



eClassroom

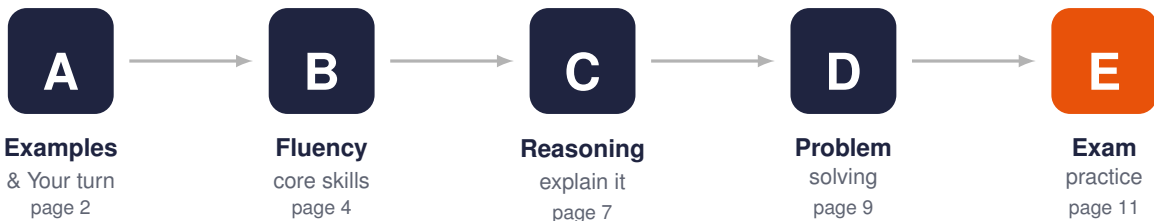
GCSE MATHEMATICS • ALGEBRA

Proof

Worksheet

Pearson Edexcel GCSE (1MA1) & International GCSE (4MA1) • No calculator

Free Proof worksheet for GCSE and iGCSE Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: disprove a statement with a counterexample • explain why examples do not prove a statement • write consecutive, even and odd numbers in algebra • use algebra to show that a statement is always true • *Higher:* formal proofs with $2m$ and $2n + 1$, multiples, identities, and prove or disprove.

Which questions are for me?

FOUNDATION Everyone starts here. These skills appear on both tiers.

HIGHER Higher only: formal algebraic proof with odd and even numbers, multiples, identities and prove or disprove. Grades 6 to 9.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Foundation marks	23	10	13	15
Higher marks	16	14	17	15
My score				

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A**Worked examples & Your turn**

Read each example carefully. Cover it up, then answer the question beside it using the same method.

FOUNDATION

Everyone starts here

Worked example A1

Show that the statement “every multiple of 4 is a multiple of 8” is false.

One counterexample is enough. 12 is a multiple of 4 but $12 \div 8 = 1.5$

12

Your turn A1

Show that the statement “every odd number greater than 1 is prime” is false.

Worked example A2

Show that the sum of any four consecutive integers is even.

Write them as $n, n+1, n+2, n+3$

$4n+6 = 2(2n+3)$, which is 2 times a whole number

always even

Your turn A2

Show that the sum of any five consecutive integers is a multiple of 5

Worked example A3

Explain why $2n+7$ is odd for every whole number n .

$2n$ is even. An even number plus 7 is odd.

$2n+7 = 2(n+3)+1$

one more than an even number

Your turn A3

Explain why $6n+4$ is even for every whole number n .



HIGHER*Higher only: formal proof***Worked example A4: odd and even**

Prove that the product of two odd numbers is odd.

Use different letters: $2m + 1$ and $2n + 1$

$$\begin{aligned}(2m + 1)(2n + 1) &= 4mn + 2m + 2n + 1 \\ &= 2(2mn + m + n) + 1\end{aligned}$$

odd

Your turn A4

Prove that the sum of two odd numbers is even.

Worked example A5: showing a multiple

Prove that $(n + 5)^2 - (n + 1)^2$ is a multiple of 8 for every integer n .

Expand, simplify, then factorise out the 8

$$n^2 + 10n + 25 - n^2 - 2n - 1 = 8n + 24$$

$8(n + 3)$

Your turn A5

Prove that $(2n + 3)^2 - (2n - 1)^2$ is a multiple of 8 for every integer n .

Worked example A6: an identity

Show that $(x + 3)(x - 1) - (x + 1)^2 \equiv -4$

Expand both products and simplify the left-hand side.

$$x^2 + 2x - 3 - (x^2 + 2x + 1)$$

-4

Your turn A6

Show that $(x + 5)(x - 2) - (x + 3)(x - 1) \equiv x - 7$

Worked example A7: a square

Prove that the product of two consecutive integers, plus the larger one, is a square number.

$$n(n + 1) + (n + 1) = (n + 1)(n + 1)$$

$(n + 1)^2$

Your turn A7

Prove that the product of two consecutive even numbers is a multiple of 8



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

FOUNDATION

Everyone starts here

B1 Each statement is false. Give a counterexample.

(a) All prime numbers are odd.

(b) All multiples of 6 are multiples of 12

(c) Half of a whole number is a whole number.

(d) The product of two numbers is bigger than their sum.

[4 marks]

B2 n is an integer. Write an expression for

(a) the next integer

(b) an even number

(c) the next even number after $2n$

(d) an odd number

[4 marks]

B3 Is each statement always, sometimes or never true?

(a) odd + odd is even

(b) even $\times$ odd is odd

(c) a multiple of 3 is odd

(d) the square of an even number is even

[4 marks]

B4 n is an integer. Show that

(a) $2n + 4$ is even

(b) $3n + 6$ is a multiple of 3

(c) $5(n + 2) - 10$ is a multiple of 5

(d) $4n + 2$ is even but not a multiple of 4

[4 marks]



FOUNDATION *continued: consecutive numbers and identities***B5** n is an integer. Show that

(a) the sum of any three consecutive integers is a multiple of 3

(b) the sum of any two consecutive integers is odd

(c) the sum of any two consecutive even numbers is not a multiple of 4

[3 marks]**B6** Is each statement true for every value of x ? If it is not, give a value of x that shows this.

(a) $3(x - 2) + 6 = 3x$

(b) $(x + 3)^2 = x^2 + 9$

(c) $x(x - 2) + 1 = (x - 1)^2$

(d) $(x + 1)(x - 1) = x^2 - 1$

[4 marks]**HIGHER** *Higher tier only***B7** Prove that

(a) an even number times any integer is even

(b) the square of an odd number is odd

(c) an odd number minus an even number is odd

(d) the square of an even number is a multiple of 4

[4 marks]**B8** n is an integer. Prove that each expression is a multiple of the number in brackets.

