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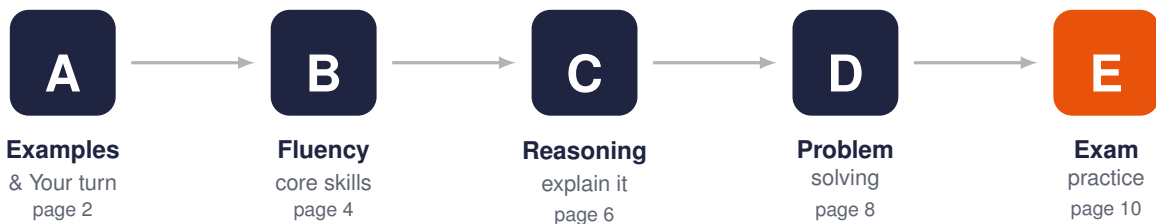
A-LEVEL MATHEMATICS • PURE 2

The Binomial Expansion

Worksheet

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Free The Binomial Expansion worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: use Pascal's triangle and " C_r " • expand $(a + bx)^n$ • find a single term or coefficient • *Stretch:* unknown constants and powers, products of brackets, estimates and their errors.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Unknown constants and unknown powers, products of brackets, and estimates with their errors.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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Name

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A**Worked examples & Your turn**

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE*Secure the method***Worked example A1**

Find the values of 6C_2 and 8C_3

$${}^nC_r = \frac{n!}{r!(n-r)!}, \text{ or read it from Pascal's triangle.}$$

$$\frac{6 \times 5}{2} = 15, \quad \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$$

15, 56

Your turn A1

Find the values of 7C_3 and 9C_2

Worked example A2

Find the first three terms, in ascending powers of x , of $(1 + 3x)^6$

Keep $3x$ in a bracket every time.

$$1 + 6(3x) + 15(3x)^2$$

$1 + 18x + 135x^2$

Your turn A2

Find the first three terms, in ascending powers of x , of $(2 - x)^5$

Worked example A3

Find the coefficient of x^3 in the expansion of $(2 + x)^7$

The term in x^3 is ${}^7C_3 2^4 x^3$

$$35 \times 16$$

560

Your turn A3

Find the coefficient of x^4 in the expansion of $(3 - 2x)^6$



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STRETCH

Worked example A4: an unknown constant

The coefficient of x^2 in $(1 + kx)^5$ is 90, and $k > 0$.
Find k

The x^2 term is ${}^5C_2(kx)^2 = 10k^2x^2$

$$10k^2 = 90, \text{ so } k^2 = 9$$

$$k = 3$$

Your turn A4

The coefficient of x in $(3 + kx)^4$ is -216 . Find k

Worked example A5: a product of brackets

Find the coefficient of x^2 in $(1 + 2x)(1 - x)^6$
 $(1 - x)^6 = 1 - 6x + 15x^2 + \dots$

Two ways to make x^2 : $1 \times 15x^2$ and $2x \times (-6x)$

$$15 - 12 = 3$$

Your turn A5

Find the coefficient of x^2 in $(3 - x)(1 + 2x)^5$

Worked example A6: an estimate

Expand $\left(1 + \frac{x}{10}\right)^8$ to the term in x^2 , and use it to
estimate 1.01^8

