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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

# Pure Mathematics 4

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA14 paper, with a full mark scheme

| Chapters (tap a chapter to jump to it)                                                                                 | Qs | Marks | Page    | My score |
|------------------------------------------------------------------------------------------------------------------------|----|-------|---------|----------|
| <b>1 Proof</b><br>Proof by contradiction: irrational numbers, primes, integers, inequalities                           | 10 | 73    | own PDF |          |
| <b>2 Partial Fractions</b><br>Linear and repeated factors, improper fractions                                          | 10 | 78    | own PDF |          |
| <b>3 Coordinate Geometry in the <math>(x, y)</math> Plane</b><br>Parametric equations and conversion to Cartesian form | 10 | 76    | own PDF |          |
| <b>4 Binomial Expansion</b><br>$(a + bx)^n$ for rational $n$ , validity, using partial fractions                       | 10 | 81    | own PDF |          |
| <b>5 Differentiation</b><br>Parametric and implicit differentiation, rates of change                                   | 10 | 81    | own PDF |          |
| <b>6 Integration</b><br>Volumes of revolution, substitution, parts, partial fractions, differential equations          | 10 | 86    | own PDF |          |
| <b>7 Vectors</b><br>3D vectors, vector equations of lines, intersections, scalar product                               | 10 | 82    | 2       |          |

[Full mark scheme](#)

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## 7

## CHAPTER 7 • KEY SKILLS

## Vectors

Specification P4: Vectors • 10 questions • 82 marks • about 98 minutes

## 1 Vectors and distance

$\vec{AB} = \mathbf{b} - \mathbf{a}$  (the end minus the start).  
 $|\mathbf{x}\mathbf{i} + \mathbf{y}\mathbf{j} + \mathbf{z}\mathbf{k}| = \sqrt{x^2 + y^2 + z^2}$ . A unit vector is  $\frac{\mathbf{a}}{|\mathbf{a}|}$

## 2 Equation of a line

$\mathbf{r} = \mathbf{a} + \lambda \mathbf{d}$ : a point on the line plus a multiple of the direction. Always write “ $\mathbf{r} =$ ”.

## 3 Points on a line

Set  $\mathbf{a} + \lambda \mathbf{d}$  equal to the point: one component gives  $\lambda$ , the others must agree.

## 4 Intersecting lines

Equate the lines, solve two components for  $\lambda$  and  $\mu$ , then use the third to check (or to find a constant).

## 5 Parallel or skew

Parallel:  $\mathbf{d}_1 = k\mathbf{d}_2$ . Skew: not parallel and no point of intersection. State both, with reasons.

## 6 Scalar product and angles

$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$ , with both vectors pointing out of the angle. A negative value means an obtuse angle.

## 7 Perpendiculars

$\mathbf{a} \cdot \mathbf{b} = 0$ . The foot  $X$  from  $P$ :  $\vec{PX} \cdot \mathbf{d} = 0$  (not  $\vec{OX} \cdot \mathbf{d}$ ). The reflection is  $\mathbf{p}' = 2\mathbf{x} - \mathbf{p}$

## 8 Areas

Triangle:  $\frac{1}{2}|\mathbf{a}||\mathbf{b}|\sin \theta$ ; parallelogram:  $|\mathbf{a}||\mathbf{b}|\sin \theta$ . Exact  $\sin \theta = \sqrt{1 - \cos^2 \theta}$

## 9 Distances along a line

$|\lambda \mathbf{d}| = |\lambda||\mathbf{d}|$ : a distance along a line gives two points, one each side. Draw a diagram.

**Skills map** every skill above is tested at least once; interleaved skills come from GCSE or other chapters

| Skill                                                                 | Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | Q8 | Q9 | Q10 |
|-----------------------------------------------------------------------|----|----|----|----|----|----|----|----|----|-----|
| Marks                                                                 | 6  | 5  | 6  | 5  | 6  | 10 | 10 | 11 | 12 | 11  |
| 1 Vectors and distance                                                | •  | •  |    |    | •  | •  | •  |    | •  | •   |
| 2 Equation of a line                                                  |    | •  |    |    |    |    | •  |    |    |     |
| 3 Points on a line                                                    |    | •  |    |    |    |    |    | •  |    | •   |
| 4 Intersecting lines                                                  |    |    | •  | •  |    |    |    | •  | •  |     |
| 5 Parallel or skew                                                    |    |    |    | •  |    |    |    |    |    |     |
| 6 Scalar product and angles                                           | •  |    |    |    |    | •  |    | •  | •  | •   |
| 7 Perpendiculars                                                      |    |    |    |    | •  |    | •  |    | •  | •   |
| 8 Areas                                                               |    |    |    |    |    | •  | •  | •  | •  | •   |
| 9 Distances along a line                                              |    |    |    |    |    |    |    | •  |    |     |
| Interleaved: quadratics and completing the square (P1)                |    |    |    |    | •  |    |    |    |    | •   |
| Interleaved: exact trigonometry, Pythagoras, circle theorems (P1, P2) | •  |    |    |    |    | •  |    | •  | •  | •   |
| Interleaved: proof and minimising with calculus (Chapters 1, 5)       |    |    |    | •  | •  |    |    |    |    |     |

