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INTERNATIONAL A LEVEL MATHEMATICS • PURE 2

Pure Mathematics 2

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA12/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA12 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Algebraic Methods Algebraic fractions, the factor and remainder theorems, proof	10	66	2	
2 Coordinate Geometry in the (x, y) Plane Equations of circles, tangents, chords and intersections	10	68	23	
3 Exponentials and Logarithms Laws of logarithms, exponential equations, modelling	10	68	45	
4 The Binomial Expansion Expanding $(a + bx)^n$, coefficients and approximations	10	65	67	
5 Sequences and Series Arithmetic and geometric series, sigma notation, recurrence	10	69	87	
6 Trigonometric Identities and Equations $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\sin^2 \theta + \cos^2 \theta = 1$, solving equations	10	66	111	
7 Differentiation Increasing and decreasing functions, stationary points, optimisation	10	67	131	
8 Integration Definite integrals, areas under curves, the trapezium rule	10	67	152	

[Full mark scheme](#)

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CHAPTER 1 • KEY SKILLS

1

Algebraic Methods

Specification P2: Proof, Algebra • 10 questions • 66 marks • about 80 minutes

1 Solving polynomial equations

Once $f(x)$ is factorised, set each factor equal to zero. Take out a common factor; never divide it away, or you lose a root.

2 Dividing polynomials

Divide by $(x \pm p)$ by long division or by inspection: $f(x) = (x - p)q(x) + r$.

3 The factor theorem

$f(a) = 0 \Leftrightarrow (x - a)$ is a factor.

For $(2x - 3)$ use $f\left(\frac{3}{2}\right) = 0$, and say so.

4 The remainder theorem

The remainder when $f(x)$ is divided by $(ax - b)$ is $f\left(\frac{b}{a}\right)$.

5 Unknown coefficients

Each factor or remainder gives one equation. Solve two equations simultaneously.

6 Factorising completely

Divide out the linear factor, then factorise the quadratic. If $b^2 - 4ac < 0$ it does not factorise.

7 Proof by deduction

Start from true facts, e.g. $(x - y)^2 \geq 0$, and give a reason for every step. State signs before dividing.

8 Proof by exhaustion

Check every possible case, then write a conclusion.

9 Disproof by counter-example

One example where the conditions hold but the statement is false. Show the working.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	4	5	4	5	8	8	8	9	10
1 Solving polynomial equations							•			•
2 Dividing polynomials		•				•	•		•	
3 The factor theorem	•		•			•	•		•	•
4 The remainder theorem				•		•				•
5 Unknown coefficients	•		•	•		•				•
6 Factorising completely	•		•			•	•		•	•
7 Proof by deduction								•		
8 Proof by exhaustion					•					
9 Disproof by counter-example					•			•		
Interleaved: quadratics and the discriminant (P1)						•	•			•
Interleaved: sketching cubics, inequalities (P1)									•	
Interleaved: differentiation, surds (P1)								•		•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA12 paper, where one part leads into the next and skills from different topics are combined.

Must learn. A factor theorem answer needs “ $f(a) = 0$, so $(x - a)$ is a factor”. “Factorise completely” means write the full product. A proof needs a concluding statement.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = x^3 + ax^2 + bx + 6$$

where a and b are constants. Given that $(x - 3)$ and $(x + 2)$ are factors of $f(x)$,

(a) find the value of a and the value of b ,

(4)

(b) hence write $f(x)$ as a product of three linear factors.

(1)

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Question 1 continued

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(Total for Question 1 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

2. Find the quotient and the remainder when $2x^3 - 3x^2 - 11x + 7$ is divided by $(x - 3)$

(4)

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(Total for Question 2 is 4 marks)



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3. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

$$f(x) = 2x^3 + 3x^2 + ax - 6$$

where a is a constant. Given that $(2x + 1)$ is a factor of $f(x)$,

(a) use the factor theorem to show that $a = -11$ (2)

(b) Hence factorise $f(x)$ completely. (3)

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Question 3 continued

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4.

$$f(x) = 2x^3 - x^2 + kx + 4$$

where k is a constant. When $f(x)$ is divided by $(x - 2)$ the remainder is 10

(a) Find the value of k .

(2)

(b) Find the remainder when $f(x)$ is divided by $(2x + 1)$

(2)

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5. (i) A student writes the statement

“ $n^2 + n + 11$ is a prime number for all positive integers n ”

Prove, by counter-example, that this statement is not true.

(2)

- (ii) Given that n is an integer with $1 \leq n \leq 5$, prove by exhaustion that $n^3 - n$ is a multiple of 6

(3)

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Question 5 continued

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(Total for Question 5 is 5 marks)



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6. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

$$f(x) = (x + 1)(2x^2 - x + k) - 9$$

where k is a constant.