$$1 + 0.8x + 0.28x^2$$

$$1 + \frac{x}{10} = 1.01 \text{ when } x = 0.1$$

$$1 + 0.08 + 0.0028 = 1.0828$$

Your turn A6

Expand $(1 - 2x)^7$ to the term in x^2 , and use it to
estimate 0.98^7

Worked example A7: an unknown power

The coefficient of x^2 in $(1 + x)^n$ is 28. Find n

$${}^nC_2 = \frac{n(n-1)}{2}$$

$$n(n-1) = 56, \text{ and } n \text{ is a positive integer}$$

$$n = 8$$

Your turn A7

The coefficient of x^2 in $(1 + 2x)^n$ is 180. Find n



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Find the value of

(a) 5C_2

(b) ${}^{10}C_3$

(c) 8C_0

(d) ${}^{12}C_{11}$

[4 marks]**B2** Use the row 1, 4, 6, 4, 1 of Pascal's triangle to expand fully

(a) $(1+x)^4$

(b) $(1-x)^4$

(c) $(1+2x)^4$

(d) $(2+x)^4$

[4 marks]**B3** Find the first three terms, in ascending powers of x , of

(a) $(1+x)^8$

(b) $(1-3x)^8$

(c) $(2+x)^8$

(d) $\left(2-\frac{x}{2}\right)^8$

[4 marks]**B4** Find the coefficient of x^3 in the expansion of

(a) $(1+x)^9$

(b) $(1-x)^9$

(c) $(1+2x)^9$

(d) $(3+x)^9$

[4 marks]

CORE

continued: estimates and terms

B5 Given that $(1+x)^{10} \approx 1+10x+45x^2$ for small x , estimate

(a) 1.01^{10}

(b) 0.99^{10}

(c) 1.02^{10}

[3 marks]

B6 Write down

(a) the 4th term of $(1+x)^7$

(b) the constant term of $(x+2)^5$

(c) the term in x^4 of $(x-1)^6$

(d) the last term of $(3-x)^4$

[4 marks]

STRETCH

B7 Find the possible values of the constant k in each case.

(a) The coefficient of x in $(1+kx)^6$ is 18

(b) The coefficient of x^2 in $(1+kx)^6$ is 135

(c) The coefficients of x and x^2 in $(1+kx)^6$ are 18 and 135

(d) The coefficient of x^3 in $(2+kx)^5$ is -320

[4 marks]

B8 Find the coefficient of x^2 in

(a) $(1+x)(1+x)^5$

(b) $(1+3x)(1-x)^5$

(c) $(2-x)(1+2x)^4$

(d) $(1+x^2)(1+x)^6$

[4 marks]

B9 Find the positive integer n in each case.

(a) ${}^nC_2 = 15$

(b) The coefficient of x in $(1+3x)^n$ is 27

(c) The coefficient of x^2 in $(1+x)^n$ is 45

(d) The coefficient of x^2 in $(1+3x)^n$ is 135

[4 marks]

B10 $\left(1+\frac{x}{4}\right)^8 = 1+2x+\frac{7}{4}x^2+\dots$

(a) State the value of x that gives an estimate of 1.05^8

(b) Use the first three terms to estimate 1.05^8

(c) Find the percentage error of your estimate, to 3 significant figures.

(d) Explain why the same three terms would give a poor estimate of 2^8

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Sam writes the second term of $(2 - 3x)^4$ as $+96x$
Explain Sam's mistake, and write the term correctly. [2 marks]
- C2** Lily writes the third term of $(1 + 2x)^5$ as ${}^5C_2 \times 2x^2 = 20x^2$
Explain Lily's mistake, and write the term correctly. [2 marks]
- C3** Kai is asked for the coefficient of x^2 in $(2 - x)(1 + 3x)^5$. He multiplies the x^2 term of $(1 + 3x)^5$ by 2 and gives 180
Explain what Kai has missed, and find the correct coefficient. [3 marks]
- C4** Jo says the 3rd term of $(1 + 2x)^6$ is the term in x^3
Explain Jo's mistake, and find the 3rd term. [3 marks]



STRETCH

- C5** Asked for the exact coefficient of x^3 in $\left(1 + \frac{x}{3}\right)^7$, Max writes 1.30

Explain why Max would lose the mark, and give the exact coefficient.

[3 marks]

- C6** Ali uses $(1+x)^{10} \approx 1 + 10x + 45x^2$ with $x = 0.1$ to estimate 1.1^{10} , and gets 2.45. The true value is 2.594 to 3 decimal places.

Explain why the estimate is poor, and how it could be improved.

