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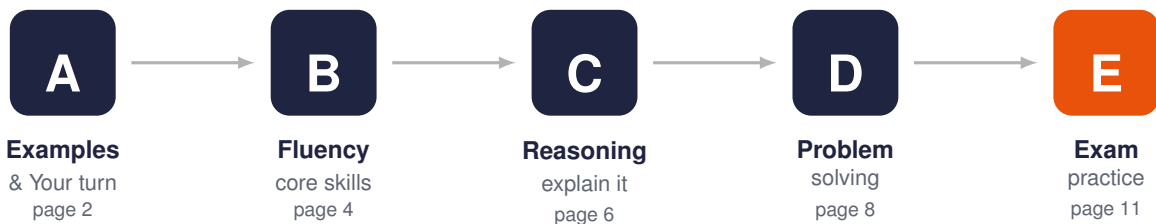
A-LEVEL MATHEMATICS • PURE 2

Sequences and Series

Worksheet

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Free Sequences and Series worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: arithmetic and geometric terms and sums • sums to infinity • recurrence relations and sigma notation • *Stretch:* least n , periodic sums, models and proofs.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Least n by logs and inequalities, periodic sums, models in context, and arithmetic and geometric together.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

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A**Worked examples & Your turn**

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

For the arithmetic sequence 5, 9, 13, ... find the 20th term and the sum of the first 20 terms.

$$a = 5, d = 4: u_n = a + (n-1)d \text{ and } S_n = \frac{n}{2}(2a + (n-1)d)$$

$$u_{20} = 5 + 19 \times 4, S_{20} = 10(10 + 76)$$

81, 860

Your turn A1

For the arithmetic sequence 40, 37, 34, ... find the 15th term and the sum of the first 15 terms.

Worked example A2

For the geometric sequence 3, 6, 12, ... find the 8th term and the sum of the first 8 terms.

$$a = 3, r = 2: u_n = ar^{n-1} \text{ and } S_n = \frac{a(r^n - 1)}{r - 1}$$

$$u_8 = 3 \times 2^7, S_8 = 3(2^8 - 1)$$

384, 765

Your turn A2

For the geometric sequence 64, -32, 16, ... find the 7th term and the sum of the first 7 terms.

Worked example A3

$u_1 = 2$ and $u_{n+1} = 2u_n + 3$. Find u_2 , u_3 and u_4

Substitute each term to get the next one.

$$u_2 = 2(2) + 3, u_3 = 2(7) + 3, u_4 = 2(17) + 3$$

7, 17, 37

Your turn A3

$u_1 = 5$ and $u_{n+1} = 10 - 2u_n$. Find u_2 , u_3 and u_4



STRETCH

Worked example A4: a sum to infinity

Find the sum to infinity of $24 + 18 + 13.5 + \dots$

$$r = \frac{18}{24} = 0.75, \text{ and } |r| < 1, \text{ so it exists: } S_{\infty} = \frac{a}{1-r}$$

$$\frac{24}{0.25} = 96$$

Your turn A4

Find the sum to infinity of $50 - 20 + 8 - \dots$

Worked example A5: sigma notation

Evaluate $\sum_{r=1}^{20} (4r - 1)$

An arithmetic series: first term 3, last term 79, 20 terms.

$$\frac{20}{2}(3 + 79)$$

$$820$$

Your turn A5

Evaluate $\sum_{r=5}^{30} (2r + 3)$

Worked example A6: the least n

Find the least n for which the sum of the first n terms of $2 + 6 + 18 + \dots$ exceeds 10^6

$$S_n = 3^n - 1 > 10^6, \text{ so } n > \frac{\log(10^6 + 1)}{\log 3} = 12.6\dots$$

n is a whole number, so round up.

$$n = 13$$

Your turn A6

Find the least n for which the sum of the first n terms of $5 + 10 + 20 + \dots$ exceeds 20 000

Worked example A7: two conditions

An arithmetic sequence has $u_4 = 17$ and $u_{10} = 41$. Find a and d

$$a + 3d = 17 \text{ and } a + 9d = 41$$

Subtract: $6d = 24$

$$a = 5, \quad d = 4$$

Your turn A7

An arithmetic sequence has $u_5 = 23$ and $S_8 = 196$. Find a and d



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method*

B1 For the arithmetic sequence 7, 11, 15, ...

