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INTERNATIONAL A LEVEL MATHEMATICS • PURE 3

Pure Mathematics 3

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA13/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA13 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Algebraic Methods Algebraic fractions, improper fractions and algebraic division	10	69	own PDF	
2 Functions and Graphs Modulus, composite and inverse functions, transformations	10	76	own PDF	
3 Trigonometric Functions sec, cosec, cot, identities, inverse trigonometric functions	10	72	own PDF	
4 Trigonometric Addition Formulae Addition and double-angle formulae, $R\cos(\theta \pm \alpha)$	10	75	own PDF	
5 Exponentials and Logarithms e^x , $\ln x$, modelling, logarithmic graphs	10	78	own PDF	
6 Differentiation Chain, product and quotient rules, trigonometric and exponential functions	10	76	own PDF	
7 Integration Standard integrals, $f(ax+b)$, trigonometric identities, reverse chain rule	10	77	2	
8 Numerical Methods Locating roots, iteration	10	77	own PDF	

[Full mark scheme](#)

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CHAPTER 7 • KEY SKILLS

7

Integration

Specification P3: Integration • 10 questions • 77 marks • about 90 minutes

1 Standard integrals

$$\int e^x dx = e^x + c, \quad \int \frac{1}{x} dx = \ln|x| + c,$$

$$\int \cos x dx = \sin x + c,$$

$$\int \sec^2 x dx = \tan x + c$$

2 Functions of $(ax + b)$ Integrate as usual, then divide by a :

$$\int \frac{1}{2x-3} dx = \frac{1}{2} \ln|2x-3| + c$$

3 Reverse chain rule

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$
 check that the numerator is (a multiple of) the derivative of the denominator.

4 Trig identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x),$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x),$$

$$\tan^2 x = \sec^2 x - 1, \quad \sin x \cos x = \frac{1}{2} \sin 2x$$

5 Improper fractions

Divide first (Chapter 1), then integrate

$$\text{each term: } \frac{C}{x+a} \text{ gives } C \ln|x+a|$$

6 Exact definite integrals

Substitute both limits; use the laws of logarithms to write one ln, e.g.

$$2 \ln 5 - 2 \ln 3 = 2 \ln \frac{5}{3}$$

7 Areas

$$\text{Area} = \int_a^b y dx;$$
 below the x -axis the integral is negative, so take its size.

8 Unknown limits

Evaluate in terms of the unknown and solve; reject values outside the domain.

9 Using “hence”

A derivative found earlier gives an integral: if $\frac{d}{dx} F(x) = f(x)$ then

$$\int f(x) dx = F(x) + c$$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	6	10	10	10	10	11
1 Standard integrals	•	•	•				•	•	•	•
2 Functions of $(ax + b)$	•	•			•	•	•		•	•
3 Reverse chain rule				•				•		
4 Trig identities			•				•			
5 Improper fractions					•	•				
6 Exact definite integrals		•		•	•	•	•	•	•	•
7 Areas						•		•		•
8 Unknown limits									•	
9 Using “hence”							•	•		
Interleaved: algebraic division (Chapter 1)					•	•				
Interleaved: addition formulae and identities (Chapters 3, 4)			•				•	•		
Interleaved: e, ln and differentiation (Chapters 5, 6)						•		•	•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA13 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Include “+c” for indefinite integrals. When integrating $\frac{1}{ax+b}$ do not lose the factor $\frac{1}{a}$. Give exact answers as a single logarithm where asked.



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1. Find

(a) $\int \left(3e^{2x} - \frac{4}{x} \right) dx$ (2)

(b) $\int \frac{5}{2x-3} dx$ (2)

(c) $\int \sec^2(4x) dx$ (1)

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Question 1 continued

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(Total for Question 1 is 5 marks)



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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Find the exact value of

(a) $\int_1^e \left(2x + \frac{3}{x}\right) dx$ (3)

(b) $\int_0^{\ln 2} e^{2x} dx$ (2)

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Question 2 continued

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(Total for Question 2 is 5 marks)



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3. Using appropriate trigonometric identities, find

(a) $\int (\sin x + \cos x)^2 dx$

(3)

(b) $\int \tan^2 x dx$

(2)

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Question 3 continued

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(Total for Question 3 is 5 marks)



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4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Find $\int \frac{6x}{3x^2 + 1} dx$

(2)

(b) Find the exact value of $\int_0^{\frac{\pi}{3}} \frac{\sin x}{2 + \cos x} dx$, giving your answer as a single logarithm.

(3)

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Question 4 continued

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(Total for Question 4 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Write $\frac{2x^2+5x-1}{x+3}$ in the form $Ax+B+\frac{C}{x+3}$, where A , B and C are constants.

