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INTERNATIONAL A LEVEL MATHEMATICS • DECISION MATHEMATICS 1

Decision 1

Chapter 7: Linear Programming

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WDM11/01) • Calculator allowed • Answer book pages included

Multi-step exam questions at the difficulty of the real WDM11 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)		Qs	Marks	Page	My score
1	**Algorithms** Flow charts, bubble sort, quick sort, bin packing, binary search	10	80	separate file	
2	**Graphs and Networks** Vocabulary, types of graph, degrees, adjacency and distance matrices	10	80	separate file	
3	**Algorithms on Graphs** Kruskal, Prim (networks and matrices), Dijkstra	10	80	separate file	
4	**Route Inspection** Eulerian graphs, pairing odd vertices, start and finish points	10	80	separate file	
5	**The Travelling Salesman Problem** Classical and practical problems, upper and lower bounds	10	80	separate file	
6	**Critical Path Analysis** Activity networks, event times, floats, Gantt charts, scheduling	10	80	separate file	
7	**Linear Programming** Formulation, feasible regions, objective lines, integer solutions	10	80	2	

Full mark scheme Free courses More worksheets

Name ..... Class ..... Date .....

CHAPTER 7 • KEY SKILLS

7 Linear Programming

Specification D1: Linear programming • 10 questions • 80 marks • about 96 minutes

1 Decision variables

Define each variable precisely, with units: "x is the number of trays of muffins made each day".

2 Objective

Write "Maximise $P = \ldots$" or "Minimise $C = \ldots$" in full; do not rescale it.

3 Constraints

One inequality per resource or condition; percentages and ratios as inequalities; simplify to integer coefficients.

4 Fractions of a resource

If a machine makes 400 of A or 300 of B in a day: $\frac{x}{400} + \frac{y}{300} \leqslant 1$

5 Drawing

Draw each boundary line accurately, shade the unwanted side (or the region) and label the feasible region R

6 Objective line

Draw $ax + by = k$ for a convenient k; slide it parallel to reach the last point of R (first point for a minimum).

7 Vertex method

Find every vertex exactly by simultaneous equations and evaluate the objective at each one.

8 Integer solutions

Test integer points near the optimal vertex that lie inside R (for a minimum, on the feasible side of every line).

9 Changing the objective

A vertex stays optimal while the objective's gradient lies between the gradients of the two lines meeting there.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	6	6	6	6	10	10	10	10	10
1 Decision variables	●					●			●	
2 Objective	●					●			●	
3 Constraints	●					●	●		●	
4 Fractions of a resource						●				
5 Drawing		●		●				●	●	●
6 Objective line			●						●	●
7 Vertex method		●	●	●	●		●	●		●
8 Integer solutions					●		●		●	●
9 Changing the objective								●		●
Interleaved: straight lines and gradients (P1)		●	●	●			●	●	●	●
Interleaved: simultaneous equations (P1)		●	●	●	●		●	●	●	●
Interleaved: inequalities and percentages (P1, GCSE)	●					●	●		●	

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WDM11 paper, where one part leads into the next.

Answer book. After each question is a copy of the page the real D1 answer book prints for it: a labelled grid (Diagram 1) or the figure reprinted with the region R.

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1. A bakery makes trays of muffins and trays of scones. Each tray of muffins needs 20 minutes of oven time and 3 kg of flour; each tray of scones needs 15 minutes of oven time and 2 kg of flour. Each day the oven is available for 6 hours and 50 kg of flour is available. The number of trays of muffins must be no more than three times the number of trays of scones. The profit is £12 on a tray of muffins and £9 on a tray of scones. The bakery wishes to maximise its daily profit.

(a) Define the decision variables.

(1)

(b) Write down the objective.

(1)

(c) Write down the constraints, simplifying each one so that it has integer coefficients.

(4)

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Question 1 continued **D1 ANSWER BOOK**

In the exam this page is printed in the separate D1 answer book. Write your working here, on the copy provided.

Decision variables: _____

Objective: _____

Subject to: _____

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(Total for Question 1 is 6 marks)

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2. A linear programming problem has the constraints

$$x + 2y \leqslant 16, \qquad 3x + 2y \leqslant 24, \qquad x \leqslant 2y, \qquad x,\ y \geqslant 0$$

(a) Represent these constraints on Diagram 1 in the answer book. Make your shading clear and label the feasible region R.