- (a) Find the 10th term.
- (b) Find the 25th term.
- (c) Find which term is equal to 199
- (d) Find how many terms are less than 500

[4 marks]

B2 Find the sum of

- (a) $1 + 2 + 3 + \dots + 100$
- (b) $2 + 4 + 6 + \dots + 200$
- (c) $1 + 3 + 5 + \dots + 99$
- (d) the first 30 terms of $7 + 11 + 15 + \dots$

[4 marks]

B3 Find

- (a) the 6th term of 2, 6, 18, ...
- (b) the 10th term of 2, 6, 18, ...
- (c) the 7th term of 81, 27, 9, ...
- (d) the 9th term of 1, -2, 4, ...

[4 marks]

B4 Find the sum of the first

- (a) 10 terms of $1 + 2 + 4 + \dots$
- (b) 8 terms of $3 + 6 + 12 + \dots$
- (c) 6 terms of $81 + 27 + 9 + \dots$
- (d) 8 terms of $1 - 2 + 4 - \dots$

[4 marks]

CORE*continued: sums to infinity and recurrences***B5** Find the sum to infinity, if it exists, of the geometric series with

(a) $a = 10, r = \frac{1}{2}$

(b) $a = 12, r = -\frac{1}{3}$

(c) $a = 4, r = \frac{3}{2}$

[3 marks]**B6** $u_1 = 3$ and $u_{n+1} = 4 - u_n$ **(a)** Find u_2, u_3 and u_4 **(b)** State the order of this periodic sequence, giving a reason.

(c) Find $\sum_{n=1}^{20} u_n$

(d) Find $\sum_{n=1}^{21} u_n$

[4 marks]**STRETCH****B7** Evaluate

(a) $\sum_{r=1}^{10} 3r$

(b) $\sum_{r=1}^{10} (3r + 2)$

(c) $\sum_{r=11}^{20} (3r + 2)$

(d) $\sum_{r=1}^8 2^r$

[4 marks]**B8** Find the least value of n for which**(a)** the sum of the first n terms of $4 + 7 + 10 + \dots$ is greater than 1000**(b)** the n th term of $3, 3.6, 4.32, \dots$ is greater than 100**(c)** the n th term of $500, 450, 405, \dots$ is less than 10**(d)** the sum of the first n terms of $2 + 4 + 8 + \dots$ is greater than 10 000**[4 marks]****B9** Find the values asked for in each geometric sequence.**(a)** $u_2 = 12$ and $u_5 = 96$: find a and r **(b)** $u_3 = 18$ and $u_6 = -486$: find a and r **(c)** $S_\infty = 60$ and $a = 15$: find r **(d)** $S_\infty = 60$ and $r = -\frac{1}{4}$: find a **[4 marks]****B10** State whether each sequence is increasing, decreasing or periodic, giving a reason. For a periodic sequence, state its order.

(a) $u_n = 3n - 5$

(b) $u_n = 0.5^n$

(c) $u_1 = 2, u_{n+1} = -u_n$

(d) $u_1 = 1, u_{n+1} = -\frac{1}{u_n + 1}$

[4 marks]

C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE*Secure the method*

- C1** Sam writes the 10th term of $3, 6, 12, \dots$ as 3×2^{10}
Explain Sam's mistake, and find the 10th term. [2 marks]
- C2** Lily finds the sum to infinity of $2 + 3 + 4.5 + \dots$ as $\frac{2}{1 - 1.5} = -4$
Explain why Lily's answer is wrong. [2 marks]
- C3** Kai finds the sum of the first 20 terms of $5 + 8 + 11 + \dots$ using $S_n = \frac{n}{2}(a + (n - 1)d)$
Explain Kai's mistake, and find the correct sum. [3 marks]
- C4** For $2 + 6 + 10 + \dots$, $S_n = 2n^2$. Jo finds the least n with $S_n > 1000$ by solving $2n^2 = 1000$, getting $n = 22.4$, and gives $n = 22$
Explain Jo's mistake, and give the correct value of n [3 marks]



STRETCH

- C5** Max finds the sum to infinity of $64 - 32 + 16 - \dots$ as $\frac{64}{1 - \frac{1}{2}} = 128$

Explain Max's mistake, and find the correct sum to infinity.

[3 marks]

- C6** $u_1 = 2$ and $u_{n+1} = 5 - u_n$. Ali writes "the sequence is periodic of order 2" with no working.

Show that Ali is correct, giving a reason, and find $\sum_{n=1}^{51} u_n$

[3 marks]

- C7** A population of 2000 animals is modelled to grow by 3% each year. Ali says the population after 6 years is 2000×1.03^5

Explain Ali's mistake, and find the population after 6 years to the nearest whole number.