**Exam difficulty.** Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

**Must learn.** Draw a diagram. Use direction vectors (not position vectors) for angles, and vectors pointing out of the angle. Finish every argument: “so the lines do not intersect”, “not parallel, so skew”. Give coordinates when coordinates are asked for.



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1.

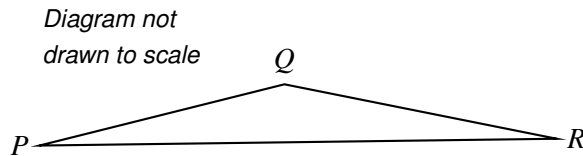


Figure 1

Figure 1 shows a triangle  $PQR$ . Relative to a fixed origin  $O$ ,

$$\overrightarrow{OP} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} \quad \overrightarrow{OQ} = 4\mathbf{i} + 3\mathbf{j} \quad \overrightarrow{OR} = 6\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

- (a) Find  $\overrightarrow{QP}$  and  $\overrightarrow{QR}$  (2)
- (b) Find the size of angle  $PQR$ , giving your answer in degrees to one decimal place. (3)
- (c) State, with a reason, which angle of triangle  $PQR$  is the largest. (1)



## Question 1 continued

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(Total for Question 1 is 6 marks)



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2. Relative to a fixed origin  $O$ , the point  $A$  has coordinates  $(2, -1, 3)$  and the point  $B$  has coordinates  $(4, 3, -1)$ . The line  $l$  passes through  $A$  and  $B$ .

(a) Find a vector equation of  $l$ .

(2)

The point  $C(p, 9, q)$  lies on  $l$ .

(b) Find the value of  $p$  and the value of  $q$ .

(2)

(c) Find a unit vector parallel to  $l$ .

(1)

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## Question 2 continued

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(Total for Question 2 is 5 marks)



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3. Relative to a fixed origin  $O$ , the lines  $l_1$  and  $l_2$  are given by the equations

$$l_1: \mathbf{r} = (\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k}) \quad l_2: \mathbf{r} = (3\mathbf{i} + q\mathbf{j} + 8\mathbf{k}) + \mu(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$$

where  $\lambda$  and  $\mu$  are scalar parameters and  $q$  is a constant. Given that  $l_1$  and  $l_2$  intersect,

- (a) find the value of  $q$ ,

(4)

- (b) find the coordinates of the point of intersection.

(2)

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## Question 3 continued

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(Total for Question 3 is 6 marks)



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4. Relative to a fixed origin  $O$ , the lines  $l_1$  and  $l_2$  are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \quad l_2: \mathbf{r} = \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

where  $\lambda$  and  $\mu$  are scalar parameters.

Prove that  $l_1$  and  $l_2$  are skew.

(5)

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## Question 4 continued

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(Total for Question 4 is 5 marks)



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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Relative to a fixed origin  $O$ , the line  $l$  has equation

$$\mathbf{r} = (-4\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}) + \lambda(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$$

The point  $P$  lies on  $l$ .

(a) Show that  $OP^2 = 9\lambda^2 - 36\lambda + 56$

(2)

(b) Hence, by completing the square, find the coordinates of the point on  $l$  that is closest to  $O$ , and the shortest distance from  $O$  to  $l$ , giving the distance in the form  $a\sqrt{b}$

(4)

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## Question 5 continued

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(Total for Question 5 is 6 marks)



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6. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

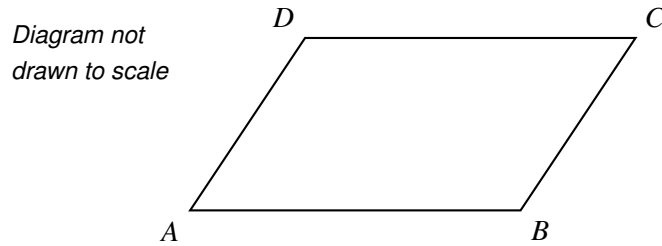


Figure 2

Figure 2 shows a parallelogram  $ABCD$ . Relative to a fixed origin  $O$ , the points  $A$ ,  $B$  and  $C$  have coordinates  $(1, 2, 0)$ ,  $(3, 1, 2)$  and  $(4, 3, 5)$  respectively.