(4)

(b) Write down the coordinates of the vertices of R.

(2)

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Question 2 continued **D1 ANSWER BOOK**

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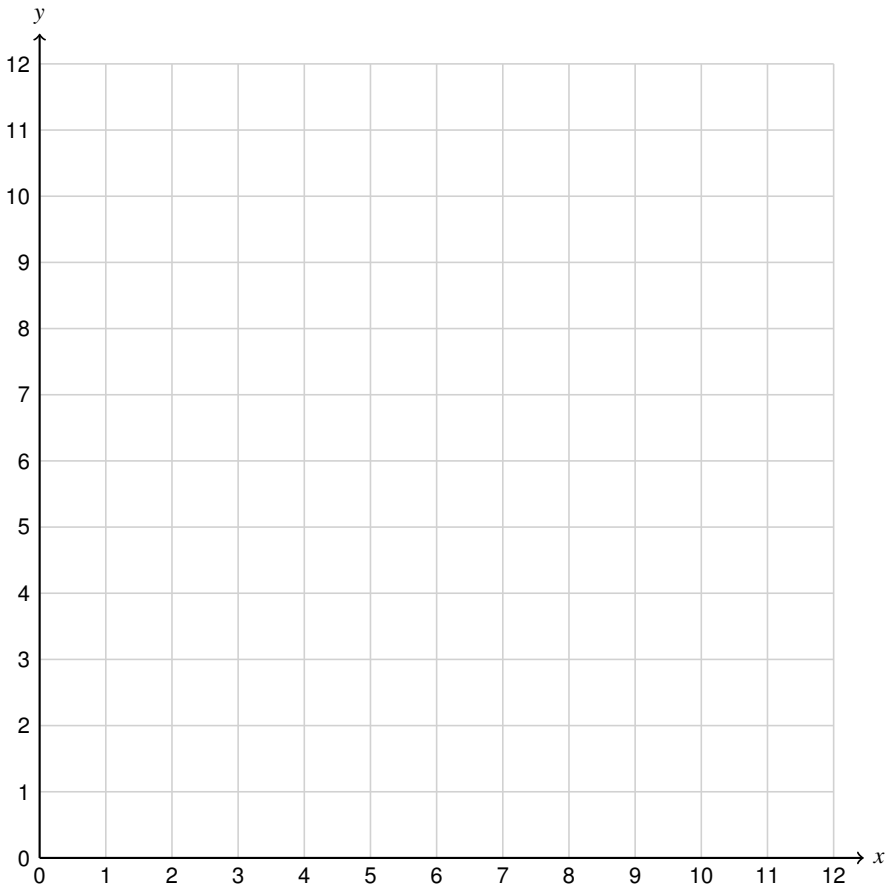

Diagram 1

(Total for Question 2 is 6 marks)

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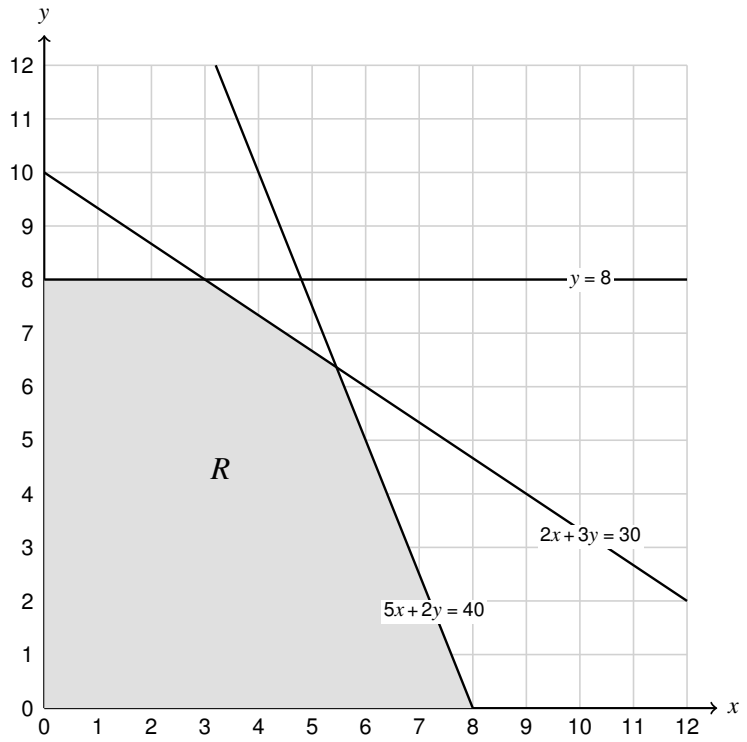

