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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 2 (WFM02)

Further Pure 2 Practice Paper 2

Mark Scheme

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM02/01) • 75 marks

Every question marked line by line the way an examiner marks it, with notes on the common errors

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It depends on the method mark before it.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao / cso	Correct answer only / correct solution only, with no errors anywhere in the working.
ft / oe	Follow through from your own earlier answer / or any correct equivalent form.

"Solutions relying on calculator technology are not acceptable." Questions 3 and 6 say this. There, a correct final answer with no algebraic working scores **no marks**. Elsewhere a calculator may be used, but the method must still be seen.

Rough grade guide (out of 75): A* **64** • A **58** • B **51** • C **44**

Based on recent WFM02 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

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What to revise next

Write in your score for each question. Where you dropped marks, tap the chapter to practise ten more exam questions on it.

Q	What it tested	Marks	Practise it	Score
1	A modulus inequality from a sketch, including every negative x	7	Ch 1 Inequalities	
2	A given substitution, then an integrating factor	8	Ch 5 First-Order DEs	
3	Three partial fractions; differences two terms apart; a limit	9	Ch 2 Series	
4	A line mapped to a circle; which side a region goes to	9	Ch 4 Further Argand Diagrams	
5	A Taylor series built from a differential equation	10	Ch 7 Maclaurin and Taylor Ch 5 First-Order DEs	
6	Fourth roots in exact form; the square they make	10	Ch 3 Complex Numbers	
7	A particular integral that clashes with the complementary function	11	Ch 6 Second-Order DEs	
8	The area inside two polar curves, split where they meet	11	Ch 8 Polar Coordinates	
	Total	**75**		

Where did the marks go? At grade A in FP2 the lost marks are rarely a missing method. More often a step was taken without checking it: a particular integral already in the complementary function, a region shaded on the wrong side of a circle, the wrong curve used on part of a polar area, or the negative values of x left out of an inequality. Find the line where your working first differs from the scheme, then practise that chapter.

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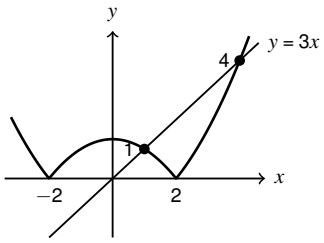
Question 1

7 marks

Part	Scheme	Marks
(a)	$y = \lvert x^2 - 4 \rvert$: a W shape touching the x-axis at $(\pm 2, 0)$ and meeting the y-axis at $(0, 4)$ $y = 3x$: a straight line through the origin, crossing the curve twice for $x > 0$  	B1 B1 (2)
(b)	$x^2 - 4 = 3x$ gives $x = 4$ ($x = -1$ rejected); $4 - x^2 = 3x$ gives $x = 1$ ($x = -4$ rejected) $\boxed{x = 1,\ x = 4}$	M1 A1 (2)
(c)	The curve is above the line outside the interval $[1, 4]$, including every $x < 0$ $\boxed{x < 1}$ $\boxed{x > 4}$ *Giving $-4 < x < 1$ or $x < -1$ misses the fact that $\lvert x^2 - 4 \rvert \geqslant 0 > 3x$ for every negative x.*	M1 A1 A1 (3)
		Total for Question 1: 7 marks

Question 2

8 marks

Part	Scheme	Marks
(a)	$y = z^{-1}$, $\frac{dy}{dx} = -z^{-2}\frac{dz}{dx}$	B1
	$-xz^{-2}\frac{dz}{dx} + z^{-1} = x^3z^{-2}$	M1
	Multiply by $-\frac{z^2}{x}$: $\frac{dz}{dx} - \frac{z}{x} = -x^2$	A1*
		(3)
(b)	Integrating factor $e^{-\int \frac{1}{x}dx} = \frac{1}{x}$	B1
	$\frac{z}{x} = \int -x\,dx = -\frac{1}{2}x^2 + c$	M1 A1
	$y = \frac{1}{cx - \frac{1}{2}x^3}$; $x = 1, y = 1$: $c = \frac{3}{2}$	M1
	$\boxed{y = \frac{2}{3x - x^3}}$	A1
	The answer must be for y, not z.	
		(5)
		Total for Question 2: 8 marks

