



eClassroom

A-LEVEL MATHEMATICS • PURE 1

Integration

Worked Solutions

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Integration worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 1 Integration worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

Integration revision engine

More worksheets



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

A**Your turn answers**

Compare each line of your working with the solution.

CORE

$$\text{A1} \quad \frac{12x^4}{4} + \frac{6x^2}{2} - 7x + c = \boxed{3x^4 + 3x^2 - 7x + c}$$

$$\text{A2} \quad \frac{8x^{-2}}{-2} - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + c = \boxed{-4x^{-2} - 4x^{\frac{3}{2}} + c}$$

$$\text{A3} \quad f(x) = x^4 + 3x + c; f(-1) = 1 - 3 + c = 6, \text{ so } c = 8: \boxed{f(x) = x^4 + 3x + 8}$$

STRETCH

$$\text{A4} \quad (x-4)(3x+1) = 3x^2 - 11x - 4: \boxed{x^3 - \frac{11}{2}x^2 - 4x + c}$$

$$\text{A5} \quad \frac{4x - 3\sqrt{x}}{\sqrt{x}} = 4x^{\frac{1}{2}} - 3: \boxed{\frac{8}{3}x^{\frac{3}{2}} - 3x + c}$$

$$\text{A6} \quad y = 4x^{\frac{3}{2}} - x^{-1} + c; \text{ at } (4, 20): 32 - \frac{1}{4} + c = 20, \text{ so } c = -\frac{47}{4}: \boxed{y = 4x^{\frac{3}{2}} - \frac{1}{x} - \frac{47}{4}}$$

$$\text{A7} \quad f'(x) = 4x^3 - 2x + c_1, c_1 = 3; f(x) = x^4 - x^2 + 3x + c_2, c_2 = -1: \boxed{f(x) = x^4 - x^2 + 3x - 1}$$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $\frac{1}{5}x^5 + c$ (b) $-5x^{-1} + c$ (c) $\frac{3}{5}x^{\frac{5}{3}} + c$ (d) $12x^{\frac{1}{2}} + c$

Raise the power by 1 and divide by the new power, whether it is negative or a fraction.

B2 (a) $\frac{2}{3}x^{\frac{3}{2}} + c$ (b) $x^{-3}: -\frac{1}{2}x^{-2} + c$ (c) $2x^{-\frac{1}{2}}: 4x^{\frac{1}{2}} + c$ (d) $x^{\frac{3}{2}}: \frac{2}{5}x^{\frac{5}{2}} + c$

Every integrand must be a sum of powers of x before the rule can be used.

B3 (a) $\frac{1}{3}x^3 + x^2 + c$ (b) $\frac{1}{3}x^3 + 2x^2 + 4x + c$ (c) $\frac{1}{3}x^3 - 4x + c$

(d) $x + 4x^{\frac{1}{2}} + 4: \frac{1}{2}x^2 + \frac{8}{3}x^{\frac{3}{2}} + 4x + c$

The middle term of a square is easy to forget: $(\sqrt{x} + 2)^2$ is not $x + 4$

B4 $f(x) = x^2 + c$ (a) $x^2 + 3$ (b) $x^2 + 2$ (c) $x^2 - 1$ (d) $x^2 - 6$

The gradient function fixes the shape; the point only moves the curve up or down.

B5 (a) $1 + x^{-2}: x - x^{-1} + c$ (b) $2x^2 - 1: \frac{2}{3}x^3 - x + c$ (c) $x^{\frac{1}{2}} + x^{-\frac{1}{2}}: \frac{2}{3}x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$

Divide every term on the top by the bottom.

B6 (a) (i) $6x$ (ii) $x^3 + c$ (b) (i) $-8x^{-3}$ (ii) $-4x^{-1} + c$

(c) (i) $\frac{1}{2}x^{-\frac{1}{2}}$ (ii) $\frac{2}{3}x^{\frac{3}{2}} + c$ (d) (i) $-\frac{3}{2}x^{-\frac{5}{2}}$ (ii) $-2x^{-\frac{1}{2}} + c$

Differentiating lowers the power and multiplies; integrating raises it and divides.

