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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE MATHEMATICS 1

Further Pure 1

Chapter 3: Numerical Solutions of Equations

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WFM01/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WFM01 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Complex Numbers** Arithmetic, conjugates, Argand diagrams, modulus and argument, roots of polynomial equations	10	80	separate file	
2	**Roots of Quadratic Equations** Sums and products of roots, symmetric functions, equations with transformed roots	10	80	separate file	
3	**Numerical Solutions of Equations** Interval bisection, linear interpolation, Newton–Raphson	10	80	2	
4	**Coordinate Systems** Parametric equations, parabolas, rectangular hyperbolas, tangents and normals	10	80	separate file	
5	**Matrices** Matrix arithmetic, determinants, inverses of 2×2 matrices	10	80	separate file	
6	**Transformations Using Matrices** Linear transformations, combined transformations, area scale factor, inverses	10	80	separate file	
7	**Series** Standard sums of r, r^2 and r^3, sigma notation	10	80	separate file	
8	**Proof** Mathematical induction for sums, divisibility, matrices and sequences	10	80	separate file	

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CHAPTER 3 • KEY SKILLS

3 Numerical Solutions of Equations

Specification FP1: Numerical solution of equations • 10 questions • 80 marks • about 96 minutes

1 Change of sign

If f is continuous on $[a, b]$ and $f(a)$, $f(b)$ have opposite signs, there is a root in (a, b). State all three parts.

2 Interval bisection

Evaluate f at the midpoint; keep the half with the sign change. Each step halves the width.

3 Linear interpolation

Join $(a, f(a))$ to $(b, f(b))$:

$\alpha \approx a + \dfrac{|f(a)|}{|f(a)| + |f(b)|}(b - a)$, by similar triangles.

4 Newton–Raphson

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$: a minus sign. Show $f(x_n)$ and $f'(x_n)$ before the new value.

5 Differentiating powers

$\frac{d}{dx}x^{\frac{3}{2}} = \frac{3}{2}x^{\frac{1}{2}}$ and $\frac{d}{dx}\left(\frac{2}{x}\right) = -\frac{2}{x^2}$.

Rewrite roots and fractions as powers first.

6 Checking accuracy

To show $\alpha = 1.83$ to 2 d.p., evaluate $f(1.825)$ and $f(1.835)$, show a sign change and conclude "so $\alpha = 1.83$ to 2 d.p.".

7 When methods fail

A sign change across a discontinuity need not mean a root.

Newton–Raphson fails if $f'(x_0) = 0$ and is unreliable near a turning point.

8 Working shown

Give each iterate to the accuracy asked; with no working the method marks are lost.

9 Comparing methods

Interpolation uses the sizes of $f(a)$ and $f(b)$; bisection ignores them.

Newton–Raphson usually converges fastest.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Change of sign	•	•		•	•	•	•	•		•
2 Interval bisection	•						•			
3 Linear interpolation		•					•			•
4 Newton–Raphson			•	•		•		•	•	•
5 Differentiating powers			•	•		•		•	•	•
6 Checking accuracy					•	•				
7 When methods fail						•		•		
8 Working shown	•	•	•	•	•	•	•	•	•	•
9 Comparing methods		•					•		•	
Interleaved: differentiation of powers (P1)			•	•		•		•	•	•
Interleaved: exponentials and graphs of functions (P1, P2)							•	•		
Interleaved: tangents and quadratics (P1)						•		•	•	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WFM01 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Every sign-change answer needs the two values, "sign change", "continuous" and a conclusion. In Newton–Raphson show $f(x_0)$, $f'(x_0)$ and the substitution, not just the answer.

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1.

$$f(x) = x^3 - 2x - 5$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$.

(2)

(b) Use interval bisection twice to find an interval of width 0.25 that contains α.

(4)

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Question 1 continued

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(Total for Question 1 is 6 marks)

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2.

$$f(x) = x^2 - 3x - 1$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[3, 4]$.

(2)

(b) Use linear interpolation once on the interval $[3, 4]$ to find an approximation to α.

(3)

(c) Given that $\alpha = \frac{3+\sqrt{13}}{2}$, state whether your approximation is an over-estimate or an under-estimate.

