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GCSE Mathematics

Primes, Factors & Multiples

Worked Solutions

Pearson Edexcel GCSE & iGCSE Mathematics

Section A — Foundation — Worked Solutions

[Fluency] Question 1

Find all pairs that multiply to 24: 1×24, 2×12, 3×8, 4×6

∴ **1, 2, 3, 4, 6, 8, 12, 24**

[Fluency] Question 2

Multiply 7 by 1, 2, 3, 4, 5: 7, 14, 21, 28, 35

∴ **7, 14, 21, 28, 35**

[Fluency] Question 3

Test divisibility: 51 ÷ 3 = 17 (exact)

So 51 = 3 × 17 → 51 has factors other than 1 and itself.

∴ **51 is not prime. 51 = 3 × 17**

[Fluency] Question 4

36 ÷ 2 = 18 → 18 ÷ 2 = 9 → 9 ÷ 3 = 3 → 3 ÷ 3 = 1

∴ **$36 = 2^2 \times 3^2$**

[Fluency] Question 5

$18 = 2 \times 3^2$ $24 = 2^3 \times 3$

HCF: lowest power of each common factor = 2 × 3

∴ **HCF(18, 24) = 6**

[Fluency] Question 6

$4 = 2^2$ $6 = 2 \times 3$

LCM: highest power of each factor = $2^2 \times 3$

∴ **LCM(4, 6) = 12**

[Reasoning] Question 7

60 ÷ 2 = 30 → 30 ÷ 2 = 15 → 15 ÷ 3 = 5 → 5 ÷ 5 = 1

∴ **$60 = 2^2 \times 3 \times 5$**

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[Reasoning] Question 8

$12 = 2^2 \times 3$ $18 = 2 \times 3^2$

HCF = lowest powers: $2 \times 3 = 6$

LCM = highest powers: $2^2 \times 3^2 = 36$

$\therefore$ **HCF = 6 LCM = 36**

[Problem Solving] Question 9

$8 = 2^3$ $12 = 2^2 \times 3$ $20 = 2^2 \times 5$

LCM = $2^3 \times 3 \times 5 = 120$

$\therefore$ **All three buses leave together after 120 minutes.**

[Problem Solving] Question 10

Use the relationship: HCF × LCM = product of the two numbers

$6 \times 60 = 12 \times n \rightarrow 360 = 12n \rightarrow n = 30$

Check: HCF(12, 30) = 6 ✓ LCM(12, 30) = 60 ✓

$\therefore$ **The other number is 30.**

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Section B — Higher — Worked Solutions

[Fluency] Question 1

$180 \div 2 = 90 \rightarrow 90 \div 2 = 45 \rightarrow 45 \div 3 = 15 \rightarrow 15 \div 3 = 5 \rightarrow 5 \div 5 = 1$

$\therefore$ **$180 = 2^2 \times 3^2 \times 5$**

[Fluency] Question 2

$84 = 2^2 \times 3 \times 7$ $126 = 2 \times 3^2 \times 7$

HCF $= 2 \times 3 \times 7 = 42$

LCM $= 2^2 \times 3^2 \times 7 = 252$

$\therefore$ **HCF = 42 LCM = 252**

[Fluency] Question 3

$120 = 2^3 \times 3 \times 5$ $168 = 2^3 \times 3 \times 7$

Common prime factors: 2^3 and 3

HCF $= 2^3 \times 3 = 8 \times 3$

$\therefore$ **HCF(120, 168) = 24**

[Reasoning] Question 4

$a = 2^3 \times 3^2 = 72$ $b = 2^2 \times 3 \times 5 = 60$

(i) HCF $= 2^2 \times 3 =$ **12**

(ii) LCM $= 2^3 \times 3^2 \times 5 =$ **360**

(iii) $a \times b = 72 \times 60 = 4320$ HCF $\times$ LCM $= 12 \times 360 = 4320$ ✓

$\therefore$ **(i) HCF = 12 (ii) LCM = 360 (iii) 4320 = 4320 ✓**

[Reasoning] Question 5

n and n+1 are consecutive integers.

Consecutive integers always include one even number and one odd number.

An even number multiplied by any integer gives an even result.

$\therefore$ **n(n+1) is always even, because one of n or n+1 must be even.**

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[Reasoning] Question 6

$p = 2^3 \times 3 \times 5^2 = 600$ $q = 2^2 \times 3^3 = 108$

(a) HCF $= 2^2 \times 3 =$ **12**

(b) LCM $= 2^3 \times 3^3 \times 5^2 =$ **5400**

(c) $p \times q$ = HCF × LCM = 12 × 5400 = **64 800**

∴ **(a) 12 (b) 5400 (c) 64 800**

[Problem Solving] Question 7

$24 = 2^3 \times 3$ $36 = 2^2 \times 3^2$ $48 = 2^4 \times 3$

LCM $= 2^4 \times 3^2 = 144$ minutes

144 minutes = 2 hours 24 minutes after 9:00 am

∴ **All three alarms next sound together at 11:24 am.**

[Problem Solving] Question 8

HCF × LCM = m × n → 8 × 240 = 48 × n

n = 1920 ÷ 48 = 40

Check: HCF(48, 40) = 8 ✓ LCM(48, 40) = 240 ✓

Also check n = 8×3 = 24: HCF(48,24)=24 ✗ (not 8) so n=40 only.

∴ **The other number is 40.**

[Problem Solving] Question 9

Factorise: $n^2 - 1 = (n-1)(n+1)$

For any integer $n > 2$: both $(n-1) > 1$ and $(n+1) > 1$

So n^2-1 has factors $(n-1)$ and $(n+1)$ in addition to 1 and itself.

A prime number has exactly two factors, so n^2-1 cannot be prime.

∴ **$n^2-1 = (n-1)(n+1)$ always has at least 4 factors for $n > 2$, so is never prime.**

[Problem Solving] Question 10

$m = 2^a \times 3^b$ $n = 2^c \times 3^d$ with $a > c$ and $b < d$

(a) HCF $= 2^{\min(a,c)} \times 3^{\min(b,d)} = 2^c \times 3^b$

(b) LCM $= 2^{\max(a,c)} \times 3^{\max(b,d)} = 2^a \times 3^d$

(c) HCF × LCM $= 2^{c+a} \times 3^{b+d}$ and $m \times n = 2^{a+c} \times 3^{b+d}$

Since a+c = c+a and b+d = b+d, these are equal. ✓

∴ **(a) $2^c \times 3^b$ (b) $2^a \times 3^d$ (c) m × n = HCF × LCM ✓**

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