STRETCH

B7 (a) $y = 2x^{\frac{3}{2}} + c, 16 + c = 10: y = 2x^{\frac{3}{2}} - 6$ (b) $y = -x^{-1} + c, -1 + c = 0: y = 1 - \frac{1}{x}$

(c) $y = 2x^{\frac{1}{2}} + c, 6 + c = 5: y = 2\sqrt{x} - 1$ (d) $y = x^2 - x^3 + c, 4 - 8 + c = 0: y = x^2 - x^3 + 4$

Substitute the point into the integrated equation, never into $\frac{dy}{dx}$



- B8** (a) $f' = 6x$, so $3x^2$ (b) $f' = 3x^2 + 1$, so $x^3 + x$ (c) $f' = 4x^3 - 4$, so $x^4 - 4x + 3$
 (d) $f' = -x^{-2} + 1$, so $x^{-1} + x - 2$

Each integration needs its own constant, found from its own condition.

- B9** (a) $f = \frac{1}{2}kx^2$, $2k = 8$: $k = 4$ (b) $f = \frac{2}{3}kx^{\frac{3}{2}}$, $\frac{16}{3}k = 32$: $k = 6$
 (c) $f = x^3 + kx$, $1 + k = 5$: $k = 4$ (d) $f = -kx^{-1} + c$, $c = k$, $-\frac{k}{2} + k = 3$: $k = 6$

The first point fixes the constant of integration; the second fixes k

- B10** (a) $x^2 + 1$: 17 (b) $2x^{\frac{3}{2}}$: 16 (c) $3 - \frac{1}{x}$: $\frac{11}{4}$ (d) $12\sqrt{x} - 10$: 14

The constant can be positive, negative or zero; find it before predicting.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

- C1** Sam has left out the constant of integration: $\int 4x^3 dx = x^4 + c$. Any value of c works, for example $y = x^4 + 1$
 and $y = x^4 - 5$

- C2** You cannot integrate a fraction by integrating the top and the bottom. Divide first: $1 + x^{-2}$, so $x - x^{-1} + c$

- C3** Kai used $y = 0$, but the curve passes through $(2, 5)$, so $y = 5$ when $x = 2$: $5 = 8 - 8 + c$, $c = 5$
 $y = x^3 - 4x + 5$

- C4** Jo lowered the power. Integrating raises it by 1: $\frac{6x^{-2}}{-2} = -3x^{-2} + c$

Check: $\frac{d}{dx}(-3x^{-2}) = 6x^{-3}$, the original integrand.

STRETCH

- C5** Each integration needs its own constant. $f'(x) = 3x^2 + c_1$, and $f'(1) = 4$ gives $c_1 = 1$
 $f(x) = x^3 + x + c_2$, and $f(1) = 3$ gives $c_2 = 1$: $f(x) = x^3 + x + 1$



C6 Every such curve is $y = 2x^2 - 3x + c$: the curve $y = 2x^2 - 3x$ moved c units parallel to the y -axis.

Through $(1, 6)$: $2 - 3 + c = 6$, $c = 7$: **a translation of 7 units up**

C7 $\frac{d}{dx}(x + x^{-1} + c) = 1 - x^{-2} = \frac{x^2 - 1}{x^2}$ **as required**

Integration is the reverse of differentiation, so if an answer differentiates back to the integrand, it is correct.

C8 (a) $f'(1) = 3 - 6 + k = 0$: **$k = 3$**

(b) $f(x) = x^3 - 3x^2 + 3x + c$, $f(0) = 2$: **$f(x) = x^3 - 3x^2 + 3x + 2$**

(c) $(x - 1)^3 + 3 = x^3 - 3x^2 + 3x - 1 + 3 = x^3 - 3x^2 + 3x + 2$ **as required**

$f'(x) = 3x^2 - 6x + 3 = 3(x - 1)^2$, a square times 3, so **$f'(x) \geq 0$** for all x

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $y = x^4 - 3x^2 + c$; at $(2, 7)$: $16 - 12 + c = 7$, $c = 3$: **$y = x^4 - 3x^2 + 3$**