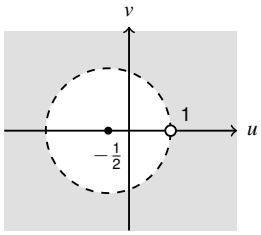
Question 3

9 marks

Part	Scheme	Marks
(a)	$\frac{A}{r} + \frac{B}{r+2} + \frac{C}{r+4}$, $4 = A(r+2)(r+4) + Br(r+4) + Cr(r+2)$	M1
	Two of $A = \frac{1}{2}$, $B = -1$, $C = \frac{1}{2}$	A1
	$\boxed{\frac{1}{2r} - \frac{1}{r+2} + \frac{1}{2(r+4)}}$	A1
		(3)
(b)	$\frac{1}{2}\sum\left[\left(\frac{1}{r} - \frac{1}{r+2}\right) - \left(\frac{1}{r+2} - \frac{1}{r+4}\right)\right]$, or terms listed for $r = 1, 2, 3, 4, \ldots, n$	M1
	$\frac{1}{2}\left[\left(1 + \frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right) - \left(\frac{1}{3} + \frac{1}{4} - \frac{1}{n+3} - \frac{1}{n+4}\right)\right]$	A1
	$\frac{1}{2}\left(\frac{3}{2} - \frac{7}{12}\right) = \frac{11}{24}$	M1
	$\boxed{a = \frac{11}{24}}$	A1
		(4)
(c)	Each of $\frac{1}{n+1}, \ldots, \frac{1}{n+4} \to 0$	M1
	$\boxed{\frac{11}{24}}$	A1
		(2)
		Total for Question 3: 9 marks

Question 4

9 marks

Part	Scheme	Marks
(a)	$w(z-\mathrm{i}) = z+2\mathrm{i},\ z = \dfrac{\mathrm{i}(w+2)}{w-1}$	**M1**
	$w = u+\mathrm{i}v:\ z = \dfrac{\mathrm{i}(u+2+\mathrm{i}v)(u-1-\mathrm{i}v)}{(u-1)^2+v^2}$	**M1**
	$= \dfrac{3v+\mathrm{i}\left(u^2+u-2+v^2\right)}{(u-1)^2+v^2}$	**A1**
	Real axis: $\mathrm{Im}\, z = 0,\ u^2+u+v^2-2=0$	**M1**
	$\left(u+\frac{1}{2}\right)^2+v^2 = \frac{9}{4}$	**A1**
	Centre $-\frac{1}{2}$ (that is, $(-\frac{1}{2}, 0)$), radius $\frac{3}{2}$	**A1**
	The point $w = 1$ lies on the circle but is not an image: it would need $z \to \infty$.	**(6)**
(b)	$y > 0$ gives $\mathrm{Im}\, z > 0$, so $u^2+u+v^2-2 > 0$ (or test $z = 2\mathrm{i}$: $w = 4$)	**M1**
	The region is outside the circle	**A1**
	Outside of the circle shaded, circle dashed or not included 	**B1**
		(3)
		Total for Question 4: 9 marks

Question 5

10 marks

Part	Scheme	Marks
(a)	$\frac{d^2y}{dx^2} = 2y\frac{dy}{dx} - 1,\ \frac{d^3y}{dx^3} = 2\left(\frac{dy}{dx}\right)^2 + 2y\frac{d^2y}{dx^2}$	M1
	$x = 1$: $y' = 4 - 1 = 3$, $y'' = 2(2)(3) - 1 = 11$	A1
	$y''' = 2(9) + 2(2)(11) = 62$	A1
		(3)
(b)	$y = y_1 + y_1'(x-1) + \frac{y_1''}{2!}(x-1)^2 + \frac{y_1'''}{3!}(x-1)^3$	M1
	$2 + 3(x-1)$	A1
	$+\frac{11}{2}(x-1)^2$	A1
	$+\frac{31}{3}(x-1)^3$	A1
		(4)
(c)	$2 + 0.3 + \frac{11}{2}(0.01) + \frac{31}{3}(0.001)$	M1
	2.365	A1
		(2)
(d)	$x = 2$ is not close to $x = 1$, so $(x-1)$ is not small and the neglected terms are not small (in fact the solution is not defined up to $x = 2$)	B1
		(1)
		Total for Question 5: 10 marks

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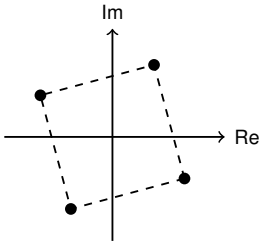
Question 6