- (a) Write down the remainder when $f(x)$ is divided by $(x + 1)$ (1)

Given that $(2x - 1)$ is a factor of $f(x)$,

- (b) show that $k = 6$ (2)

- (c) (i) Write $f(x)$ in the form $(2x - 1)g(x)$, where $g(x)$ is a quadratic expression. (3)

- (ii) Hence show that the equation $f(x) = 0$ has only one real root. (2)



Question 6 continued

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(Total for Question 6 is 8 marks)



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7. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

$$f(x) = x^3 + 2x^2 - 5x - 6$$

- (a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$.

(2)

- (b) Factorise $f(x)$ completely.

(2)

- (c) Using your answer to part (b), solve the equation

$$f(x) = x + 3$$

giving your answers in exact form.

(4)

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Question 7 continued

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Question 8 continued

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9. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

$$f(x) = x^3 - 3x^2 - 4x + 12$$

- (a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$. (2)
- (b) Factorise $f(x)$ completely. (2)
- (c) Sketch the curve with equation $y = f(x)$, showing clearly the coordinates of the points where the curve meets the coordinate axes. (3)
- (d) Using your answers to parts (b) and (c), find the set of values of x for which $f(x) \geq 0$ (2)



Question 9 continued

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Question 9 continued

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(Total for Question 9 is 9 marks)



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Solutions relying on calculator technology are not acceptable.

where p and q are constants. The curve with equation $y = f(x)$ touches the x -axis at the point $(2, 0)$.

- (c)** Write $f(x)$ as a product of linear factors. **(1)**

When $f(x)$ is divided by $(x - k)$, where k is a non-zero constant, the remainder is 12

- (d)** Find the exact possible values of k . **(3)**



Question 10 continued

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 1 IS 66 MARKS

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CHAPTER 2 • KEY SKILLS

2

Coordinate Geometry in the (x, y) Plane

Specification P2: Circles • 10 questions • 68 marks • about 80 minutes

1 Equation of a circleCentre (a, b) , radius r :

$$(x - a)^2 + (y - b)^2 = r^2$$

2 Centre and radiusComplete the square in x and in y .e.g. $x^2 + y^2 - 4x + 2y = 4$ is

$$(x - 2)^2 + (y + 1)^2 = 9: \text{centre } (2, -1),$$

$$r = 3$$

3 Midpoints and distancesThe centre is the midpoint of a diameter. r is the distance from the centre to any point on the circle.**4 Tangents**

A tangent is perpendicular to the radius at the point of contact: gradient

$$= -\frac{1}{m_{\text{radius}}}$$

5 Chords

The perpendicular bisector of any chord passes through the centre.

6 Angle in a semicircleIf angle $ACB = 90^\circ$, then AB is a diameter of the circle through A , B and C .**7 Lines meeting circles**

Substitute the line into the circle.

 $b^2 - 4ac > 0$: two points; $= 0$: tangent; < 0 : no points.**8 Two circles**Compare the distance d between the centres with $r_1 + r_2$ (and $|r_1 - r_2|$).

Draw a sketch.

9 Tangents from a pointTangent length² = (distance to centre)² - r^2 , by Pythagoras.**Skills map** every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	4	5	6	5	5	8	8	8	9	10
1 Equation of a circle	•		•		•			•		
2 Centre and radius		•				•			•	
3 Midpoints and distances	•		•	•	•	•		•		•
4 Tangents		•	•				•			•
5 Chords					•				•	
6 Angle in a semicircle								•		
7 Lines meeting circles				•			•		•	
8 Two circles						•				
9 Tangents from a point										•
Interleaved: straight lines and perpendicular gradients (P1)			•		•			•	•	
Interleaved: quadratics and the discriminant (P1)				•			•		•	
Interleaved: radians: sectors and segments (P1)									•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA12 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Watch the signs: $(x + 2)^2$ means the centre has $x = -2$. The radius is $\sqrt{r^2}$. When comparing distances, write the values and a conclusion.



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1. The points $A(-1, 4)$ and $B(5, -4)$ are the ends of a diameter of the circle C .
Find an equation for C .

(4)

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(Total for Question 1 is 4 marks)



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2. The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

- (a) Find the coordinates of the centre of C and the radius of C .

(3)

- (b) Show that C touches the x -axis, and state the coordinates of the point where it does so.

(2)

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Question 2 continued

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3. A circle C has centre $(-1, 3)$. The point $P(3, 0)$ lies on C .

(a) (i) Find the radius of C .

(1)

(ii) Find an equation for C .

(2)

(b) Find an equation of the tangent to C at P , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(3)

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Question 3 continued

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(Total for Question 3 is 6 marks)



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4. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**The circle C has equation $x^2 + y^2 = 25$ and the line l has equation $y = x + 1$ The line l meets C at the points A and B .**(a)** Using algebra, find the coordinates of A and B .**(4)****(b)** Find the exact length of the chord AB .**(1)**

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Question 4 continued

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(Total for Question 4 is 5 marks)



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5. The points $A(-2, 1)$ and $B(4, 5)$ lie on the circle C .

(a) Find an equation of the perpendicular bisector of AB .

(3)

Given that the centre of C lies on the x -axis,

(b) find an equation for C .

