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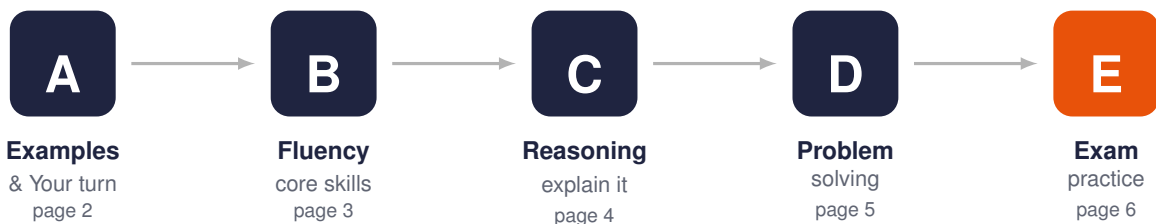
A-LEVEL MATHEMATICS • PURE 2

Coordinate Geometry

Worked Solutions

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Coordinate Geometry worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 2 Coordinate Geometry worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

Exam questions by topic

Coordinate Geometry revision engine

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

A1 $(x+4)^2 - 16 + (y-1)^2 - 1 + 1 = 0$, so $(x+4)^2 + (y-1)^2 = 16$: centre $(-4, 1)$, radius 4

A2 $r^2 = 5^2 + 12^2 = 169$: $(x+2)^2 + (y-3)^2 = 169$

A3 Radius gradient $\frac{3-1}{2+2} = \frac{1}{2}$, so the tangent has gradient -2 : $y-3 = -2(x-2)$: $2x+y-7=0$

STRETCH

A4 Centre $(-1, -1)$; $r^2 = (-4+1)^2 + (1+1)^2 = 13$: $(x+1)^2 + (y+1)^2 = 13$

A5 $x^2 + (x+2)^2 = 10$, so $2x^2 + 4x - 6 = 0$, $x^2 + 2x - 3 = 0$, $x = 1$ or -3 ; $y = x + 2$: $(1, 3)$, $(-3, -1)$

A6 $5x^2 + 4kx + k^2 - 5 = 0$; $16k^2 - 20(k^2 - 5) = 0$, so $4k^2 = 100$: $k = \pm 5$

A7 Half-chord 3 , $r = \sqrt{13}$: distance $\sqrt{13-9} = 2$ below the centre: $y = -3$



B

Fluency solutions

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

- B1** (a) $(3, -1)$, 4 (b) $(-2, 0)$, $\sqrt{7}$ (c) $x^2 + (y - 5)^2 = 25$: $(0, 5)$, 5 (d) $(x + 2)^2 + (y - 3)^2 = 16$: $(-2, 3)$, 4

The signs in the brackets are the opposite of the centre's coordinates.

- B2** (a) $x^2 + y^2 = 36$ (b) $(x - 2)^2 + (y + 3)^2 = 16$ (c) $r^2 = 9 + 16$: $(x - 2)^2 + (y + 3)^2 = 25$ (d) centre $(2, 1)$: $(x - 2)^2 + (y - 1)^2 = 25$

Every equation needs a centre and r^2 ; a point or a diameter gives r

- B3** Compare $(x - 2)^2 + (y - 1)^2$ with 25: (a) 25: on (b) 8: inside (c) 25: on (d) 41: outside

- B4** (a) radius gradient $\frac{4}{3}$, tangent $-\frac{3}{4}$: $3x + 4y = 25$ (b) $3x - 4y + 25 = 0$ (c) radius horizontal: $x = 5$
(d) radius vertical: $y = -5$

At the ends of the axes the tangent is vertical or horizontal.

- B5** (a) $x^2 = 16$: $(4, 2)$, $(-4, 2)$ (b) $x^2 + 2x - 8 = 0$, and $y = x + 2$: $(2, 4)$, $(-4, -2)$ (c) $x^2 + 8x + 16 = 0$, and $y = 2x + 10$: $(-4, 2)$, a tangent

A repeated root means the line touches the circle.

- B6** (a) $2x^2 = 9$: 2 (b) $2x^2 + 12x + 27 = 0$, $144 - 216 < 0$: 0 (c) $x^2 = 0$: 1 (d) $2x^2 + 6\sqrt{2}x + 9 = 0$, $72 - 72 = 0$: 1

The discriminant counts the points: positive 2, zero 1, negative 0.

