eClassroom

A-LEVEL MATHEMATICS • STATISTICS 2

# Approximations

Worksheet

Pearson Edexcel International A Level (WST02) • Calculator and statistical tables allowed: show every step

Free Approximations worksheet for A-Level and International A-Level Maths, with full worked solutions


**A** **Examples** & Your turn page 2

**B** **Fluency** core skills page 4

**C** **Reasoning** explain it page 6

**D** **Problem** solving page 7

**E** **Exam** practice page 9


Tap a section to jump straight to it

**By the end of this worksheet you can:** approximate a binomial by a Poisson • approximate a binomial or a Poisson by a normal distribution • apply continuity corrections • choose and justify an approximation • *Stretch:* counting failures when $p$ is close to 1, unknown parameters and limits found from a given probability.

## Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Counting failures, running an approximation backwards from a probability, and choosing between approximations.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

## Track your progress

| Section | B | C | D | E |
|---|---|---|---|---|
| Core marks | 23 | 10 | 13 | 22 |
| Stretch marks | 16 | 14 | 17 | 14 |
| My score | | | | |


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# A Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

**CORE** *Secure the method*

**Worked example A1: Poisson for binomial**

$X \sim \text{B}(200, 0.015)$. Use a suitable approximation to find $\text{P}(X \leqslant 2)$

*$n$ large, $p$ small:* $X \approx \text{Po}(np) = \text{Po}(3)$

Poisson tables: $\text{P}(X \leqslant 2)$

0.4232

**Your turn A1**

$X \sim \text{B}(160, 0.025)$. Use a suitable approximation to find $\text{P}(X \geqslant 3)$

**Worked example A2: normal for binomial**

$X \sim \text{B}(100, 0.4)$. Use a normal approximation to find $\text{P}(X \leqslant 45)$

*$Y \sim \text{N}(40, 24)$; "$\leqslant 45$" becomes $Y < 45.5$*

$$z = \frac{45.5 - 40}{\sqrt{24}} \approx 1.12$$

0.8686

**Your turn A2**

$X \sim \text{B}(80, 0.45)$. Use a normal approximation to find $\text{P}(X > 40)$

**Worked example A3: normal for Poisson**

$X \sim \text{Po}(36)$. Use a normal approximation to find $\text{P}(X < 30)$

*$Y \sim \text{N}(36, 36)$; "$< 30$" is "$\leqslant 29$", so $Y < 29.5$*

$$z = \frac{29.5 - 36}{6} \approx -1.08$$

0.1401

**Your turn A3**

$X \sim \text{Po}(50)$. Use a normal approximation to find $\text{P}(X \geqslant 60)$

STRETCH

**Worked example A4: a single value**

$X \sim$ Po(25). Use a normal approximation to find P($X = 25$)

*$X = 25$ becomes* $24.5 < Y < 25.5$

$z = \pm\dfrac{0.5}{5} = \pm 0.1;\ 2(0.5398) - 1$

0.0796

**Your turn A4**

$X \sim$ B(60, 0.5). Use a normal approximation to find P($X = 30$)

**Worked example A5: $p$ close to 1**

97% of seeds germinate. 200 are sown. Find the probability that at least 195 germinate.

*Count the failures:* $F \sim$ B(200, 0.03) $\approx$ Po(6)

at least 195 germinate means $F \leqslant 5$

0.4457

**Your turn A5**

99% of parcels arrive on time. 300 parcels are sent. Find the probability that more than 296 arrive on time.

**Worked example A6: an interval**

$X \sim$ B(150, 0.3). Use a normal approximation to find P($40 \leqslant X \leqslant 50$)

$Y \sim$ N(45, 31.5); $39.5 < Y < 50.5$, $z = \pm 0.98$

$2(0.8365) - 1 = 0.6730$

**Your turn A6**

$X \sim$ Po(81). Use a normal approximation to find P($75 < X < 90$)

**Worked example A7: choosing**

$X \sim$ B(400, 0.01). Choose a suitable approximation, give a reason, and find P($X > 6$)

*$n$ large and $p$ small, so* Po(4)

$1 - \text{P}(X \leqslant 6) = 1 - 0.8893$

0.1107

**Your turn A7**

$X \sim$ B(500, 0.3). Choose a suitable approximation, give a reason, and find P($X < 140$)

