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INTERNATIONAL A LEVEL MATHEMATICS • PURE 3

Pure Mathematics 3

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA13/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA13 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Algebraic Methods Algebraic fractions, improper fractions and algebraic division	10	69	own PDF	
2 Functions and Graphs Modulus, composite and inverse functions, transformations	10	76	own PDF	
3 Trigonometric Functions sec, cosec, cot, identities, inverse trigonometric functions	10	72	own PDF	
4 Trigonometric Addition Formulae Addition and double-angle formulae, $R\cos(\theta \pm \alpha)$	10	75	own PDF	
5 Exponentials and Logarithms e^x , $\ln x$, modelling, logarithmic graphs	10	78	own PDF	
6 Differentiation Chain, product and quotient rules, trigonometric and exponential functions	10	76	own PDF	
7 Integration Standard integrals, $f(ax+b)$, trigonometric identities, reverse chain rule	10	77	own PDF	
8 Numerical Methods Locating roots, iteration	10	77	2	

[Full mark scheme](#)

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CHAPTER 8 • KEY SKILLS

Numerical Methods

Specification P3: Numerical methods • 10 questions • 77 marks • about 90 minutes

1 Locating roots

If f is continuous on $[a, b]$ and $f(a), f(b)$ have different signs, there is a root between a and b . Write all three parts.

2 When it fails

A sign change across an asymptote (e.g. $\frac{1}{x-2}$ or $\tan x$ at $\frac{\pi}{2}$) does not show a root: f is not continuous there.

3 Rearranging

Show $f(x) = 0$ can be written as $x = g(x)$ step by step, e.g.
 $x^3 - 3x - 5 = 0 \Rightarrow x = \sqrt[3]{3x+5}$

4 Iteration

$x_{n+1} = g(x_n)$: use the calculator's ANS key; give each iterate to the accuracy asked.

5 Root to a given accuracy

For $\alpha = 1.749$ (3 d.p.) use the interval $[1.7485, 1.7495]$: find the sign at both ends and conclude.

6 Convergence

Iteration converges when $|g'(x)| < 1$ near the root. A different rearrangement may be needed for another root.

7 Roots from calculus

A stationary point satisfies $f'(x) = 0$; a tangent through the origin satisfies $f(\alpha) = \alpha f'(\alpha)$

8 Intersections

Curves meet where $f(x) - g(x) = 0$: locate the roots of this new function.

9 Context

Answer with units and the accuracy asked, e.g. a time in hours to 3 significant figures.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	5	11	10	10	10	11
1 Locating roots	•	•			•	•			•	•
2 When it fails			•						•	
3 Rearranging	•			•		•	•	•	•	•
4 Iteration	•			•	•	•	•	•	•	•
5 Root to a given accuracy		•				•	•		•	
6 Convergence				•						•
7 Roots from calculus						•	•	•		
8 Intersections					•					•
9 Context								•		
Interleaved: differentiation (Chapter 6)						•	•	•		
Interleaved: trigonometric functions (Chapters 3, 4)							•		•	
Interleaved: e, ln and inverse functions (Chapters 2, 5)		•			•	•		•		•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA13 paper, where one part leads into the next and skills from different topics are combined.

Must learn. For a sign change, give both values (or their signs), state that f is continuous on the interval, and conclude "so there is a root". Work in radians for trigonometric functions.



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1.

$$f(x) = x^3 - 3x - 5$$

(a) Show that the equation $f(x) = 0$ has a root between $x = 2$ and $x = 3$ (2)

(b) Show that $f(x) = 0$ can be written as $x = \sqrt[3]{3x+5}$ (1)

(c) Using $x_{n+1} = \sqrt[3]{3x_n+5}$ with $x_1 = 2$, find the values of x_2 , x_3 and x_4 , giving your answers to 4 decimal places. (2)

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Question 1 continued

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(Total for Question 1 is 5 marks)



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2.

$$f(x) = e^x - x - 4$$

- (a) Show that the equation $f(x) = 0$ has a root α in the interval $[1, 2]$

(2)

- (b) By choosing a suitable interval, show that $\alpha = 1.749$ correct to 3 decimal places.