(2)

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Question 5 continued

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(Total for Question 5 is 5 marks)



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6. The circle C_1 has equation $x^2 + y^2 - 4x + 6y - 12 = 0$
The circle C_2 has equation $x^2 + y^2 - 20x - 18y + 145 = 0$

(a) Find the centre and the radius of C_1 .

(3)

(b) Find the centre and the radius of C_2 .

(2)

(c) Show that C_1 and C_2 do not meet.

(3)

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Question 6 continued

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(Total for Question 6 is 8 marks)



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Solutions relying on calculator technology are not acceptable.

The line l has equation $y = 2x + k$, where k is a constant.

$$5x^2 + (4k - 10)x + k^2 - 2k - 10 = 0$$

(3)

(b) find the two possible values of k .

(3)

(c) For the positive value of k , find the coordinates of the point where l touches C .

(2)

Question 7 continued

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(Total for Question 7 is 8 marks)



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8. The points A , B and C have coordinates $(-1, 2)$, $(7, 6)$ and $(5, 0)$ respectively.

(a) Show that angle $ACB = 90^\circ$

(3)

(b) Hence find an equation of the circle that passes through A , B and C .

(3)

The point D lies on the circle and CD is a diameter.

(c) Find the coordinates of D .

(2)

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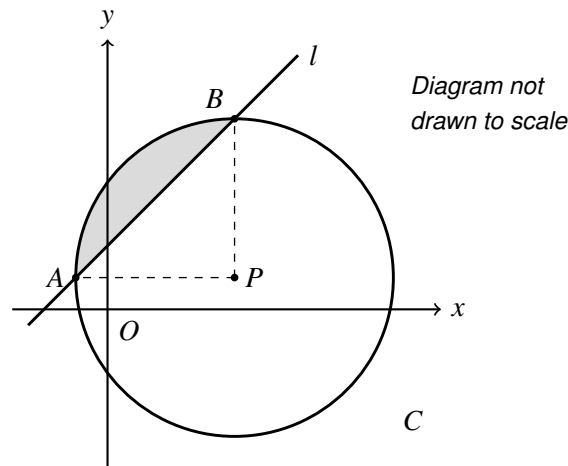


Figure 1

Figure 1 shows the circle C with equation

$$x^2 + y^2 - 8x - 2y - 8 = 0$$

and centre P . The line l with equation $y = x + 2$ meets C at the points A and B .

(a) Find the coordinates of P and the radius of C .

(2)

(b) Using algebra, find the coordinates of A and B .

(3)

(c) Show that angle $APB = \frac{\pi}{2}$ radians.

(2)

The minor segment cut off by the chord AB is shown shaded in Figure 1.

(d) Find the exact area of the shaded segment.

(2)



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Question 9 continued

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Question 9 continued

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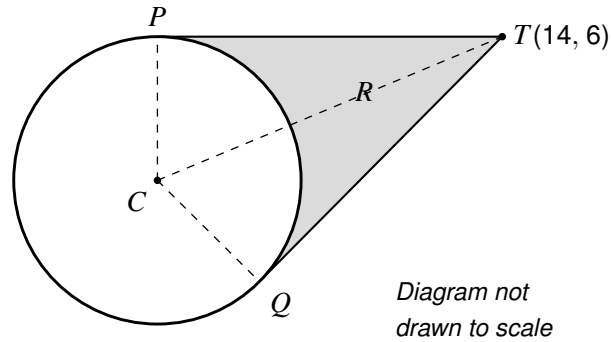


Figure 2

The circle C has equation $(x - 2)^2 + (y - 1)^2 = 25$ and centre C .

The point T has coordinates $(14, 6)$. The lines TP and TQ are tangents to the circle at P and Q , as shown in Figure 2.

(a) Show that the length of the tangent TP is 12 (3)

(b) Find the area of the quadrilateral $CPTQ$. (2)

(c) Find the size of angle PCQ , in radians, to 3 significant figures. (2)

The region R , shown shaded in Figure 2, is bounded by the tangents TP and TQ and the minor arc PQ .

(d) Find the area of R , giving your answer to 3 significant figures. (3)



Question 10 continued

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Question 10 continued

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TOTAL FOR CHAPTER 2 IS 68 MARKS

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CHAPTER 3 • KEY SKILLS

3

Exponentials and Logarithms

Specification P2: Exponentials, Logarithms • 10 questions • 68 marks • about 80 minutes

1 Graphs of $y = a^x$

Passes through (0, 1) with asymptote $y = 0$. Transformations move the asymptote: $y = 4 - 2^x$ has asymptote $y = 4$.

2 Index and log form

$$a^x = b \Leftrightarrow \log_a b = x$$

$$\text{e.g. } \log_2 8 = 3, \log_9 3 = \frac{1}{2}$$

3 Laws of logarithms

$$\log xy = \log x + \log y,$$

$$\log \frac{x}{y} = \log x - \log y, \log x^k = k \log x$$

$$\log(x + y) \neq \log x + \log y$$

4 Log equations

Combine into one log each side, then remove the logs. Check each answer: you can only take the log of a positive number.