[3 marks]

- C7** Explain, using Pascal's triangle or the formula for nC_r , why the coefficient of x^3 in $(1+x)^7$ is equal to the coefficient of x^4 , and find this coefficient.

[3 marks]

- C8** (a) Show that ${}^5C_2 + {}^5C_3 = {}^6C_3$

(1)

- (b) Prove that, for every integer $n \geq 3$, ${}^nC_2 + {}^nC_3 = {}^{n+1}C_3$

(3)

- (c) Explain how this result appears in Pascal's triangle.

(1)

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

- D1** Find the coefficient of x^4 in the binomial expansion of

$$\left(3 - \frac{x}{3}\right)^8$$

giving your answer in its simplest form.

[3 marks]

- D2** (a) Find the first four terms, in ascending powers of x , of the binomial expansion of $\left(1 + \frac{x}{2}\right)^8$, giving each term in simplest form.

(3)

(b) Hence find the coefficient of x^3 in the expansion of $(2 - x)\left(1 + \frac{x}{2}\right)^8$, showing how each part of the coefficient arises.

(1)

[4 marks]

- D3** Find the coefficient of x^2 in the expansion of

$$(2 + x)(1 - 3x)^4$$

showing clearly how each part of the coefficient arises.

[3 marks]

- D4** The coefficient of x^2 in the expansion of

$$(3 - kx)(1 + 2x)^7$$

is 210, where k is a constant. Find the value of k .

[3 marks]



STRETCH

- D5** (a) Find the first three terms, in ascending powers of x , of the expansion of $(2+ax)^6$, where a is a positive constant, giving each term in terms of a

(2)

Given that the coefficient of x in the expansion of $\left(1 + \frac{1}{x}\right)(2+ax)^6$ is 432,

- (b) find the value of a , explaining why any other solution of your equation is rejected.

(2)

[4 marks]

- D6** (a) Find the first three terms, in ascending powers of x , of $\left(1 - \frac{x}{5}\right)^{10}$

(2)

- (b) Use your answer, with a suitable value of x , to estimate 0.98^{10} , and find the percentage error of your estimate to 3 significant figures.

(2)

[4 marks]

- D7** In the expansion of $(1+kx)^8$, where k is a non-zero constant, the coefficients of x^2 and x^3 are equal.

- (a) Find the value of k

(3)

- (b) Find the coefficient of x^4 in the expansion, giving your answer as an exact fraction in its simplest form.

(1)

[4 marks]

- D8** (a) Find the expansion of $(1+x)^4(1-x)^5$ in ascending powers of x , up to and including the term in x^3 , showing clearly how each coefficient is formed.

(4)

- (b) Hence estimate the value of $1.1^4 \times 0.9^5$

(1)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA12. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of $(3 + 2x)^6$

giving each term in simplest form.

.....
(4)

This expansion is to be used to find an approximation for 3.02^6

- (b) State the value of x that should be used. (There is no need to carry out this calculation.)

.....
(1)

(Total for Question E1 is 5 marks)



E2 The binomial expansion, in ascending powers of x , of

$$(2 + px)^5$$

where p is a non-zero constant, can be written in the form $A + Bx + Cx^2 + Dx^3 + \dots$

(a) Find the value of A

.....

(1)

Given that $B + C = 0$,

(b) find the value of p

.....

(3)

(c) find the value of D

.....

(2)

(Total for Question E2 is 6 marks)



STRETCH

- E3** (a) Find the first 4 terms, in ascending powers of x , of the binomial expansion of

$$\left(1 - \frac{x}{2}\right)^{10}$$

giving each term in simplest form.

.....
(3)

In the expansion of $(a + bx)\left(1 - \frac{x}{2}\right)^{10}$, where a and b are constants, the coefficient of x is -7 and the coefficient of x^2 is $\frac{15}{2}$

- (b) Find the value of a and the value of b

.....
(3)

- (c) Use your answer to part (a) with $x = 0.1$ to estimate 0.95^{10} , giving your answer to 4 decimal places.

.....
(2)

(Total for Question E3 is 8 marks)

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