(3)

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Question 2 continued

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(Total for Question 2 is 5 marks)



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3.

$$f(x) = \frac{1}{x-2} + 3 \quad x \neq 2$$

(a) Show that $f(1.9)$ and $f(2.1)$ have opposite signs. (1)

(b) Explain why the equation $f(x) = 0$ does not have a root between 1.9 and 2.1 (2)

(c) Find the exact root of $f(x) = 0$ (2)

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Question 3 continued

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(Total for Question 3 is 5 marks)



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4. The equation $x^2 - 5x + 3 = 0$ has roots α and β , where $0 < \alpha < 1$ and $4 < \beta < 5$. The iteration formula

$$x_{n+1} = \frac{x_n^2 + 3}{5}$$

is used to try to find the roots.

- (a) Using $x_1 = 1$, find x_2 , x_3 and x_4 (2)
- (b) Using $x_1 = 4.5$, find x_2 and x_3 to 4 decimal places, and describe what happens. (2)
- (c) Suggest a rearrangement of the equation that could be used to find β by iteration. (1)



Question 4 continued

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5. The curves with equations $y = \ln x$ and $y = 3 - x$ meet at the point with x coordinate α .

(a) Show that α lies between 2 and 3

(3)

(b) Using $x_{n+1} = 3 - \ln x_n$ with $x_1 = 2$, find x_2 and x_3 to 4 decimal places.

(2)

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Question 5 continued

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(Total for Question 5 is 5 marks)



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6. The curve C has equation $y = e^x + x^2$

The tangent to C at the point P with x coordinate α passes through the origin.

(a) Find $\frac{dy}{dx}$

(1)

(b) Show that $x = \alpha$ is a solution of the equation $e^x(1 - x) - x^2 = 0$

(3)

(c) Show that α lies between 0.5 and 1

(2)

(d) Show that the equation in part (b) can be written as $x = 1 - x^2e^{-x}$

(1)

(e) Using $x_{n+1} = 1 - x_n^2e^{-x_n}$ with $x_1 = 0.7$, find x_2 and x_3 to 4 decimal places.

(2)

(f) Show that $\alpha = 0.739$ correct to 3 decimal places.

(2)

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 11 marks)



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7.

$$f(x) = x^2 \cos x \quad 0 < x < \frac{\pi}{2}$$

The curve $y = f(x)$ has a maximum point at $x = \beta$.

- (a) Find $f'(x)$. (2)
- (b) Show that β satisfies the equation $x = \arctan\left(\frac{2}{x}\right)$ (2)
- (c) Using $x_{n+1} = \arctan\left(\frac{2}{x_n}\right)$ with $x_1 = 1$, find x_2 , x_3 and x_4 to 4 decimal places. (3)
- (d) By considering the sign of $f'(x)$ on a suitable interval, show that $\beta = 1.077$ correct to 3 decimal places. (3)

Question 7 continued

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Question 7 continued

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- 8.** The concentration, C mg/l, of a drug in a patient's blood t hours after it is given is modelled by

$$C = 8(e^{-0.2t} - e^{-t}) \quad t \geq 0$$

- (a)** Use calculus to find the exact time at which the concentration is greatest.

(3)

The drug is effective while $C > 2$. The concentration first reaches 2 mg/l when $t = 0.394$

- (b)** Show that the later time T at which $C = 2$ satisfies $T = -5 \ln(0.25 + e^{-T})$

(2)

- (c) Using $T_{n+1} = -5 \ln(0.25 + e^{-T_n})$ with $T_1 = 6$, find T_2 and T_3 to 4 decimal places.

(2)

- (d)** Hence find, to the nearest minute, the length of time for which the drug is effective.

(3)

Question 8 continued

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9.

$$f(x) = \tan x - 3x \quad 0 < x < \pi, x \neq \frac{\pi}{2}$$

- (a) Show that the equation $f(x) = 0$ has a root between 1.2 and 1.4 (2)
- (b) Show that $f(1.5)$ and $f(1.6)$ have opposite signs, and explain why this does not show that there is a root between 1.5 and 1.6 (2)
- (c) Using $x_{n+1} = \arctan(3x_n)$ with $x_1 = 1.3$, find x_2 and x_3 to 4 decimal places. (3)
- (d) Show that the root in part (a) is 1.324 correct to 3 decimal places. (3)



Question 9 continued

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$$f(x) = e^x - 2 \quad x \in \mathbb{R}$$

(3)

- (3)**

(2)

(3)

Question 10 continued

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