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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 3 (WFM03)

Further Pure 3 Practice Paper 2

Edexcel IAL Further Pure Mathematics 3

AIMING FOR A

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM03/01) • Calculator allowed

A full 75-mark paper, harder than a typical series, with one printed result and chapters combined, and a separate mark scheme

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 8 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	An identity and a hyperbolic equation	7	
2	An integral by a hyperbolic substitution	8	
3	A 3×3 transformation and the image of a line	9	
4	Two planes and a skew line	9	
5	Arc length and surface area	10	
6	A normal to a hyperbola and a locus	10	
7	An eigenvector with an unknown entry	11	
8	A reduction formula for $\cosh^n x$	11	
	Total	**75**	

Rough grade guide (out of 75): A* **63** • A **57** • B **50** • C **43**

Based on recent WFM03 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Starting from the definitions of $\cosh x$ and $\sinh x$ in terms of exponentials, express $\cosh 2x$ in terms of $\sinh x$

(3)

(b) Hence solve the equation

$$\cosh 2x - 3\sinh x = 3$$

giving your answers as exact logarithms.

(4)

(Total for Question 1 is 7 marks)

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2. Use a suitable substitution to find the exact value of

$$\int_0^4 \sqrt{x^2+9}\,\mathrm{d}x$$

giving your answer in the form $a+b\ln 3$, where a and b are rational numbers.

(8)

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(Total for Question 2 is 8 marks)

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3.

$$\mathbf{M} = \begin{pmatrix} k & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & k \end{pmatrix}$$

where k is a constant.

(a) Find $\det \mathbf{M}$ in terms of k, and explain why $\mathbf{M}$ is non-singular for every real value of k

(3)

A solid of volume 2 cm^3 is transformed by $\mathbf{M}$ into a solid of volume 38 cm^3

(b) Given that $k > 0$, find the value of k

(2)

Using this value of k, the line l with equation

$$\frac{x-1}{2} = \frac{y}{-1} = \frac{z+2}{1}$$

is transformed by $\mathbf{M}$ into the line l'

(c) Find a Cartesian equation of l'

(4)

(Total for Question 3 is 9 marks)

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4. The planes Π_1 and Π_2 have equations

$$\Pi_1 : x + y - z = 1 \qquad \Pi_2 : 2x - y + z = 5$$

(a) Find a vector equation of the line L in which Π_1 and Π_2 meet.

(5)

The line m has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mu(\mathbf{i} + 2\mathbf{k})$

(b) Find the shortest distance between L and m, giving your answer as an exact surd.

(4)

(Total for Question 4 is 9 marks)

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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has parametric equations

$$x = 2t^3, \qquad y = 3t^2, \qquad 0 \leqslant t \leqslant 1$$

(a) Find the exact length of C

(4)

(b) The curve C is rotated through 2π radians about the x-axis. Find the exact area of the surface generated.

(6)

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Question 5 continued

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(Total for Question 5 is 10 marks)

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6.

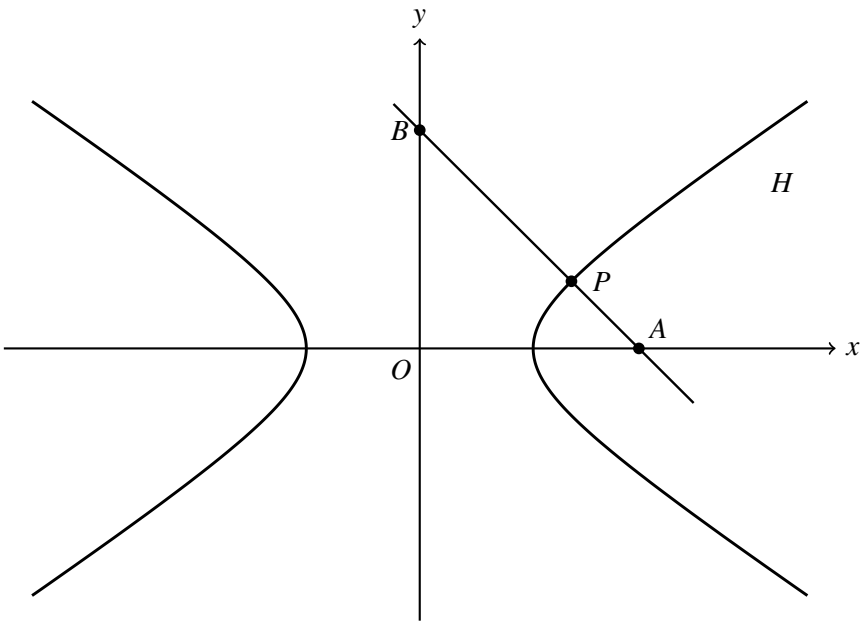

Figure 1

Figure 1 shows the hyperbola H with equation $\frac{x^2}{9} - \frac{y^2}{4} = 1$. The point $P(3\cosh t, 2\sinh t)$, $t > 0$, lies on H. The normal to H at P meets the x-axis at A and the y-axis at B

(a) Find an equation of the normal to H at P, giving your answer in the form $px\sinh t + qy\cosh t = r\sinh t\cosh t$, where p, q and r are integers.

(5)

(b) Find the coordinates of A and B

(2)

(c) Find a Cartesian equation of the locus of the midpoint of AB as t varies, stating any restrictions on x and y

(3)

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Question 6 continued

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(Total for Question 6 is 10 marks)

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7.

$$\mathbf{A} = \begin{pmatrix} 3 & a & 1 \\ a & 2 & a \\ 1 & a & 3 \end{pmatrix}$$

where a is a constant. Given that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$

(a) find the value of a and the corresponding eigenvalue

(3)

(b) find the other two eigenvalues of $\mathbf{A}$

(4)

(c) find an orthogonal matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^{\mathrm{T}}\mathbf{A}\mathbf{P} = \mathbf{D}$

(4)

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Question 7 continued

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(Total for Question 7 is 11 marks)

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8.

$$I_n = \int_0^{\ln 2} \cosh^n x \, \mathrm{d}x, \qquad n \geqslant 0$$

(a) Show that, for $n \geqslant 2$,

$$nI_n = \frac{3}{4}\left(\frac{5}{4}\right)^{n-1} + (n-1)I_{n-2}$$

(6)

(b) Hence find the exact value of I_4

(5)

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Question 8 continued

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(Total for Question 8 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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