Figure 1

Figure 1 shows the feasible region R for a linear programming problem. The objective is to maximise $P = 3x + 4y$

(a) Use an objective line to identify the optimal vertex of R. Make your objective line clear.

(3)

(b) Find the exact coordinates of this vertex.

(2)

(c) Find the exact maximum value of P.

(1)

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Question 3 continued **D1 ANSWER BOOK**

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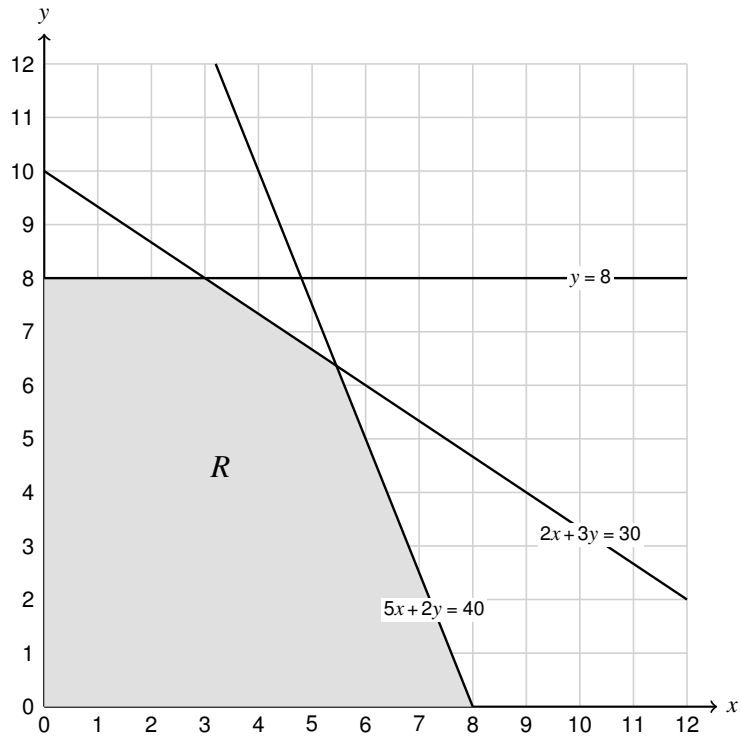

Diagram 2

(Total for Question 3 is 6 marks)

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4. Minimise $C = 5x + 2y$ subject to

$$x + y \geqslant 8, \qquad 3x + y \geqslant 12, \qquad x + 3y \geqslant 12, \qquad x,\ y \geqslant 0$$

(a) Represent the constraints on Diagram 1 in the answer book, labelling the feasible region R.

(2)

(b) Use the vertex method to find the minimum value of C and the values of x and y at which it occurs.

(4)

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Question 4 continued **D1 ANSWER BOOK**