STRETCH

- B7** (a) $x = 4$ (b) $y = 3$ (c) $(4, 3)$ (d) $r = \sqrt{16 + 9} = 5$; angle $BAC = 90^\circ$ (the axes), so *BC is a diameter*

Two perpendicular bisectors of chords meet at the centre.

- B8** (a) $k = 5$ or $k = -1$ (b) $k = 4$ or $k = -2$ (c) $2x^2 + 2kx + k^2 - 18 = 0$, $4k^2 - 8(k^2 - 18) = 0$: $k = \pm 6$
(d) $10x^2 + 6kx + k^2 - 10 = 0$, $36k^2 - 40(k^2 - 10) = 0$: $k = \pm 10$

Horizontal and vertical tangents are one radius from the centre; sloping ones need the discriminant.



- B9** (a) $9 + 16 = 25$, so O is on C . $y = 0$: $(x - 3)^2 = 9$, $x = 0$ or 6 : **6** (b) $x = 0$: $(y - 4)^2 = 16$, $y = 0$ or 8 : **8**
 (c) **4**, the y coordinate of the centre (d) $A(6, 0)$, $B(0, 8)$: angle $AOB = 90^\circ$, so **AB is a diameter** ($AB = 10 = 2r$)

A right angle at a point on the circle stands on a diameter.

- B10** (a) $m_{AC} = \frac{1}{2}$, $m_{BC} = -2$, product -1 : **angle $ACB = 90^\circ$** (b) $\frac{1}{2} \times 10 \times 4 =$ **20**
 (c) QR is a diameter: centre is its midpoint, **$(4, 5)$** (d) $r^2 = 9 + 16$: **$(x - 4)^2 + (y - 5)^2 = 25$**

The hypotenuse of a right-angled triangle is a diameter of its circle.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

- C1** $(x + 3)^2 + (y - 4)^2 = 25$: the signs change, so the centre is **$(-3, 4)$** , radius **5**

- C2** The right-hand side is r^2 , not r : $r = \sqrt{20} =$ **$2\sqrt{5}$**

- C3** Completing the square adds 9 and 1 to the left-hand side, so they must be added to the right too:
 $(x - 3)^2 + (y + 1)^2 = 15 + 9 + 1 = 25$: **$r = 5$**

- C4** The tangent is perpendicular to the radius, so its gradient is $-\frac{3}{4}$, not $\frac{4}{3}$
 $y - 2 = -\frac{3}{4}(x - 4)$, so $4y - 8 = -3x + 12$: **$3x + 4y - 20 = 0$**

STRETCH

- C5** Centres $(0, 0)$ and $(6, 8)$, radii 2 and 8. Distance between centres $\sqrt{36 + 64} = 10$
 $10 = 2 + 8$, the sum of the radii, so **the circles touch (externally)**
- C6** $(2 + 1)^2 + (7 - 3)^2 = 9 + 16 = 25$, so A is on the circle. The centre $(-1, 3)$ is the midpoint of the diameter:
 other end $= (2 \times (-1) - 2, 2 \times 3 - 7) =$ **$(-4, -1)$**



C7 $x^2 + (7 - x)^2 = 25$, so $2x^2 - 14x + 24 = 0$, $x^2 - 7x + 12 = 0$, $x = 3$ or 4

two points, (3, 4) and (4, 3): the line is a chord, not a tangent

C8 (a) At (0, 0): $0 + 0 - 0 - 0 = 0$

as required

(b) It touches the x -axis, so $r = b$: $(x - a)^2 + (y - b)^2 = b^2$, so $x^2 - 2ax + a^2 + y^2 - 2by = 0$

as required

(c) (0, 1): $1 - 2b + a^2 = 0$; (2, 1): $5 - 4a - 2b + a^2 = 0$. Subtracting: $4 - 4a = 0$, so $a = 1$, then $b = 1$:

$(x - 1)^2 + (y - 1)^2 = 1$

D

Problem solving solutions

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 (a) $(x - 2)^2 + (y + 3)^2 = 12 + 4 + 9 = 25$: centre **(2, -3)**, radius **5**

(b) $(7 - 2)^2 + (1 + 3)^2 = 41 > 25$, so the distance from the centre is greater than 5: **outside**