5 Solving $a^x = b$

$$\text{Take logs of both sides: } x = \frac{\log b}{\log a}$$

$$\text{e.g. } 3^x = 20 \text{ gives } x = 2.73$$

6 Hidden quadratics

$$4^x = (2^x)^2 \text{ and } 2^{x+1} = 2 \times 2^x: \text{ let } y = 2^x.$$

$$\text{Remember } 2^x > 0.$$

7 Change of base

$$\log_a b = \frac{\log_c b}{\log_c a}, \text{ so } \log_a b = \frac{1}{\log_b a} \text{ and}$$

$$\log_4 x = \frac{1}{2} \log_2 x$$

8 Intersections of exponentials

Set $a^x = k b^x$, collect the powers:

$$\left(\frac{a}{b}\right)^x = k, \text{ then take logs.}$$

9 Substitution in log equations

“Hence” may mean replacing x by $2y$ (or similar) in an equation you have already solved.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	4	5	6	5	5	8	8	8	9	10
1 Graphs of $y = a^x$				•			•			
2 Index and log form	•		•	•	•	•		•	•	•
3 Laws of logarithms	•		•		•			•		•
4 Log equations			•		•			•		•
5 Solving $a^x = b$		•		•		•	•			
6 Hidden quadratics						•			•	
7 Change of base					•				•	
8 Intersections of exponentials							•			
9 Substitution in log equations										•
Interleaved: quadratics, surds, inequalities (P1)			•			•	•	•	•	•
Interleaved: graph transformations and sketching (P1)				•			•			
Interleaved: simultaneous equations (P1)								•		

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA12 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Use the power law *before* combining logs. Write the number on the other side as a log ($2 = \log_3 9$). Check every answer in the original equation.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Write down the value of $\log_3 81$

(1)

(b) Express $2\log_a 5 + \log_a 4 - \log_a 10$ as a single logarithm, simplifying your answer.

(3)

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(Total for Question 1 is 4 marks)



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2. Find, giving each answer to 3 significant figures, the value of x for which

(a) $5^x = 12$

(2)

(b) $3^{2x-1} = 7^x$

(3)

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Question 2 continued

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(Total for Question 2 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Solve

$$\log_3(2x+3) + \log_3(x-1) = 2 + \log_3(x+1)$$

giving your answer in the form $a + \sqrt{b}$, where a and b are integers. Explain why any other value must be rejected.

(6)

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Question 3 continued

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4. (a) Sketch the curve with equation $y = 4 - 2^x$
On your sketch show clearly the coordinates of the points where the curve meets the coordinate axes and state the equation of the asymptote.

(3)

- (b) Find the exact x coordinate of the point where the curve meets the line $y = 1$

(2)

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Question 4 continued

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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Show that $\log_4 x = \frac{1}{2} \log_2 x$

(2)

(b) Hence solve $\log_2 x + \log_4 x = 6$

(3)

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Question 5 continued

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(Total for Question 5 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

6. In this question you must show all stages of your working.

(a) Given that $y = 2^x$, show that the equation

$$2^{2x+1} - 11 \times 2^x + 12 = 0$$

can be written as $2y^2 - 11y + 12 = 0$

(2)

(b) Hence solve $2^{2x+1} - 11 \times 2^x + 12 = 0$, giving any non-integer answer to 3 significant figures.

(4)

(c) Hence find the set of values of x for which $2^{2x+1} - 11 \times 2^x + 12 < 0$

(2)

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Question 6 continued

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(Total for Question 6 is 8 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

The curves C_1 and C_2 have equations $y = 5^x$ and $y = 2^{x+1}$ respectively. The curves meet at the point P .

- (a) On the same axes, sketch C_1 and C_2 , showing clearly the coordinates of the points where each curve meets the y -axis. (3)
- (b) Show that the x coordinate of P is
$$\frac{\log 2}{\log 2.5}$$
 (3)
- (c) Hence find the set of values of x for which $5^x > 2^{x+1}$, giving the boundary value to 3 significant figures. (2)



Question 7 continued

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(Total for Question 7 is 8 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

8. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Given that $\log_a x = p$ and $\log_a y = q$, express $\log_a \left(\frac{x^2 \sqrt{y}}{a} \right)$ in terms of p and q .

(3)

(b) Solve the simultaneous equations

$$\log_2(x + y) = 3$$

$$\log_2 x + \log_2 y = \log_2 15$$

(5)

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(Total for Question 8 is 8 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

9. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given that $a > 1$, $y > 0$ and $y \neq 1$,

(a) show that the equation $\log_a y + 2 = 3 \log_y a$ can be written as

$$(\log_a y)^2 + 2 \log_a y - 3 = 0$$

(3)

(b) Hence find the possible values of y , in terms of a .

(3)

Given also that the product of the two possible values of y is $\frac{1}{16}$

(c) find the value of a .

(3)

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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 9 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Solutions relying on calculator technology are not acceptable.