10 marks

Part	Scheme	Marks
(a)	$r = \sqrt{64+192} = 16,\ \theta = -\pi + \frac{\pi}{3} = -\frac{2\pi}{3}$	M1
	$\boxed{w = 16\left(\cos\left(-\frac{2\pi}{3}\right) + \mathrm{i}\sin\left(-\frac{2\pi}{3}\right)\right)}$	A1
		(2)
(b)	$\lvert z\rvert = 16^{\frac{1}{4}} = 2$	B1
	$\arg z = \dfrac{-\frac{2\pi}{3} + 2k\pi}{4}$	M1
	$\arg z = -\frac{\pi}{6},\ \frac{\pi}{3},\ \frac{5\pi}{6},\ -\frac{2\pi}{3}$	M1
	Two roots correct in the form $a+b\mathrm{i}$	A1
	$\boxed{\sqrt{3}-\mathrm{i},\ 1+\sqrt{3}\mathrm{i},\ -\sqrt{3}+\mathrm{i},\ -1-\sqrt{3}\mathrm{i}}$	A1
		(5)
(c)	Four points on a circle of radius 2, at right angles to each other	B1
		
	A square with diagonals 4: $\frac{1}{2} \times 4 \times 4$ (or side $2\sqrt{2}$)	M1
	$\boxed{8}$	A1
		(3)
	Total for Question 6: 10 marks	

Question 7

11 marks

Part	Scheme	Marks
(a)	$m^2 - m - 6 = 0$, $m = 3$ or $m = -2$: e^{3x} is in the complementary function, so λe^{3x} gives 0	**B1**
	Try $y = \lambda x e^{3x}$	**M1**
	$y' = \lambda e^{3x}(1 + 3x)$, $y'' = \lambda e^{3x}(6 + 9x)$	**M1**
	$\lambda e^{3x}[(6 + 9x) - (1 + 3x) - 6x] = 5\lambda e^{3x} = 10e^{3x}$	**A1**
	$\boxed{y = 2xe^{3x}}$	**A1**
		(5)
(b)	Complementary function $Ae^{3x} + Be^{-2x}$	**M1**
	$\boxed{y = Ae^{3x} + Be^{-2x} + 2xe^{3x}}$	**A1**
		(2)
(c)	$x = 0$, $y = 0$: $A + B = 0$	**B1**
	$y' = 3Ae^{3x} - 2Be^{-2x} + 2e^{3x} + 6xe^{3x}$	**M1**
	$x = 0$: $3A - 2B + 2 = 1$, so $A = -\frac{1}{5}$, $B = \frac{1}{5}$	**M1**
	$\boxed{y = -\frac{1}{5}e^{3x} + \frac{1}{5}e^{-2x} + 2xe^{3x}}$	**A1**
		(4)
	Total for Question 7: 11 marks	

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Question 8

11 marks

Part	Scheme	Marks
(a)	$1+\cos\theta = 3\cos\theta$	M1
	$\cos\theta = \frac{1}{2},\ \theta = \pm\frac{\pi}{3}$	A1
	$\boxed{A\left(\frac{3}{2}, \frac{\pi}{3}\right),\ B\left(\frac{3}{2}, -\frac{\pi}{3}\right)}$	A1
		(3)
(b)	By symmetry, $2\times[C_1$ from 0 to $\frac{\pi}{3}$ + C_2 from $\frac{\pi}{3}$ to $\frac{\pi}{2}]$	M1
	$\frac{1}{2}\int(1+\cos\theta)^2\,\mathrm{d}\theta = \frac{1}{2}\int\left(\frac{3}{2}+2\cos\theta+\frac{1}{2}\cos 2\theta\right)\mathrm{d}\theta$	M1
	$\frac{1}{2}\left[\frac{3}{2}\theta+2\sin\theta+\frac{1}{4}\sin 2\theta\right]_0^{\pi/3}$	A1
	$=\frac{\pi}{4}+\frac{9\sqrt{3}}{16}$	A1
	$\frac{1}{2}\int_{\pi/3}^{\pi/2} 9\cos^2\theta\,\mathrm{d}\theta = \frac{9}{4}\left[\theta+\frac{1}{2}\sin 2\theta\right]_{\pi/3}^{\pi/2}$	M1
	$=\frac{3\pi}{8}-\frac{9\sqrt{3}}{16}$	A1
	$2\left(\frac{\pi}{4}+\frac{9\sqrt{3}}{16}+\frac{3\pi}{8}-\frac{9\sqrt{3}}{16}\right)$	M1
	$\boxed{\frac{5\pi}{4}}$	A1
	Using C_1 (or C_2) all the way from 0 to $\frac{\pi}{2}$ is the common error: C_2 is the inner curve for $\theta > \frac{\pi}{3}$.	
		(8)
		Total for Question 8: 11 marks

TOTAL FOR PAPER: 75 MARKS