D2 (a) $(x - 3)^2 + (y + 2)^2 = 3 + 9 + 4 = 16$: **(3, -2)**, **$r = 4$**

(b) $y = 0$: $(x - 3)^2 + 4 = 16$, $(x - 3)^2 = 12$: **$x = 3 \pm 2\sqrt{3}$**

D3 (a) Centre (1, 7), $r^2 = (5 - 1)^2 + (10 - 7)^2 = 25$: **$(x - 1)^2 + (y - 7)^2 = 25$**

(b) Straight below the centre, 5 down: **(1, 2)**

D4 (a) Half-chord 4 and radius 5, so the chord is $\sqrt{25 - 16} = 3$ below the centre: **$y = -5$**

(b) $(x - 1)^2 + 9 = 25$, $x = -3$ or 5 : **$M(-3, -5)$** , **$N(5, -5)$**

STRETCH

D5 (a) Touching both axes, the centre is r from each axis: (r, r) . Through (2, 1): $(2 - r)^2 + (1 - r)^2 = r^2$, so $r^2 - 6r + 5 = 0$

(b) $(r - 1)(r - 5) = 0$: **$(x - 1)^2 + (y - 1)^2 = 1$** and **$(x - 5)^2 + (y - 5)^2 = 25$**

D6 (a) Midpoint (4, 6); $m_{AB} = -\frac{1}{3}$, so the bisector has gradient 3: $y - 6 = 3(x - 4)$: **$y = 3x - 6$**

(b) $3x - 6 = 4 - \frac{1}{3}x$, so $x = 3$, $y = 3$: centre **(3, 3)**; $r^2 = (1 - 3)^2 + (7 - 3)^2 = 20$: **$(x - 3)^2 + (y - 3)^2 = 20$**



D7 (a) Midpoint of AB is $(2, 4) = P$, so AB is a diameter; $m_{AC} = -\frac{1}{7}$, $m_{BC} = 7$, product -1 : angle $ACB = 90^\circ$

(b) $AC = \sqrt{50}$, $BC = \sqrt{50}$: area $= \frac{1}{2} \times \sqrt{50} \times \sqrt{50} = 25$

D8 (a) $(x-1)^2 + (y-2)^2 = 20 + 1 + 4 = 25$: $(1, 2)$, $r = 5$

(b) Distance between centres $\sqrt{8^2 + 6^2} = 10$. Touching externally: $5 + r = 10$, so $r = 5$

Touching internally (C_1 inside C_2): $r - 5 = 10$, so $r = 15$

E

Exam practice solutions

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $r^2 = (7-3)^2 + (2+1)^2 = 25$

M1

$$(x-3)^2 + (y+1)^2 = 25$$

A1

(b) Radius gradient $\frac{2+1}{7-3} = \frac{3}{4}$, so the tangent has gradient $-\frac{4}{3}$

M1

$$y-2 = -\frac{4}{3}(x-7)$$

dM1

$$4x+3y-34=0$$

A1

E2 (a) $(x+3)^2 + (y-2)^2 = 12+9+4 = 25$: (i) $(-3, 2)$ (ii) 5

B1 B1

(b) $(x+3)^2 + (x-2)^2 = 25$, so $2x^2 + 2x - 12 = 0$

M1: substitutes $y = x$

$$x^2 + x - 6 = 0, x = 2 \text{ or } -3$$

A1

$$A(2, 2), B(-3, -3)$$

A1

(c) $AB = \sqrt{25+25} = 5\sqrt{2}$

B1



STRETCH

- E3** (a) $(x-2)^2 + (y-1)^2 = 10$: centre $(2, 1)$, $r = \sqrt{10}$; $(6-2)^2 + (3-1)^2 = 20 > 10$, so T is outside B1 B1
- (b) Tangent: $y = m(x-6) + 3$. Substitute: $(x-2)^2 + (m(x-6)+2)^2 = 10$ M1
 $(1+m^2)x^2 - (12m^2 - 4m + 4)x + 36m^2 - 24m - 2 = 0$ A1
 One root: $b^2 - 4ac = -8(m-3)(3m+1) = 0$, so $3m^2 - 8m - 3 = 0$ as required A1*
- (c) $(3m+1)(m-3) = 0$, $m = 3$ or $-\frac{1}{3}$ M1
 $3 \times \left(-\frac{1}{3}\right) = -1$: the tangents are perpendicular A1
- (d) Each radius meets its tangent at 90° and the tangents meet at 90° ; the tangent length is $\sqrt{20-10} = \sqrt{10} = r$: a square of side $\sqrt{10}$, area 10 B1