- $$\log_2(x^2) - \log_2(x-1) = 3 \quad x > 1$$

can be written as $x^2 - 8x + 8 = 0$

(3)

- (b)** Hence solve $\log_2(x^2) - \log_2(x-1) = 3$, giving your answers in the form $a \pm b\sqrt{2}$, where a and b are integers. Show that each answer is valid.

(4)

- (c)** Hence, or otherwise, solve

$$\log_2(4y^2) - \log_2(2y - 1) = 3 \quad y > \frac{1}{2}$$

giving your answers in exact form.

(3)

Question 10 continued

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Question 10 continued

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(Total for Question 10 is 10 marks)**TOTAL FOR CHAPTER 3 IS 68 MARKS****Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

CHAPTER 4 • KEY SKILLS

4

The Binomial Expansion

Specification P2: Binomial expansion • 10 questions • 65 marks • about 80 minutes

1 Binomial coefficients

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

e.g. $\binom{6}{2} = 15$

2 Expanding $(a + bx)^n$

$$(a + bx)^n =$$

$$a^n + \binom{n}{1}a^{n-1}(bx) + \binom{n}{2}a^{n-2}(bx)^2 + \dots$$

Keep bx in brackets, including its sign.

3 A single term

The term in x^r is $\binom{n}{r}a^{n-r}(bx)^r$. The *coefficient* is the number only.

4 Unknown constants

Write each given coefficient in terms of the unknown, form an equation and solve it. Use any given sign condition.

5 Unknown power n

$$\binom{n}{2} = \frac{n(n-1)}{2} \text{ and}$$

$$\binom{n}{3} = \frac{n(n-1)(n-2)}{6} \text{ give equations in } n.$$

6 Products of brackets

For the x^2 term of $(p + qx)(\dots)$:
 $p \times (\text{coefficient of } x^2) + q \times (\text{coefficient of } x)$.

7 Approximations

Choose x so the bracket equals the number, e.g. $0.99 = 1 - \frac{x}{4}$ gives $x = 0.04$. Higher powers of small x are tiny.

8 Powers of x and $\frac{1}{x}$

Write the general term as a single power x^k ; the constant term has $k = 0$.
 k must come from a whole number r .

9 Symmetry

In $(1+x)^n + (1-x)^n$ the odd powers cancel; in $(1+x)^n - (1-x)^n$ the even powers cancel.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	4	4	5	5	4	8	8	8	9	10
1 Binomial coefficients		•	•				•	•		
2 Expanding $(a + bx)^n$	•			•		•			•	•
3 A single term		•	•			•		•		
4 Unknown constants			•			•				•
5 Unknown power n							•			
6 Products of brackets					•					•
7 Approximations				•						
8 Powers of x and $\frac{1}{x}$								•		
9 Symmetry									•	
Interleaved: surds and indices (P1)								•	•	
Interleaved: quadratics and hidden quadratics (P1)			•			•	•		•	
Interleaved: simultaneous equations (P1)							•			•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA12 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Put brackets round the whole of bx before raising it to a power. Give coefficients exactly (as fractions, not rounded decimals) unless told otherwise.



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1. Find the first 4 terms, in ascending powers of x , of the binomial expansion of

$$(2 + 3x)^5$$

giving each term in its simplest form.

(4)

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(Total for Question 1 is 4 marks)



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2. In the binomial expansion of $(3 - 2x)^7$, find

(a) the coefficient of x^3

(2)

(b) the coefficient of x^5

(2)

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(Total for Question 2 is 4 marks)



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3. In the binomial expansion of $(1 + kx)^8$, where k is a positive constant, the coefficient of x^2 is 112

(a) Find the value of k .

(3)

(b) Find the coefficient of x^3 in the expansion.

(2)

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Question 3 continued

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(Total for Question 3 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

4. (a) Find the first 3 terms, in ascending powers of x , of the binomial expansion of $\left(1 - \frac{x}{4}\right)^{10}$ giving each term in its simplest form. (3)
- (b) Use your expansion, with a suitable value of x , to find an estimate for 0.99^{10} State the value of x that you use. (2)

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Question 4 continued

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(Total for Question 4 is 5 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

5. Find the coefficient of x^2 in the expansion of

$$(3+x)(1-2x)^5$$

(4)

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(Total for Question 5 is 4 marks)



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6. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**

The binomial expansion, in ascending powers of x , of $(2 + px)^6$, where p is a non-zero constant, can be written as

$$A + Bx + Cx^2 + Dx^3 + \dots$$

(a) Write down the value of A .

(1)

Given that $2D = 15B$ and that $p < 0$,

(b) find the value of p .

(4)

(c) Find the value of C .

(2)

(d) Write down the coefficient of x^6 in the expansion.

(1)

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Question 6 continued

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(Total for Question 6 is 8 marks)**Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk

Solutions relying on calculator technology are not acceptable.