(2)

(b) Hence find the exact value of $\int_0^2 \frac{2x^2+5x-1}{x+3} dx$, giving your answer in the form $p+q\ln r$, where p , q and r are rational.

(4)

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Question 5 continued

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(Total for Question 5 is 6 marks)



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6. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

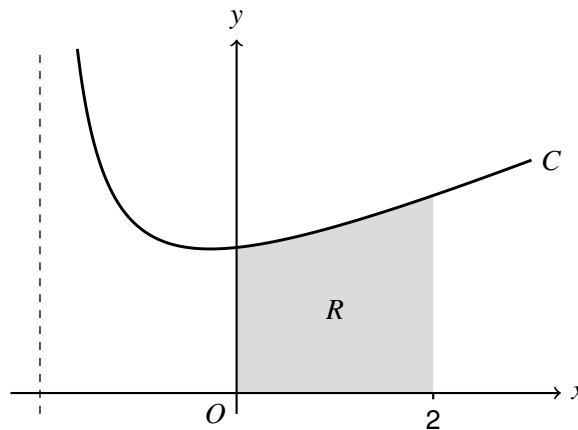


Figure 1

Figure 1 shows part of the curve C with equation

$$y = \frac{x^2 + 4x + 7}{x + 2} \quad x > -2$$

The region R , shown shaded, is bounded by C , the coordinate axes and the line $x = 2$

- (a) Show that $y = x + 2 + \frac{3}{x + 2}$ (2)
- (b) Find the exact area of R , giving your answer in the form $a + b \ln 2$ (4)
- (c) Use calculus to find the exact coordinates of the minimum point of C . (4)



Question 6 continued

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Question 6 continued

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(Total for Question 6 is 10 marks)



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7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

- (a)** Show that $(2\cos x - \sin x)^2 \equiv a + b\cos 2x + c\sin 2x$, where a , b and c are constants to be found.

(4)

- (b)** Hence find the exact value of $\int_0^{\frac{\pi}{4}} (2\cos x - \sin x)^2 dx$

(4)

- (c)** Hence write down the exact value of $\int_0^{\frac{\pi}{8}} (2 \cos 2x - \sin 2x)^2 dx$

(2)

Question 7 continued

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Question 7 continued

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(Total for Question 7 is 10 marks)



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8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Show that $\frac{d}{dx}(\ln(\sec x + \tan x)) = \sec x$, $0 < x < \frac{\pi}{2}$ (4)

(b) Hence find the exact value of $\int_0^{\frac{\pi}{4}} \sec x dx$ (2)

(c) Show that $\int_0^{\frac{\pi}{3}} \tan x dx = \ln 2$ (2)

(d) Find the exact area of the region bounded by the curves $y = \sec x$ and $y = \tan x$, the y -axis and the line $x = \frac{\pi}{4}$, giving your answer as a single logarithm. (2)



Question 8 continued

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Question 8 continued

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(Total for Question 8 is 10 marks)



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Solutions relying on calculator technology are not acceptable.

(3)

(5)

(2)

Question 9 continued

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Question 9 continued

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(Total for Question 9 is 10 marks)



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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

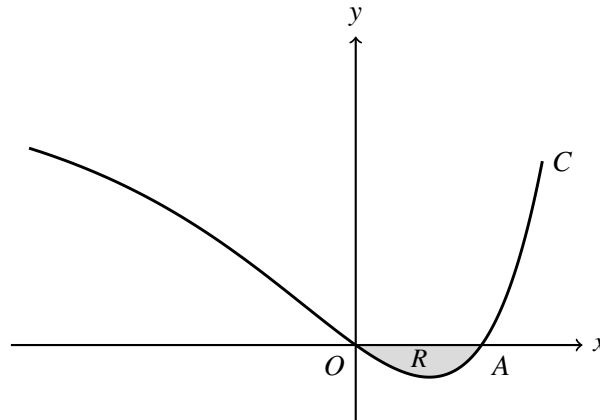


Figure 2

Figure 2 shows part of the curve C with equation

$$y = e^{2x} - 3e^x + 2 \quad x \in \mathbb{R}$$

The curve passes through the origin O and crosses the x -axis again at the point A . The region R , shown shaded, is bounded by C and the x -axis.

- (a) Show that the x coordinate of A is $\ln 2$ (3)
- (b) Write down the equation of the asymptote to C as $x \rightarrow -\infty$ (1)
- (c) Find the exact area of R , giving your answer in the form $p - \ln q$, where p and q are rational. (5)
- (d) Find an equation of the tangent to C at A . (2)



Question 10 continued

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Question 10 continued

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(Total for Question 10 is 11 marks)

TOTAL FOR CHAPTER 7 IS 77 MARKS

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