(1)

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Question 2 continued

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3.

$$f(x) = x^3 + 2x - 7$$

(a) Find $f'(x)$.

(1)

(b) Taking $x_0 = 1.5$ as a first approximation to a root of $f(x) = 0$, apply the Newton–Raphson process once to find x_1, giving your answer to 3 decimal places.

(3)

(c) Apply the process a second time to find x_2, giving your answer to 3 decimal places.

(2)

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Question 3 continued

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4.

$$f(x) = x^{\frac{3}{2}} + \frac{2}{x} - 6, \qquad x > 0$$

(a) Differentiate $f(x)$ with respect to x.

(2)

(b) The curve $y = f(x)$ crosses the x-axis close to $x = 3$. Starting from 3, carry out one step of the Newton–Raphson process, stating the new estimate correct to 3 decimal places.

(3)

(c) Explain why the curve crosses the x-axis between $x = 3$ and $x = 3.1$

(1)

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Question 4 continued

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5. The equation $x^3 + x - 8 = 0$ has a single real root α.

(a) Show that α lies in the interval [1, 2].

(2)

(b) Show that $\alpha = 1.83$ correct to 2 decimal places.

(3)

(c) Explain why checking the values of $x^3 + x - 8$ at $x = 1.83$ alone is not enough.

(1)

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Question 5 continued

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6.

$$f(x) = x^3 - 4x^2 + 2$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[3.5, 4]$.

(2)

(b) Using $x_0 = 3.5$ as a first approximation, apply the Newton–Raphson process once to find a second approximation to α, giving your answer to 3 decimal places.

(4)

(c) Given that $\alpha = 3.87$ to 2 decimal places, show that this is correct.

(3)

(d) Explain why $x_0 = \frac{8}{3}$ would not be a suitable first approximation.

(1)

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Question 6 continued

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Question 6 continued

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7.

$$f(x) = 2^x - x - 3$$

(a) Show that the equation $f(x) = 0$ has a root α in the interval $[2, 3]$.

(2)

(b) Use interval bisection twice to find an interval of width 0.25 containing α.

(3)

(c) Use linear interpolation once on the interval $[2, 3]$ to find an approximation to α, giving your answer to 3 decimal places.

(3)

(d) Given that $\alpha = 2.445$ to 3 decimal places, comment on the accuracy of each method after the work done.

(2)

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Question 7 continued

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8.

$$g(x) = x + \frac{3}{x-2}, \qquad x \neq 2$$

(a) Show that g(1) and g(3) have opposite signs.

(2)

(b) Show that the equation $g(x) = 0$ has no real roots.

(3)

(c) Explain why your answers to parts (a) and (b) do not contradict each other.

(1)

$$h(x) = x^3 - 3x + 1$$

(d) Explain why the Newton–Raphson process cannot be used with $x_0 = 1$ to find a root of $h(x) = 0$

(2)

(e) Apply the Newton–Raphson process once with $x_0 = 0.5$, giving your answer to 3 decimal places.

(2)

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Question 8 continued

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9. The Newton–Raphson process is used to find $\sqrt{10}$, as a root of $f(x) = x^2 - 10 = 0$

(a) Show that the process gives the iteration formula $x_{n+1} = \frac{1}{2}\left(x_n + \frac{10}{x_n}\right)$

(3)

(b) Starting with $x_0 = 3$, find x_1 and x_2, giving your answers to 5 decimal places.

(3)

(c) Explain, with reference to the tangent to the curve $y = f(x)$ at $x = x_n$, how the formula for x_{n+1} arises.

(2)

(d) Find the error in x_2, and comment on how quickly the process converges.

(2)

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Question 9 continued

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10.

$$f(x) = x^3 - kx + 1$$

where k is a constant. Given that f(2) = 1

(a) find the value of k.

(1)

(b) By considering values of f(x) at suitable integers, show that f(x) = 0 has three real roots, and give an interval of width 1 containing each.

(3)

(c) Taking $x_0 = 1.8$, apply the Newton–Raphson process twice to approximate the largest root, giving your answers to 4 decimal places.

(4)

(d) Use linear interpolation once on [0, 1] to approximate the root in that interval.

(2)

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TOTAL FOR CHAPTER 3 IS 80 MARKS

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