In the binomial expansion of $(1+ax)^n$, where a is a constant and n is an integer with $n > 2$, the coefficient of x is 20 and the coefficient of x^2 is 180

- (a) Show that $n = 10$, and find the value of a . (5)
- (b) Find the coefficient of x^3 in the expansion. (1)
- (c) Find the largest coefficient in the expansion, stating the power of x it belongs to. (2)

Question 7 continued

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8.

$$\left(x^2 - \frac{2}{x}\right)^6 \quad x \neq 0$$

- (a) Show that the general term of the binomial expansion can be written as $\binom{6}{r}(-2)^r x^{12-3r}$ (2)
- (b) Find the term independent of x . (2)
- (c) Find the coefficient of x^3 (2)
- (d) Explain why there is no term in x^2 in the expansion. (2)

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(Total for Question 8 is 8 marks)



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9. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Expand and simplify $(1+x)^5 + (1-x)^5$

(3)

(b) Hence show that $(1+\sqrt{2})^5 + (1-\sqrt{2})^5 = 82$

(2)

(c) Hence solve the equation

$$(1+y)^5 + (1-y)^5 = 242$$

(4)

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Question 9 continued

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(Total for Question 9 is 9 marks)



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10. In this question you must show all stages of your working.**Solutions relying on calculator technology are not acceptable.**

- (a) Find the first 3 terms, in ascending powers of x , of the binomial expansion of $(2+ax)^5$, where a is a non-zero constant. Give each term in terms of a .

(3)

In the expansion of $(2+ax)^5(1-bx)$, where b is a constant, the coefficient of x is 16 and the coefficient of x^2 is 0

- (b) Show that $b = a$ and find the value of a .

(5)

- (c) Find the exact coefficient of x^3 in the expansion of $(2+ax)^5(1-bx)$

(2)

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 4 IS 65 MARKS

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5

CHAPTER 5 • KEY SKILLS

Sequences and Series

Specification P2: 4.1 to 4.4 • 10 questions • 69 marks • about 85 minutes

1 Recurrence relations

 $u_{n+1} = f(u_n)$: substitute one term to get the next.e.g. $u_1 = 4$, $u_{n+1} = 3u_n - 4$: 4, 8, 20, ...

2 Types of sequence

Increasing: $u_{n+1} > u_n$. Decreasing: $u_{n+1} < u_n$. Periodic: the terms repeat with period k .

3 Arithmetic sequences

$$u_n = a + (n-1)d$$

Two terms give two equations in a and d .

4 Arithmetic series

$$S_n = \frac{n}{2}(2a + (n-1)d) = \frac{n}{2}(a+l)$$

Know the proof: write S_n forwards and backwards, then add.

5 Sigma notation

$$\sum_{r=1}^n r = \frac{1}{2}n(n+1). \text{ Count the terms: } \sum_{r=k}^n$$

has $n - k + 1$ terms.

6 Geometric sequences

 $u_n = ar^{n-1}$. Three consecutive terms:

$$\frac{u_2}{u_1} = \frac{u_3}{u_2}, \text{ so } u_2^2 = u_1 u_3$$

7 Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}; S_\infty = \frac{a}{1-r} \text{ only if}$$

$$|r| < 1$$

Proof: subtract rS_n from S_n .8 Finding n with logsSolve $r^n > k$ with logs. If $\log r < 0$ the inequality reverses. n is a whole number: round in the right direction.

9 Modelling

Decide whether the change is by a fixed amount (arithmetic) or a fixed percentage (geometric). Year n is the n th term.**Skills map** every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	5	5	5	5	9	8	8	9	10
1 Recurrence relations	•				•			•		
2 Types of sequence					•			•		
3 Arithmetic sequences		•					•			•
4 Arithmetic series		•		•			•			•
5 Sigma notation	•			•	•			•		
6 Geometric sequences			•			•			•	•
7 Geometric series and proof			•			•			•	•
8 Finding n with logs						•			•	•
9 Modelling							•			•
Interleaved: logarithms (Chapter 3)						•			•	•
Interleaved: quadratics and inequalities (P1)						•	•	•		
Interleaved: simultaneous equations and proof (P1, Ch 1)		•		•			•			

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA12 paper, where one part leads into the next and skills from different topics are combined.

Must learn. The proofs of both sum formulae. Check which term is “year n ”: a starting value is often u_1 , so year n uses r^{n-1} . Give n as a whole number that satisfies the condition.



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1. A sequence is defined by

$$u_1 = 4 \quad u_{n+1} = 3u_n - k$$

where k is a constant.

- (a) Find, in terms of k , the values of u_2 and u_3

(2)

- (b) Given that $u_3 = 20$, find the value of k .

(2)

- (c) Find the value of $\sum_{r=1}^4 u_r$

(1)

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2. The 4th term of an arithmetic sequence is 23 and the 10th term is 53

(a) Find the first term and the common difference.

(3)

(b) Find the sum of the first 20 terms.

