)

(e) Give one reason why a sample might not give a good representation of the views of all the students.

(1)

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Question 1 continued

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2. A random sample X_1, X_2, X_3 is taken from a population with unknown mean μ and unknown standard deviation σ.

(a) State, with a reason, which of the following are statistics.

(i) $X_1 + X_2 + X_3$ (ii) $(X_1 - \mu)^2$ (iii) $\max(X_1, X_2, X_3)$ (iv) $\dfrac{X_1 + X_2}{\sigma}$

(3)

(b) Explain what is meant by the sampling distribution of a statistic.

(1)

(c) Show that $\mathrm{E}\left(\dfrac{X_1 + X_2 + X_3}{3}\right) = \mu$

(2)

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Question 2 continued

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(Total for Question 2 is 6 marks)

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3. A bag contains a large number of counters, each numbered 1, 2 or 3. A counter is chosen at random, and its number X has the distribution

x	1	2	3
$P(X = x)$	0.5	0.3	0.2

A random sample of 2 counters is taken.

(a) List all the possible samples.

(2)

(b) Find the sampling distribution of the mean of the sample.

(4)

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Question 3 continued

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4. A warehouse stores a large number of sacks. Each sack weighs either 2 kg or 5 kg, and 70% of the sacks weigh 2 kg. A random sample of 3 sacks is chosen.

(a) Explain why the maximum weight in the sample is a statistic.

(1)

(b) Find the sampling distribution of the maximum weight M.

(2)

(c) Find the sampling distribution of the range R of the weights in the sample.

(2)

(d) Find E(R).

(1)

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Question 4 continued

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5. In a large town, 30% of residents use the library. A random sample of 10 residents is taken, and Y is the number who use the library.

(a) Write down the sampling distribution of Y.

(1)

(b) Find P($Y \geqslant 5$)

(2)

(c) Find the mean and the variance of the sample proportion $\frac{Y}{10}$

(2)

(d) In a different town the proportion p who use the library is unknown. Explain why $\frac{Y}{10}$ is a statistic but $Y - 10p$ is not.

(1)

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Question 5 continued

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6. Bag A contains a large number of 1p and 2p coins in the ratio 3 : 1. Bag B contains a large number of 2p and 5p coins in the ratio 1 : 1. Two coins are taken at random from bag A and one coin is taken at random from bag B.

(a) List all the possible samples of three coins, with their probabilities.

(3)

(b) Find the sampling distribution of the total value, T pence, of the three coins.

(3)

(c) Find the sampling distribution of the median value, M pence, of the three coins.

(3)

(d) Explain why M can take only two values.

(1)

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Question 6 continued

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7. A random variable X has the probability distribution

x	0	1	2
$P(X = x)$	0.4	0.4	0.2

(a) Find the mean μ and the variance σ^2 of X.

(2)

A random sample of 2 observations is taken from this population, and $\bar{X}$ is the sample mean.

(b) Find the sampling distribution of $\bar{X}$.

(4)

(c) Find $E(\bar{X})$ and $Var(\bar{X})$, and show that they are equal to μ and $\frac{\sigma^2}{2}$ respectively.

(4)

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Question 7 continued

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8. A polling company wants to estimate the proportion p of the voters in a city who support a new tram line. It plans to telephone a random sample of 400 voters, chosen from the city's telephone directory.

(a) Give one reason why the company uses a sample rather than a census.

(1)

(b) Identify the sampling frame, and explain why using it may give a biased estimate of p.

(2)

Let Y be the number of the 400 voters who support the tram line. The company will use $\hat{P} = \frac{Y}{400}$ to estimate p.

(c) Given that $p = 0.35$, use a suitable approximation to find the probability that $\hat{P}$ is greater than 0.4

(4)

(d) Explain why $\hat{P}$ is a statistic, but $\hat{P} - p$ is not, when p is unknown.

(2)

(e) State the mean of the sampling distribution of $\hat{P}$.