(b) $x = 0$: **$(0, 3)$**

D2 (a) **$f'(x) = 3x^{\frac{3}{2}} - 2x^{-\frac{1}{2}}$**

(b) $f(x) = \frac{6}{5}x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + c$; $f(4) = \frac{6}{5} \times 32 - 8 + c = 30$, so $c = -\frac{2}{5}$

$$f(x) = \frac{6}{5}x^{\frac{5}{2}} - 4x^{\frac{1}{2}} - \frac{2}{5}$$

D3 (a) $3 + k = 7$: **$k = 4$**

(b) $y = x^3 + 2x^2 + c$; at P : $1 + 2 + c = 4$, $c = 1$: **$y = x^3 + 2x^2 + 1$**

D4 (a) $h = 4t^{\frac{3}{2}} + c$; $h = 5$ when $t = 1$: $c = 1$: **$h = 4t^{\frac{3}{2}} + 1$**

(b) $4 \times 27 + 1 =$ **109 cm**



STRETCH

D5 (a) $f'(x) = 3x^2 + 2x^{-2} + c_1$; $f'(1) = 5 + c_1 = 0$: $f'(x) = 3x^2 + 2x^{-2} - 5$

(b) $f(x) = x^3 - 2x^{-1} - 5x + c_2$; $f(1) = -6 + c_2 = 4$: $f(x) = x^3 - \frac{2}{x} - 5x + 10$

D6 (a) The tangent's gradient is 5, so at $x = 2$: $2k - 1 = 5$: $k = 3$

(b) P is on the tangent: $y = 10 - 3 = 7$. $y = \frac{3}{2}x^2 + 4x^{-1} + c$; $6 + 2 + c = 7$, $c = -1$: $y = \frac{3}{2}x^2 + \frac{4}{x} - 1$

D7 (a) $f(x) = x^3 - 12x + c$; $8 - 24 + c = -16$, $c = 0$: $f(x) = x^3 - 12x$

(b) $x(x^2 - 12) = 0$: $x = 0$, $x = \pm 2\sqrt{3}$

D8 (a) $f'(1) = a + b = 1$; $f'(2) = 4a + 2b = 12$, so $2a + b = 6$ **as required**

(b) Subtracting: $a = 5$, $b = -4$

(c) $f(x) = \frac{5}{3}x^3 - 2x^2 + c$; $\frac{5}{3} - 2 + c = 2$, $c = \frac{7}{3}$: $f(x) = \frac{5}{3}x^3 - 2x^2 + \frac{7}{3}$



E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

- E1 (a) $f'(x) = \frac{5}{2}x^{\frac{3}{2}} - 4x^{-\frac{1}{2}}$ B1 B1
- (b) $f(x) = x^{\frac{5}{2}} - 8x^{\frac{1}{2}} + c$ M1: a power raised; A1
- $32 - 16 + c = 10$, so $c = -6$: $f(x) = x^{\frac{5}{2}} - 8x^{\frac{1}{2}} - 6$ A1: in full

- E2 (a) $f(x) = 2x^3 - x^2 + kx + c$; $(0, -3)$ gives $c = -3$ B1
- (b) $(1, 3)$: $2 - 1 + k - 3 = 3$ M1
- $k = 5$ A1
- (c) $f'(1) = 6 - 2 + 5 = 9$, so the normal has gradient $-\frac{1}{9}$ M1
- $y - 3 = -\frac{1}{9}(x - 1)$ dM1
- $x + 9y - 28 = 0$ A1

STRETCH

- E3 (a) $f(x) = \frac{2}{3}kx^{\frac{3}{2}} - 3x + c$ M1 A1
- $(1, 1)$: $\frac{2}{3}k - 3 + c = 1$, so $2k + 3c = 12$; $(4, 6)$: $\frac{16}{3}k - 12 + c = 6$, so $16k + 3c = 54$ as required A1*
- (b) Subtracting: $14k = 42$: $k = 3$, $c = 2$; $f(x) = 2x^{\frac{3}{2}} - 3x + 2$ M1 A1
- (c) $y = 2x^{\frac{3}{2}} - 3x + 2 = 2$, so $x(2\sqrt{x} - 3) = 0$ M1: factorises
- $x = 0$ or $\sqrt{x} = \frac{3}{2}$, so $x = \frac{9}{4}$ A1
- $(0, 2)$, $(\frac{9}{4}, 2)$ A1