(2)

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Question 2 continued

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3. A geometric series has first term 81 and common ratio $\frac{2}{3}$
- (a) Find the 5th term. (1)
- (b) Find the sum of the first 10 terms, giving your answer to 3 significant figures. (2)
- (c) Find the sum to infinity. (1)
- (d) Find the difference between the sum to infinity and the sum of the first 10 terms, to 3 significant figures. (1)

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Question 3 continued

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4. (a) Prove that the sum of the first n terms of an arithmetic series with first term a and common difference d is

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

(3)

- (b) Hence find $\sum_{r=1}^{40} (3r+2)$

(2)

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5. A sequence is defined by

$$u_1 = 2 \quad u_{n+1} = \frac{1}{1 - u_n}$$

- (a) Find the values of u_2 , u_3 and u_4

(2)

- (b) Describe the behaviour of the sequence.

(1)

- (c) Find the value of $\sum_{r=1}^{100} u_r$

(2)

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6. In this question you must show all stages of your working.

The first three terms of a geometric sequence are

$$(k+3) \quad 2k \quad (3k-2)$$

where k is a constant.

- (a) Show that $k^2 - 7k + 6 = 0$ (2)
- (b) Find the two possible values of k . (1)
- (c) For the value of k for which the series has a sum to infinity, find the sum to infinity. (3)
- (d) For the other value of k , find the least number of terms for which the sum of the series exceeds 1000 (3)

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Question 6 continued

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Question 6 continued

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Question 7 continued

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8. A sequence is defined by

$$u_1 = 2 \quad u_{n+1} = au_n + 2a - 1$$

where a is a constant.

- (a) Find u_2 and u_3 in terms of a , simplifying your answers.

(3)

Given that $\sum_{n=1}^3 u_n = 21$

- (b) show that $4a^2 + 5a - 21 = 0$

(2)

- (c) Given also that the sequence is increasing, find the value of a and the value of u_4

(3)

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Question 8 continued

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9. A geometric series has first term a and common ratio r , where $r \neq 1$

(a) Prove that the sum of the first n terms of the series is

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

(3)

The sum to infinity of the series is 4 times the first term, and the sum of the first three terms is 74

(b) Find the value of r and the value of a .

(3)

(c) Find the least value of n for which the sum to infinity exceeds the sum of the first n terms by less than 0.01

(3)

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Question 9 continued

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Question 9 continued

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Scheme B: £28 000 in the first year, increasing by 5% each year.

- (a) Find the salary in the 5th year under each scheme, giving scheme B to the nearest pound. (2)
- (b) Show that the total earned in the first 10 years under scheme A is £367 500 (2)
- (c) Find the total earned in the first 10 years under scheme B , to the nearest pound. (2)
- (d) Show that the total earned under scheme B first exceeds £1 000 000 after n years, where n is the least integer satisfying

$$1.05^n > \frac{39}{14}$$

and hence find n .

Question 10 continued

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Question 10 continued

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TOTAL FOR CHAPTER 5 IS 69 MARKS

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CHAPTER 6 • KEY SKILLS

6

Trigonometric Identities and Equations

Specification P2: 6.1, 6.2 • 10 questions • 66 marks • about 80 minutes

1 The tan identity

 $\tan x \equiv \frac{\sin x}{\cos x}$, so $a \sin x = b \cos x$ gives

$$\tan x = \frac{b}{a} \text{ (if } \cos x \neq 0 \text{)}$$

2 The Pythagorean identity

 $\sin^2 x + \cos^2 x \equiv 1$. Exact values from one ratio: use the quadrant to choose the sign.

3 Proving identities

Start from one side and transform it into the other. Factorise, e.g.

$$a^2 - b^2 = (a - b)(a + b).$$

4 Basic equations

Principal value from the calculator, then the second value by symmetry.

$$\sin: 180^\circ - \alpha. \quad \cos: 360^\circ - \alpha. \quad \tan: \alpha + 180^\circ$$

5 Transformed arguments

For $\cos(2x + 40^\circ)$, change the interval first: $0 \leq x < 180^\circ$ gives $40^\circ \leq 2x + 40^\circ < 400^\circ$

6 Quadratics in one function

Use $\sin^2 x = 1 - \cos^2 x$ (or the reverse) so only one function remains, then factorise.

7 Never divide by a trig function

Dividing by $\sin x$ loses the solutions of $\sin x = 0$. Factorise instead.

8 Impossible values

$-1 \leq \sin x \leq 1$ and $-1 \leq \cos x \leq 1$: reject values outside this range and say why.

9 Radians and exact answers

Give answers as multiples of π when the interval is in radians, e.g.

$$\sin x = \frac{\sqrt{3}}{2}: x = \frac{\pi}{3}, \frac{2\pi}{3}$$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	4	5	4	6	4	8	8	8	9	10
1 The tan identity	•					•	•		•	•
2 The Pythagorean identity	•			•		•		•		•
3 Proving identities				•						
4 Basic equations		•		•		•	•		•	•
5 Transformed arguments			•		•	•	•		•	•
6 Quadratics in one function						•		•	•	
7 Never divide by a trig function							•			
8 Impossible values						•		•		
9 Radians and exact answers			•					•		
Interleaved: quadratics and factorising (P1)				•		•	•	•	•	
Interleaved: trig graphs, periods and exact values (P1)	•		•				•	•		
Interleaved: area and cosine rule, surds (P1)	•									•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA12 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Both identities are *not* in the formula booklet. Give every solution in the interval and no others. Work to 1 decimal place in degrees unless told otherwise.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given that $\sin \theta = \frac{\sqrt{5}}{3}$ and that θ is obtuse, find the exact value of

(a) $\cos \theta$

(2)

(b) $\tan \theta$

(2)

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2. Solve, for $0 \leq x < 360^\circ$, giving your answers to one decimal place,

(a) $4 \sin x = 3$

(2)

(b) $3 \tan x + 5 = 0$

(3)

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Question 2 continued

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3. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Solve, for $0 \leq x < 2\pi$,

$$2 \sin \left(x - \frac{\pi}{6} \right) = \sqrt{3}$$

giving your answers as exact multiples of π .