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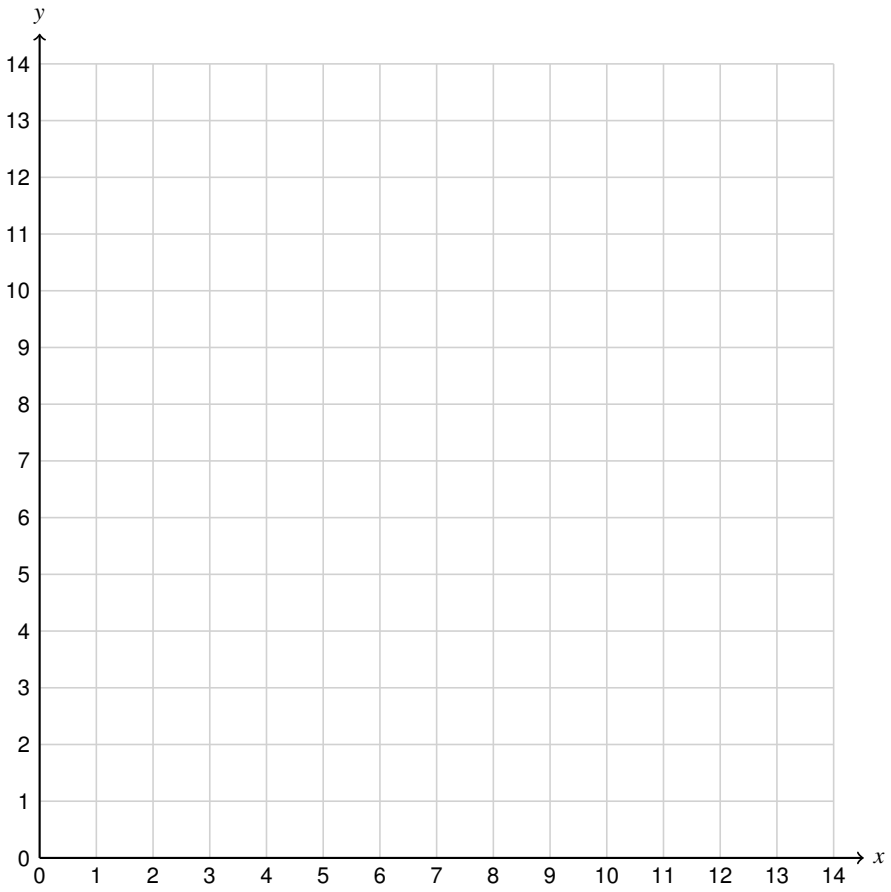

Diagram 3

(Total for Question 4 is 6 marks)

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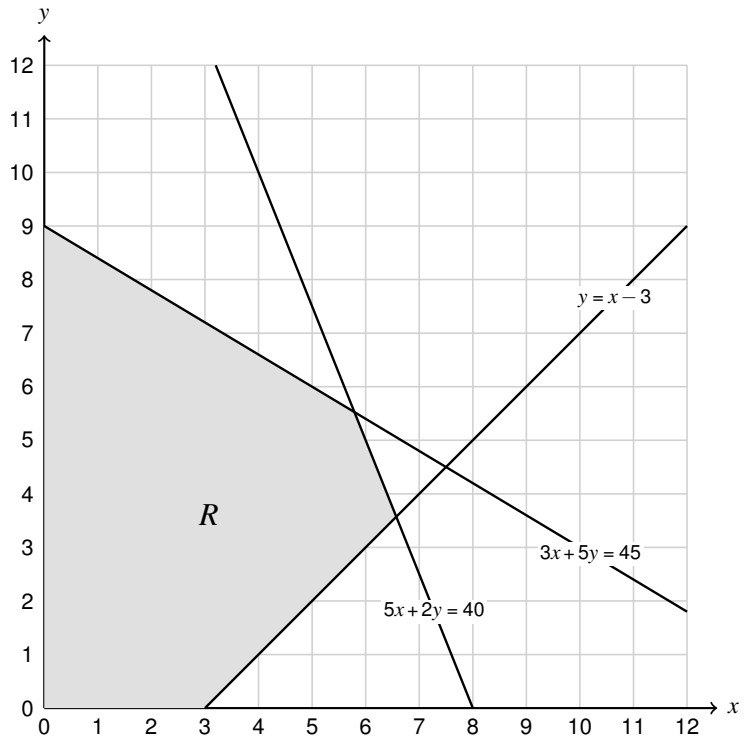

Figure 2

Figure 2 shows the feasible region R for the problem: maximise $P = 4x + 5y$ subject to $3x + 5y \leqslant 45$, $5x + 2y \leqslant 40$, $x - y \leqslant 3$, $x,\ y \geqslant 0$. The variables x and y must be integers.

(a) Find the exact coordinates of the vertex of R at which P is greatest if x and y need not be integers.

(2)

(b) Find the optimal integer solution, and the corresponding value of P. You must make your method clear.