[3 marks]

- C8** (a) Prove that the sum of the first n terms of an arithmetic series with first term a and common difference d is

$$S_n = \frac{n}{2} (2a + (n-1)d)$$

(3)

(b) Hence prove that the sum of the first n odd numbers is n^2

(2)

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

- D1** A builder stacks bricks in rows, putting 3 bricks in the top row, 10 in the next, 17 in the next, and so on, so that the numbers of bricks form an arithmetic series S given by

$$S = 3 + 10 + 17 + 24 + \dots + 360$$

- (a) Find the number of terms in the series, showing your working clearly.

(1)

- (b) Find the sum of the series, giving your answer as a whole number.

(2)**[3 marks]**

- D2** *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

In an arithmetic series,

- the fourth term is 11
- the sum of the first twelve terms is 222

- (a) Find the value of the first term and the value of the common difference.

(2)

- (b) Hence find the smallest value of n for which the sum of the first n terms is greater than 1000, showing how you choose the integer value from your solution of an inequality.

(2)**[4 marks]**

- D3** A new car is bought for £24 000. Its value is modelled to fall by 12% each year, so that its values at the end of each year form a geometric sequence.

- (a) Find, according to the model, the value of the car 5 years after it was bought, to the nearest £100

(1)

- (b) Find the number of complete years after which the value of the car is first less than £5000, showing clearly how you use logarithms.

(2)**[3 marks]**

D4 A geometric series has common ratio 0.6 and sum to infinity 75. Give each answer exactly, as an integer or a terminating decimal.

(a) Find the first term of the series.

(1)

(b) Find the sum of the first 5 terms of the series.

(1)

(c) Find the difference between the sum to infinity and the sum of the first 5 terms, and explain why this difference is always positive for this series, whatever the number of terms added.

(1)

[3 marks]

STRETCH

D5 In this question you must show all stages of your working.

A sequence is defined by $u_1 = 5$, $u_{n+1} = \frac{12}{u_n}$ for $n \geq 1$

(a) Show that the sequence is periodic, and state its order, giving a reason.

(2)

(b) Hence find the value of $\sum_{n=1}^{40} (2n + u_n)$, showing clearly how you split the sum into two parts.

(2)

[4 marks]

D6 A ball is dropped from a height of 8 m onto horizontal ground. Each time it hits the ground it rebounds to $\frac{3}{4}$ of the height from which it fell, so that the heights reached form a geometric sequence.

(a) Find the height reached after the third bounce.

(1)

(b) Assuming the ball continues to bounce in this way indefinitely, find the total vertical distance it travels, explaining why each rebound height is counted twice.

(3)

[4 marks]



D7 In this question you must show all stages of your working.

A geometric series has first term a and common ratio r . The sum of its first two terms is 15 and its sum to infinity is 27

(a) Show that $9r^2 = 4$

(2)

(b) Hence find the two possible values of a , explaining why both series have a sum to infinity.

(2)

[4 marks]

D8 The first, third and ninth terms of an arithmetic sequence with first term a and common difference d , where a and d are non-zero, form the first three terms of a geometric sequence.

(a) Show that $d = a$

(3)

(b) Find the common ratio of the geometric sequence.

(1)

(c) Given that $a = 3$, find the sum of the first 10 terms of the geometric sequence, giving your answer as an exact integer.

(1)

[5 marks]





Exam practice

Exam-style questions in the style of WMA12. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

E1 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

Given that, in a particular geometric series,

- the sum of the first three terms is 78
- the sum to infinity is 81

find, for this series,

(a) the common ratio,

.....
(3)

(b) the first term,

.....
(1)

(c) the exact sum of the first six terms, giving your answer as a fraction in its simplest form.

.....
(1)

(Total for Question E1 is 5 marks)



E2 An athlete follows a training plan. She runs 2 km on day 1, and on each following day she runs 0.25 km more than on the day before, so that the distances she runs each day form an arithmetic sequence.

(a) Find the distance she runs on day 20

.....

(1)

(b) Find the total distance she runs over the first 20 days.

.....

(2)

She keeps to the plan until her total distance is at least 400 km.

(c) Show that the number of days N she needs satisfies $N^2 + 15N - 3200 \geq 0$, and hence find the least possible value of N .

.....

(3)

(Total for Question E2 is 6 marks)



STRETCH

E3 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

A geometric sequence has second term 48 and fifth term $\frac{3}{4}$

(a) Find the common ratio and the first term of the sequence.

.....
(3)

(b) Find the sum to infinity of the associated series, giving a reason why it exists.

.....
(1)

(c) Use algebra, with logarithms, to find the least value of n for which the n th term of the sequence is less than 0.001

.....
(3)

(d) Show that the sum of the first n terms can be written as $S_n = 256(1 - 4^{-n})$

(1)

(Total for Question E3 is 8 marks)

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