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# B Fluency

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

**CORE** *Secure the method*

**B1** A discrete random variable $X$ is approximated by a normal variable $Y$. Write each probability in terms of $Y$

**(a)** $P(X \leqslant 20)$

**(b)** $P(X < 20)$

**(c)** $P(X \geqslant 20)$

**(d)** $P(X > 20)$

**[4 marks]**

**B2** $X \sim B(100, 0.03)$. Find

**(a)** $P(X = 2)$ exactly

**(b)** $P(X = 2)$ using a Poisson approximation

**(c)** $P(X \leqslant 2)$ using a Poisson approximation

**(d)** $P(X \leqslant 2)$ exactly

**[4 marks]**

**B3** $X \sim B(200, 0.5)$. Use a normal approximation to find

**(a)** $P(X \leqslant 110)$

**(b)** $P(X < 90)$

**(c)** $P(X = 100)$

**(d)** $P(95 \leqslant X \leqslant 105)$

**[4 marks]**

**B4** $X \sim Po(49)$. Use a normal approximation to find

**(a)** $P(X \geqslant 56)$

**(b)** $P(X \leqslant 40)$

**(c)** $P(45 < X < 55)$

**(d)** $P(X > 49)$

**[4 marks]**

**B5** For each distribution, state, with a reason, which approximation (if any) is suitable, and give its parameters.

**(a)** $B(500, 0.004)$

**(b)** $B(80, 0.5)$

**(c)** $B(30, 0.6)$

**[3 marks]**

**B6** Write down the approximating distribution, with its parameters, for

**(a)** $B(250, 0.36)$

**(b)** calls at 30 per hour, counted over 2.5 hours

**(c)** $B(1000, 0.002)$

**(d)** the number of failures when $X \sim B(400, 0.98)$

**[4 marks]**

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## STRETCH

**B7** 96% of eggs are not cracked. A crate contains 150 eggs. Using a suitable approximation, find the probability that

**(a)** no egg in the crate is cracked

**(b)** at least 145 eggs are not cracked

**(c)** fewer than 140 eggs are not cracked

**(d)** Explain why a normal approximation is not suitable here.

**[4 marks]**

**B8** $X \sim \text{B}(120, 0.5)$ is approximated by a normal distribution.

**(a)** Find $\text{P}(X \geqslant 66)$

**(b)** Find the least $k$ with $\text{P}(X \geqslant k) < 0.05$

**(c)** Find the greatest $m$ with $\text{P}(X \leqslant m) < 0.1$

**(d)** Find $\text{P}(m < X < k)$ for your values.

**[4 marks]**

**B9** Texts arrive at 12 per hour from one friend and, independently, at 13 per hour from another.

**(a)** Find the probability of more than 40 texts from the first friend in 3 hours.

**(b)** Find the probability of at most 30 texts from the first friend in 3 hours.

**(c)** Find the probability of at least 30 texts altogether in an hour.

**(d)** Find the probability of between 20 and 30 (inclusive) texts altogether in an hour.

**[4 marks]**

**B10** $X \sim \text{B}(40, 0.5)$

**(a)** Find $\text{P}(X \leqslant 17)$ from the binomial tables.

**(b)** Find it with a normal approximation and a continuity correction.

**(c)** Find it with a normal approximation without a continuity correction.

**(d)** Comment on your answers.

**[4 marks]**

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# C Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

## CORE *Secure the method*

**C1** $X \sim$ B(100, 0.4). Using $Y \sim$ N(40, 24), Amir writes P($X \geqslant 48$) $\approx$ P($Y \geqslant 48$) = 0.0516. Explain his error, and find the approximation. **[2 marks]**

**C2** $X \sim$ Po(50). Using $Y \sim$ N(50, 50), Bea writes P($X < 42$) $\approx$ P($Y < 42.5$). Explain her error, and find the approximation. **[2 marks]**

**C3** 95% of bolts are usable. In a box of 120, Cal uses N(114, 5.7) to find the probability that at least 117 are usable.

Explain why this is not suitable, and find the probability with a suitable method. **[3 marks]**

**C4** $X \sim$ Po(4). Dev uses N(4, 4) to estimate P($X \leqslant 2$)

Explain why this is not appropriate, and find P($X \leqslant 2$) **[3 marks]**

## STRETCH

**C5** $X \sim$ B(200, 0.4). Eve says a Poisson approximation can be used because $n$ is large.

Explain her error, and find P($X < 70$) using a suitable approximation. **[3 marks]**

**C6** $X \sim$ B(150, 0.2). Fay approximates $X$ by N(30, 30) to find P($X > 35$)

Explain her error, and find the approximation. **[3 marks]**

**C7** $X \sim$ Po(30). Gus says he cannot find P($X \leqslant 25$) because the Poisson tables stop at a mean of 10

Explain how it can be found, and find it. **[3 marks]**

**C8** $X \sim$ B($n$, 0.5), where $n$ is even, is approximated by a normal distribution $Y$ with a continuity correction.

**(a)** Show that the approximation to P$\left(X \leqslant \frac{n}{2}\right)$ is P$\left(Z < \frac{1}{\sqrt{n}}\right)$, where $Z \sim$ N(0, 1)

**(2)**

**(b)** The approximation gives P$\left(X \leqslant \frac{n}{2}\right)$ = 0.5398. Find $n$

**(3)**

**[5 marks]**

# D Problem solving

You decide the method. Read the whole question first, and show every step.