(4)

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Question 5 continued **D1 ANSWER BOOK**

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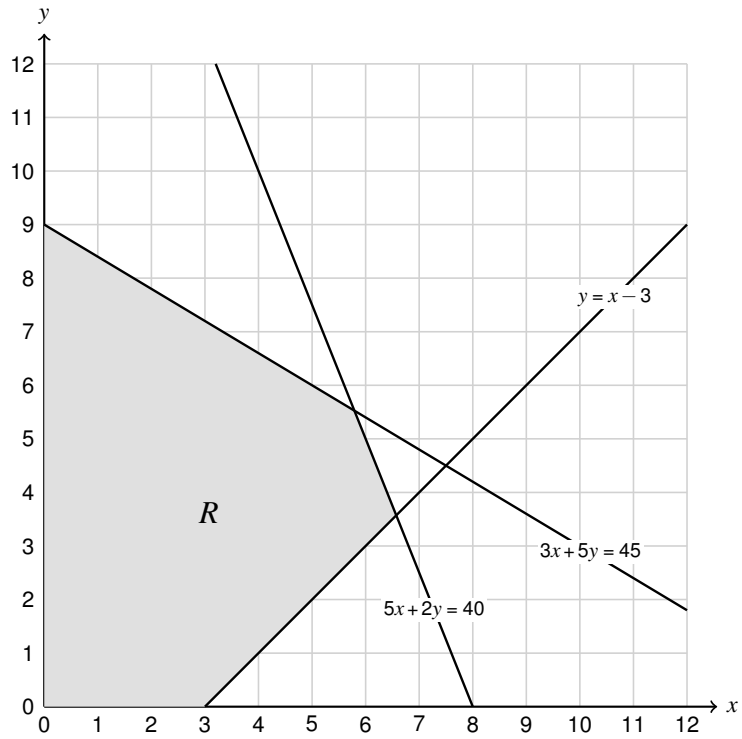

Diagram 4

(Total for Question 5 is 6 marks)

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6. A factory makes two drinks, A and B. In one day the bottling machine can fill either 400 litres of A or 300 litres of B, or any combination in proportion. At least 30% of the total output must be drink B, and at least 120 litres of A must be made each day. The profit is £0.80 per litre of A and £1.20 per litre of B. Let x and y be the numbers of litres of A and B made each day.

(a) Write down the objective.

(2)

(b) Show that the bottling machine gives the constraint $3x + 4y \leqslant 1200$

(3)

(c) Write down, simplifying where possible, two further constraints on x and y.

(3)

(d) Explain why the problem does not require integer solutions, and state one assumption of the model.

(2)

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Question 6 continued **D1 ANSWER BOOK**

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Objective: _____

Constraints: _____

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Question 6 continued

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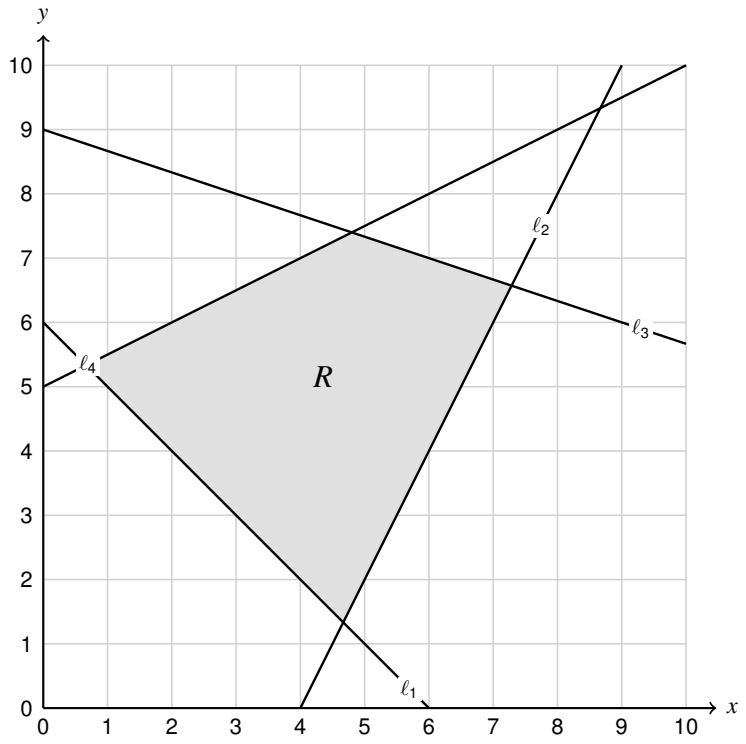

Figure 3

Figure 3 shows the lines ℓ_1: $x+y=6$, ℓ_2: $2x-y=8$, ℓ_3: $x+3y=27$ and ℓ_4: $2y-x=10$, and the feasible region R

(a) Write down the four inequalities that define R.