(1)

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Question 8 continued

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9. A fair spinner is numbered 1, 2 and 3. It is spun 3 times, and the three scores form a random sample.

(a) Explain why the range R of the three scores is a statistic.

(1)

(b) Find the sampling distribution of R.

(5)

(c) Find E(R).

(2)

(d) Given that the median of the three scores is 2, find the probability that the range is 2

(2)

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10. A large bag contains discs, each labelled 1 or 3. The proportion of discs labelled 1 is p. A random sample of 3 discs is taken, and $\bar{X}$ is the mean of the labels.

(a) Find, in terms of p, the sampling distribution of $\bar{X}$.

(4)

(b) Given that $\mathrm{P}\left(\bar{X} = 3\right) = \frac{1}{8}$, find the value of p.

(2)

(c) For this value of p, find the sampling distribution of the median of the sample.

(2)

(d) Show that, for any p, $\mathrm{E}\left(\bar{X}\right) = 3 - 2p$, and explain why this equals the mean of the population.

(2)

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Question 10 continued

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TOTAL FOR CHAPTER 6 IS 80 MARKS

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CHAPTER 7 • KEY SKILLS

7 Hypothesis Testing

Specification S2: Hypothesis testing • 10 questions • 80 marks • about 96 minutes

1 Hypotheses

State them for the parameter: $H_0: p = 0.3$, $H_1: p > 0.3$ (or λ for a Poisson rate). Never use the sample value.

2 Probability method

Find the probability of the observed value or more extreme, e.g. $P(X \geqslant 10) = 1 - P(X \leqslant 9)$, and compare with the significance level.

3 Critical regions

The values that would lead to rejecting H_0: write $X \leqslant 5$, not a probability. Show the probabilities either side of the boundary.

4 Two-tailed tests

$H_1: p \neq p_0$. Split the significance level: 10% gives 5% in each tail.

5 Actual significance

The probability of being in the critical region when H_0 is true; add both tails for a two-tailed test.

6 Poisson tests

Scale λ to the interval sampled: 1.5 per month for 4 months gives Po(6) under H_0

7 Tests with approximations

Large n or λ: use the normal approximation with a continuity correction, e.g. $P(X \leqslant 20) \rightarrow P(Y < 20.5)$

8 Conclusions

Two parts: reject or do not reject H_0, then in context with "evidence", e.g. "there is evidence that the rate has decreased".

9 Direction and limits

The tail comes from what is suspected, not the wording of a claim. If even the most extreme value is not significant, no critical region exists.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Hypotheses	•		•	•		•	•	•	•	•
2 Probability method	•			•		•		•	•	
3 Critical regions		•	•		•	•	•	•	•	•
4 Two-tailed tests			•				•			
5 Actual significance		•	•			•	•			•
6 Poisson tests				•			•	•		
7 Tests with approximations								•	•	
8 Conclusions	•	•	•	•		•	•	•	•	
9 Direction and limits					•	•				•
Interleaved: binomial and Poisson distributions (Chapters 1, 2)	•	•	•	•	•	•	•			•
Interleaved: approximations and the normal distribution (Chapter 3, S1)								•	•	
Interleaved: logarithms and inequalities (P1, P2)					•			•	•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WST02 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Write the hypotheses with the parameter, the distribution under H_0, the probability statement, the comparison, and a conclusion in context. A probability close to 1 means you have the wrong tail.

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1. A spinner is designed to land on red with probability 0.3. Asha suspects that it lands on red more often than this. She spins it 20 times and it lands on red 10 times.

Test, at the 5% level of significance, whether Asha's suspicion is supported. State your hypotheses clearly.

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Question 1 continued

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2. A test is carried out of $H_0 : p = 0.4$ against $H_1 : p < 0.4$, using a random sample of size 25 and a 5% level of significance.

(a) Find the critical region for the test.

(3)

(b) State the actual significance level of the test.

(1)

(c) The sample contains 6 successes. State, with a reason, the conclusion of the test.