## CORE *Secure the method*

**D1** A factory makes light bulbs, and 1.5% of the bulbs it makes are faulty. The bulbs are packed in boxes of 200

**(a)** Use a suitable approximation to find the probability that a box contains at least 5 faulty bulbs.

**(2)**

**(b)** Give a reason why your approximation is suitable.

**(1)**

**[3 marks]**

**D2** In a large town, 35% of voters support a new road scheme. A random sample of 120 voters is taken. Using a suitable approximation, find the probability that between 35 and 50 (inclusive) of the voters in the sample support the scheme. **[4 marks]**

**D3** On a main road, cars pass a checkpoint independently and at random, at a mean rate of 18 per hour. Using a suitable approximation, find the probability that fewer than 45 cars pass the checkpoint in a randomly chosen 3-hour period. **[3 marks]**

**D4** $X \sim \text{B}(n, p)$ has mean 120. Using a normal approximation with a continuity correction, $\text{P}(X \leqslant 110) = 0.1314$

**(a)** Show that $p = 0.4$, correct to 1 decimal place.

**(2)**

**(b)** Find $n$

**(1)**

**[3 marks]**

## STRETCH

**D5** At a stadium, 98.5% of tickets are scanned correctly at the gate. On one evening 400 tickets are scanned.

**(a)** Using a suitable approximation, find the probability that at least 396 of the tickets are scanned correctly.

**(3)**

**(b)** Explain why a normal approximation would not be suitable.

**(1)**

**[4 marks]**

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**D6** Complaints to a call centre arrive independently and at random, at a mean rate of 4 per day. A week is 7 consecutive days, and the numbers of complaints in different weeks are independent.

**(a)** Using a suitable approximation, find the probability that there are more than 35 complaints in a week.

**(2)**

**(b)** Find the probability that none of the next 10 weeks has more than 35 complaints.

**(2)**

**[4 marks]**

**D7** $X \sim \text{Po}(100)$ is approximated by a normal distribution with a continuity correction.

**(a)** Given that $\text{P}(X \geqslant t) = 0.8944$, find the integer $t$

**(3)**

**(b)** Find the approximation to $\text{P}(X = t)$

**(1)**

**[4 marks]**

**D8** A machine makes items, and each item is defective with probability 0.05, independently of all other items. They are packed in batches of 20, and a batch is rejected if it contains more than 3 defective items.

**(a)** Find the probability that a batch is rejected.

**(2)**

**(b)** 200 batches are made. Using a suitable approximation, find the probability that at most 2 batches are rejected.

**(3)**

**[5 marks]**

# E Exam practice

Full-size exam questions in the style of WST02. Write your answers in the spaces, and show every step.

CORE *Exam questions for everyone*

**E1** A survey of the students at a large college finds that 25% of the students cycle to college.

**(a)** A random sample of 20 students is taken. Find the probability that at least 8 of them cycle to college.

**(2)**

A second random sample of 300 students is taken.

**(b)** Using a suitable approximation, find the probability that more than 80 of these 300 students cycle to college.

**(4)**

**(c)** Find the probability that between 65 and 85 (inclusive) of these 300 students cycle to college.

**(3)**

**(d)** Explain why the approximation you used in part (b) is suitable.

**(2)**

**(Total for Question E1 is 11 marks)**

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**E2** A factory produces large sheets of cloth. Flaws occur randomly in the cloth at a mean rate of 0.7 per square metre.

**(a)** Find the probability that a randomly chosen piece of cloth of area 5 square metres contains at most 1 flaw.

**(2)**

A roll of the cloth has an area of 90 square metres.

**(b)** Using a suitable approximation, find the probability that the roll contains more than 75 flaws.

**(4)**

**(c)** Find the largest integer $n$ for which the probability that the roll contains fewer than $n$ flaws is less than 0.05

**(3)**

**(d)** Explain why a normal approximation is used in part (b) rather than the Poisson tables.

**(2)**

**(Total for Question E2 is 11 marks)**

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STRETCH

**E3** A railway company knows that 4% of the tickets that its inspectors check are invalid, independently of each other.

**(a)** On one day an inspector checks 250 tickets. Using a suitable approximation, find the probability that more than 15 of these tickets are invalid.

**(3)**

**(b)** Find the probability that at least 245 of these 250 tickets are valid.

**(2)**

Over a month the inspectors check 5000 tickets.

**(c)** Using a normal approximation, find the probability that more than 225 of these 5000 tickets are invalid.

**(4)**

**(d)** The company will review its fares if the number of invalid tickets in the 5000 is $k$ or more. Find the least value of $k$ for which the probability of a review is less than 0.025

**(3)**

**(e)** Explain why a normal approximation is suitable in part (c).

**(2)**

**(Total for Question E3 is 14 marks)**

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