(2)

(b) The objective is to find the maximum and the minimum values of $P=2x+3y$ on R. Find each value exactly, and the vertex at which it occurs.

(5)

(c) Given that x and y must be integers, find the minimum value of P, and the point at which it occurs.

(3)

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Question 7 continued **D1 ANSWER BOOK**

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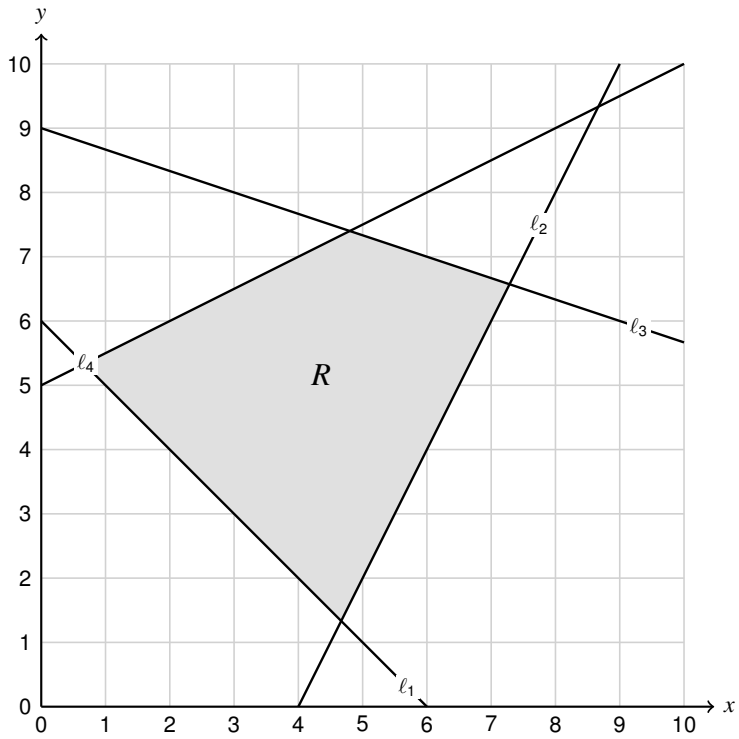

Diagram 5

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Question 7 continued

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8. A linear programming problem is: maximise $P = kx + y$, where k is a positive constant, subject to

$$x + 2y \leqslant 20, \qquad 3x + 2y \leqslant 36, \qquad y \leqslant 8, \qquad x \leqslant 10, \qquad x,\ y \geqslant 0$$

(a) Represent the constraints on Diagram 1 in the answer book, and label the feasible region R.

(3)

(b) Find the coordinates of the vertices of R.

(2)

(c) Given that the optimal point is the vertex $(8, 6)$, find the range of possible values of k.

(3)

(d) Given that $k = 1$, find the maximum value of P.

(2)

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Question 8 continued **D1 ANSWER BOOK**