(a) $(n + 2)^2 - n^2$ [4]

(b) $(n + 4)^2 - (n + 2)^2$ [4]

(c) $(3n + 1)^2 - (3n - 1)^2$ [12]

(d) $(n + 1)^3 - n^3 - 1$ [6]

[4 marks]

HIGHER*continued: identities and prove or disprove***B9** Show that

(a) $(x+2)^2 - 4 \equiv x(x+4)$

(b) $(x+1)^3 \equiv x^3 + 3x^2 + 3x + 1$

(c) $(a+b)^2 - (a-b)^2 \equiv 4ab$

(d) $(x^2+1)^2 - (x^2-1)^2 \equiv 4x^2$

[4 marks]**B10** n is a positive integer. Prove each statement, or give a counterexample to show that it is false.

(a) $n^2 + n + 1$ is always odd

(b) $2^n + 1$ is always prime

(c) the sum of three consecutive odd numbers is a multiple of 3

(d) if n is even, $n^2 + 2$ is a multiple of 4

[4 marks]



Reasoning

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

FOUNDATION

Everyone starts here

- C1** Tom says, “ $3 + 5 = 8$, so the sum of two odd numbers is always even.”
Explain why this is not a proof. [2 marks]
- C2** Sam writes two consecutive integers as n and m . Explain why this will not help him prove results about consecutive integers. [2 marks]
- C3** Explain, in words, why the product of any whole number and the next whole number is always even. [3 marks]
- C4** Show that the mean of any three consecutive integers is the middle one of the three. [3 marks]



HIGHER*Higher tier only*

- C5** Kai tries to prove that the sum of two odd numbers is even. He writes $(2n+1) + (2n+1) = 4n+2 = 2(2n+1)$
Explain the flaw in Kai's proof. [3 marks]
- C6** Prove that $n^2 - 1$ is not prime for any integer $n > 2$ [3 marks]
- C7** A two-digit number has digits a and b . Prove that the number plus the number with its digits reversed is always a multiple of 11 [3 marks]
- C8** You may use the fact that the product of any three consecutive integers is a multiple of 6
Prove that the product of any three consecutive even numbers is a multiple of 48 [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

FOUNDATION

Everyone starts here

- D1** Think of a number. Double it, add 10, halve the result, then subtract the number you first thought of.

Prove that the answer is always 5

[3 marks]

- D2** Here is part of a hundred square. A 2×2 box is drawn on it.

31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

Multiply the numbers on each diagonal of the box: 44×55 and 45×54 .

(a) Work out the difference between the two products.

(b) Prove that the difference is the same for every 2×2 box on a hundred square.

[4 marks]

- D3** Prove that the sum of any two consecutive multiples of 5 is an odd multiple of 5

[3 marks]

- D4** A rectangle has whole-number side lengths.

(a) Prove that its perimeter is always even.

(b) Show that its area can be odd.

[3 marks]



HIGHER*Higher tier only*

- D5** Prove that, for any three consecutive integers, the square of the middle one is 1 more than the product of the other two. [4 marks]
- D6** Prove that the sum of the squares of any three consecutive integers is 2 more than a multiple of 3 [4 marks]
- D7** Prove that $2^{n+2} - 2^n$ is a multiple of 3 for every positive integer n [4 marks]
- D8** (a) Prove that every odd number can be written as the difference of the squares of two consecutive integers.
- (b) Hence write 99 as the difference of two square numbers. [5 marks]



**Exam practice**

Exam-style questions. Write your answers in the spaces. Aim for about one minute per mark.

FOUNDATION*Foundation and Higher crossover*

E1 Kim says, "The sum of two prime numbers is always even."

Give an example to show that Kim is wrong.

(Total for Question E1 is 2 marks)

E2 n is an integer.

(a) Write down an expression for the next integer after n

.....

(1)

(b) Show that the sum of any three consecutive integers is a multiple of 3

(2)

(Total for Question E2 is 3 marks)

E3 Jo checks that $n^2 + 2$ is odd when $n = 1$, $n = 3$ and $n = 5$. She says this proves that $n^2 + 2$ is odd whenever n is odd.

Is Jo right? Explain your answer.

(Total for Question E3 is 2 marks)

E4 (a) Show that $73 - 37$ is a multiple of 9

(1)

(b) A two-digit number has tens digit a and units digit b . Prove that the number minus the number with its digits reversed is always a multiple of 9

(2)

(Total for Question E4 is 3 marks)



- E5** Using $2m$ for an even number and $2n + 1$ for an odd number, prove that the sum of an even number and an odd number is always odd.

(Total for Question E5 is 3 marks)

- E6** Is $(n + 1)^2 = n^2 + 1$ true for every value of n ? Show how you decide.

(Total for Question E6 is 2 marks)

HIGHER

Higher tier only

- E7** Prove that the product of two consecutive odd numbers is always odd.

(Total for Question E7 is 3 marks)

- E8** n is an integer. Prove that $(n + 3)^2 - (n - 3)^2$ is a multiple of 12

(Total for Question E8 is 3 marks)

- E9** (a) Prove that the sum of the squares of any two consecutive integers is always odd.

(2)

- (b) Hence explain why the mean of the squares of two consecutive integers is never a whole number.

(2)

(Total for Question E9 is 4 marks)



E10 Prove that the square of any odd number is 1 more than a multiple of 8

(Total for Question E10 is 5 marks)

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