- (a) Find the coordinates of  $D$ . (2)
- (b) Show that  $\cos \angle ABC = -\frac{\sqrt{14}}{7}$ . (3)
- (c) Find the exact area of the parallelogram, giving your answer in the form  $p\sqrt{q}$ , where  $p$  and  $q$  are integers. (3)
- (d) Hence find the exact shortest distance from  $A$  to the line through  $B$  and  $C$ . (2)



## Question 6 continued

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## Question 6 continued

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## Question 7 continued

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## Question 7 continued

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(Total for Question 7 is 10 marks)



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8. Relative to a fixed origin  $O$ , the lines  $l_1$  and  $l_2$  are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} -3 \\ 0 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad l_2: \mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 7 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$$

where  $\lambda$  and  $\mu$  are scalar parameters. The lines meet at the point  $P$ .

- (a) Find the coordinates of  $P$ .

(3)

The acute angle between  $l_1$  and  $l_2$  is  $\theta$ .

- (b) Show that  $\cos \theta = \frac{4}{9}$

(3)

The point  $Q$  lies on  $l_1$  where  $\lambda = 5$ . The point  $R$  lies on  $l_2$  such that  $PR = PQ$ .

- (c) Find the coordinates of the two possible positions of  $R$ .

(3)

- (d) Find the exact area of triangle  $QPR$ , explaining why it is the same for both positions of  $R$ .

(2)



## Question 8 continued

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## Question 8 continued

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(Total for Question 8 is 11 marks)



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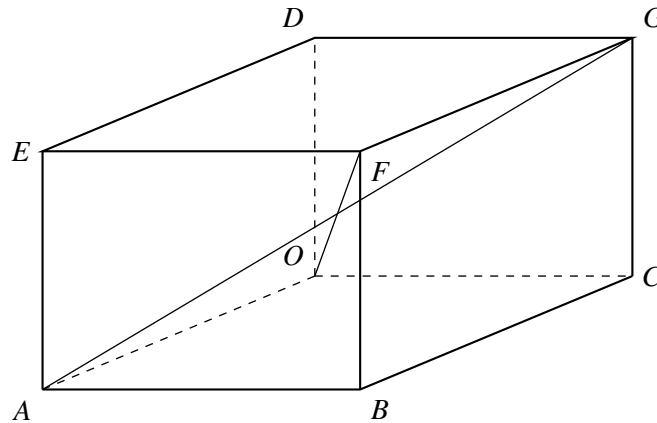


Figure 3

Figure 3 shows a cuboid  $OABCDEFG$ . Relative to the origin  $O$ , the vertices  $A$ ,  $C$  and  $D$  have position vectors  $6\mathbf{i}$ ,  $4\mathbf{j}$  and  $3\mathbf{k}$  respectively.

- (a) Write down  $\overrightarrow{OF}$  and  $\overrightarrow{AG}$  (2)
- (b) Find the acute angle between the lines  $OF$  and  $AG$ , giving your answer in degrees to one decimal place. (3)
- (c) Show that the lines  $OF$  and  $AG$  intersect, and find the coordinates of the point of intersection. (3)
- (d) Show that angle  $AOG$  is a right angle, and hence find the area of triangle  $OAG$  (2)
- (e) Hence find the exact shortest distance from  $O$  to the line  $AG$ , giving your answer in the form  $k\sqrt{61}$  (2)



## Question 9 continued

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## Question 9 continued

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(Total for Question 9 is 12 marks)



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**10. In this question you must show all stages of your working.****Solutions relying on calculator technology are not acceptable.**Relative to a fixed origin  $O$ , the line  $l$  has equation

$$\mathbf{r} = (\mathbf{j} + 2\mathbf{k}) + \lambda(\mathbf{i} + \mathbf{j})$$

The points  $A$  and  $B$  have coordinates  $(5, 1, 3)$  and  $(1, 3, 3)$  respectively.

- (a) Explain why neither  $A$  nor  $B$  lies on  $l$ .

**(1)**The point  $C$  lies on  $l$  and angle  $ACB = 90^\circ$ 

- (b) Show that the value of  $\lambda$  at  $C$  satisfies  $\lambda^2 - 4\lambda + 3 = 0$

**(4)**

- (c) Hence find the coordinates of the two possible positions of  $C$ .

**(2)**

- (d) Explain, with reference to the sphere with diameter  $AB$ , why there cannot be more than two points on  $l$  at which angle  $ACB = 90^\circ$

**(2)**

- (e) For the position of  $C$  with the smaller value of  $\lambda$ , find the area of triangle  $ACB$ .

**(2)**

## Question 10 continued

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## Question 10 continued

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(Total for Question 10 is 11 marks)

**TOTAL FOR CHAPTER 7 IS 82 MARKS**

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