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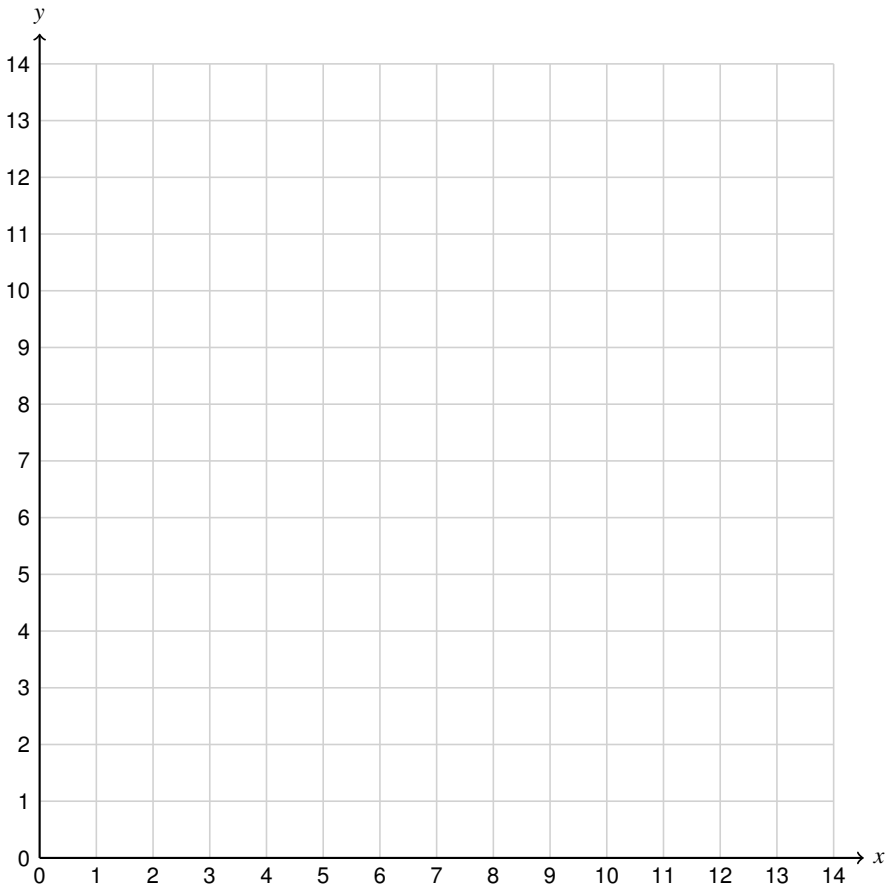

Diagram 6

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Question 8 continued

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(Total for Question 8 is 10 marks)

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9. A craftsperson makes chairs and tables. A chair takes 3 hours and uses 4 units of wood; a table takes 5 hours and uses 3 units of wood. Each week there are 45 hours and 48 units of wood available. At least twice as many chairs as tables must be made. The profit is £30 on a chair and £45 on a table.

(a) Formulate this as a linear programming problem, defining your variables.

(3)

(b) Represent the constraints on Diagram 1 in the answer book, and label the feasible region R.

(3)

(c) Using an objective line, find the numbers of chairs and tables that should be made each week to maximise the profit, and the maximum profit.

(4)

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Question 9 continued **D1 ANSWER BOOK**

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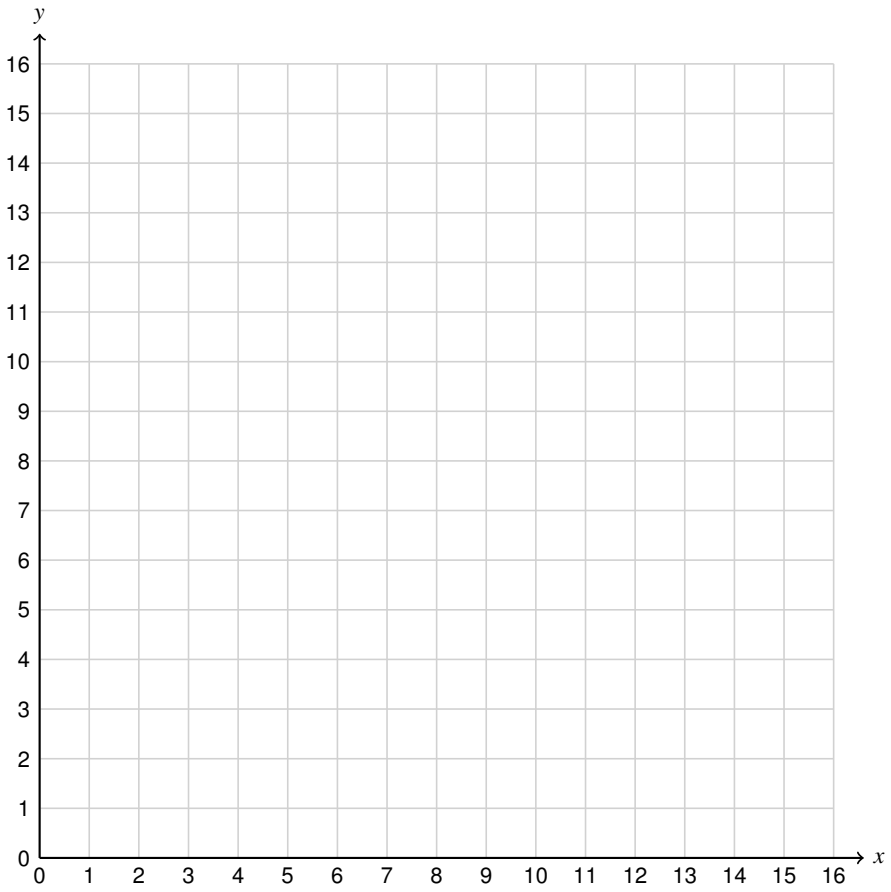