(2)

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Question 2 continued

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3. In a survey, 25% of customers are expected to choose a new product. A random sample of 20 customers is taken to test whether this proportion is different from 25%, using a 10% level of significance.

(a) Write down suitable hypotheses.

(1)

(b) Find the critical region, with the probability in each tail as close as possible to 5%.

(3)

(c) Find the actual significance level of the test.

(1)

(d) In the sample, 8 customers choose the new product. State the conclusion of the test.

(1)

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Question 3 continued

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4. Accidents at a road junction used to occur at a mean rate of 1.5 per month. After new traffic lights were installed, there were 2 accidents in a 4-month period.

(a) Write down suitable hypotheses to test whether the rate of accidents has decreased.

(1)

(b) Carry out the test at the 5% level of significance.

(4)

(c) State your conclusion in context.

(1)

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Question 4 continued

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5. A manufacturer claims that 5% of its light bulbs are faulty. A quality inspector wants to test, at the 5% level of significance, whether the proportion is lower than this, using a random sample of n bulbs.

(a) Show that, with $n = 20$, there is no critical region for the test.

(3)

(b) Find the smallest value of n for which a critical region exists.

(3)

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Question 5 continued

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6. A newspaper reports that 25% of the workers in a city cycle to work. The council believes that the proportion is in fact lower. In a random sample of 30 workers, 3 cycle to work.

(a) State suitable hypotheses for a test of the council's belief.

(2)

(b) Carry out the test at the 5% level of significance.

(4)

(c) Find the critical region for this test.

(2)

(d) Find the probability of rejecting H_0 when it is true.

(1)

(e) Explain why the alternative hypothesis is $p < 0.25$ and not $p > 0.25$

(1)

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7. Calls arrive at a help desk at a mean rate of 7 every 10 minutes. After a change to the company's website, the manager wants to test whether the rate has changed, using the number of calls in a 10-minute period and a 10% level of significance.

(a) Give a reason why a Poisson distribution is suitable for the number of calls.

(1)

(b) Write down suitable hypotheses.

(1)

(c) Find the critical region, with the probability in each tail as close as possible to 5%.

(4)

(d) Find the actual significance level of the test.

(1)

(e) In a 10-minute period, 12 calls arrive. State, in context, the conclusion of the test.

(2)

(f) State one assumption needed for the test to be valid.

(1)

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Question 7 continued

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8. A company receives emails at a mean rate of 30 per day. A spam filter is installed, and on a randomly chosen day afterwards 20 emails are received.

(a) Write down suitable hypotheses to test whether the mean number of emails per day has decreased.

(1)

(b) Explain why a normal approximation is suitable.

(1)

(c) Using a suitable approximation, carry out the test at the 5% level of significance.

(5)

(d) Using the same approximation, find the critical region for the test.

(3)

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Question 8 continued

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9. A politician claims that 40% of voters support her policy. A rival believes that the support is lower. In a random sample of 150 voters, 48 support the policy.

(a) Write down suitable hypotheses.

(1)

(b) Using a suitable approximation, test the rival's belief at the 5% level of significance.

(5)

(c) State, with a reason, whether your conclusion would be different at the 1% level of significance.

(2)

(d) Using the same approximation, find the critical region for the test at the 5% level.

(2)

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Question 9 continued

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10. In the past, 10% of the items made by a machine were defective. After the machine is serviced, a random sample of 50 items is taken to test, at the 5% level of significance, whether the proportion of defective items has decreased.

(a) Find the critical region for the test.

(3)

(b) State the actual significance level of the test.

(1)

(c) The proportion of defective items is in fact 5% after the service. Find the probability that the test concludes that the proportion has decreased.

(2)

(d) Comment on your answer to part (c), and suggest how the test could be improved.

(2)

(e) The test is carried out on 5 independent samples of 50 items when the proportion is still 10%. Find the probability that at least one of the tests wrongly concludes that the proportion has decreased.

(2)

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TOTAL FOR CHAPTER 7 IS 80 MARKS

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