Diagram 7

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Question 9 continued

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10. An animal feed is made by mixing x kg of feed A and y kg of feed B. The mix must satisfy

$$6x + 2y \geqslant 25, \qquad 2x + 4y \geqslant 21, \qquad x + y \geqslant 8, \qquad x,\ y \geqslant 0$$

Feed A costs £5 per kg and feed B costs £4 per kg. The cost is to be minimised.

(a) Represent the constraints on Diagram 1 in the answer book, and label the feasible region R.

(3)

(b) Find the exact optimal point and the minimum cost.

(3)

(c) Feed is sold in whole kilograms only. Find the cheapest mix and its cost.

(2)

(d) The cost of feed A changes to £c per kg, while feed B still costs £4 per kg. Find the range of values of c for which your answer to part (b) is still optimal.

(2)

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Question 10 continued **D1 ANSWER BOOK**

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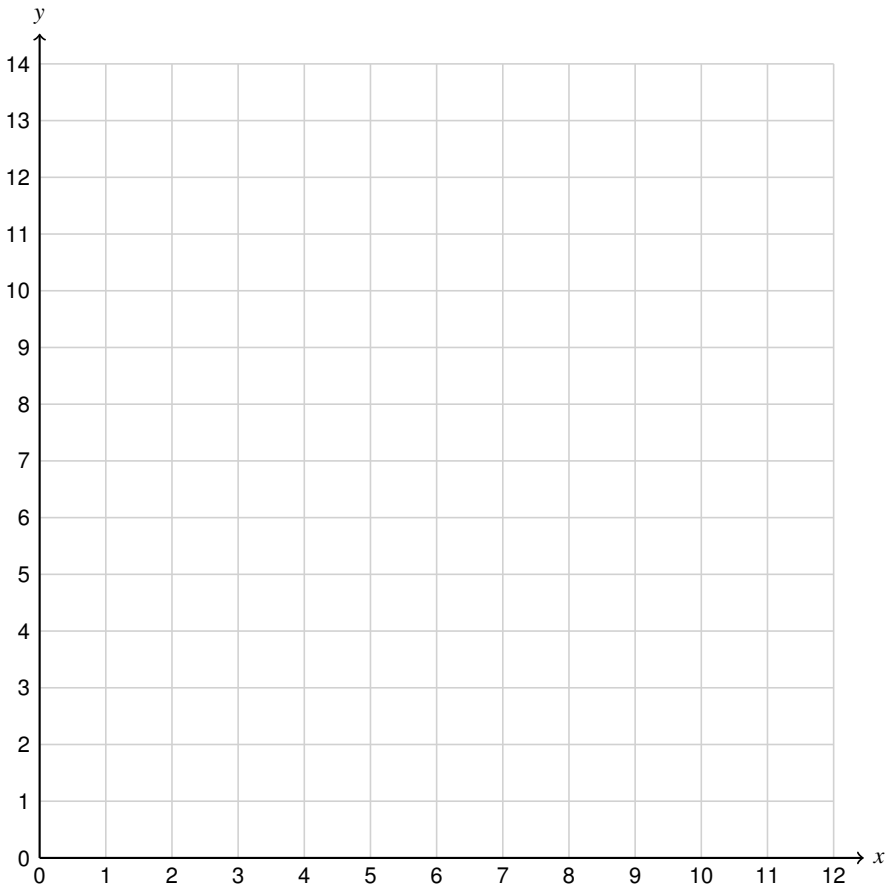

Diagram 8

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Question 10 continued

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TOTAL FOR CHAPTER 7 IS 80 MARKS

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