(4)

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4. (a) Prove that $\cos^4 x - \sin^4 x \equiv 1 - 2\sin^2 x$

(3)

- (b) Hence solve, for $0 \leq x < 360^\circ$,

$$\cos^4 x - \sin^4 x = \frac{1}{2}$$

(3)

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Question 4 continued

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(Total for Question 4 is 6 marks)



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5. Solve, for $0 \leq x < 180^\circ$,

$$\tan(2x - 30^\circ) = \sqrt{3}$$

(4)

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6. (a) Show that the equation

$$3 \tan x \sin x = 8$$

can be written as $3 \cos^2 x + 8 \cos x - 3 = 0$

(3)

- (b) Hence solve, for $0 \leq x < 360^\circ$, $3 \tan x \sin x = 8$, giving your answers to one decimal place.

(3)

- (c) Hence solve, for $0 \leq \theta < 180^\circ$, $3 \tan 2\theta \sin 2\theta = 8$, giving your answers to one decimal place.

(2)

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Question 6 continued

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7. A student is asked to solve the equation $5 \sin x = 2 \tan x$ for $0 \leq x \leq 360^\circ$. The student's working is shown below.

$$5 \sin x = \frac{2 \sin x}{\cos x} \quad \text{so} \quad 5 \sin x \cos x = 2 \sin x$$

$$\text{Divide by } \sin x: \quad 5 \cos x = 2, \quad \cos x = 0.4$$

$$x = 66.4^\circ \text{ or } x = 293.6^\circ$$

- (a) Identify the error made by the student. (2)
- (b) Solve the equation correctly, giving your answers to one decimal place where necessary. (4)
- (c) Hence state the number of solutions of $5 \sin 3x = 2 \tan 3x$ for $0 \leq x \leq 360^\circ$ (2)



Question 7 continued

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Solutions relying on calculator technology are not acceptable.

- $$4 \sin^2 x + 4 \sin x - 3 = 0$$

(2)

- (4)

- (2)

Question 8 continued

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9. (a) Show that the equation

$$2 \sin^2 x + 5 \sin x \cos x = 3 \cos^2 x$$

can be written as $2 \tan^2 x + 5 \tan x - 3 = 0$, explaining why $\cos x \neq 0$ for any solution.

(3)

- (b) Hence solve, for $0 \leq x < 360^\circ$, $2 \sin^2 x + 5 \sin x \cos x = 3 \cos^2 x$, giving your answers to one decimal place.

(4)

- (c) Given that x is an acute angle satisfying the equation, find the exact value of $\sin x$, giving your answer in the form $\frac{\sqrt{a}}{b}$, where a and b are integers.

(2)

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Question 9 continued

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Question 9 continued

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10. In the triangle ABC , $AB = 5$ cm, $AC = 8$ cm and angle $BAC = \theta$. The area of the triangle is 12 cm^2

(a) Show that $\sin \theta = \frac{3}{5}$ (1)

(b) Find the two possible exact values of $\cos \theta$. (2)

(c) Find the two possible exact lengths of BC , giving your answers in simplest surd form where appropriate. (3)

(d) For the triangle in which θ is obtuse, write down the exact value of $\tan \theta$. (1)

(e) Solve, for $0 \leq x < 360^\circ$,
$$5 \sin(x - 20^\circ) = 3 \cos(x - 20^\circ)$$

giving your answers to one decimal place. (3)



Question 10 continued

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TOTAL FOR CHAPTER 6 IS 66 MARKS

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7

CHAPTER 7 • KEY SKILLS

Differentiation

Specification P2: 7.1 • 10 questions • 67 marks • about 80 minutes

1 Stationary points

Solve $\frac{dy}{dx} = 0$, then substitute each x into y (not into $\frac{dy}{dx}$).

2 Nature of a stationary point

$\frac{d^2y}{dx^2} < 0$: maximum. $\frac{d^2y}{dx^2} > 0$: minimum. Give the value and a conclusion.

3 Increasing and decreasing

Increasing where $f'(x) > 0$; decreasing where $f'(x) < 0$. Solve the quadratic inequality.

4 Increasing for all x

Complete the square:

$f'(x) = a(x+p)^2 + q$ with $a, q > 0$ means $f'(x) > 0$ for all x .

5 Powers of x first

Write roots and fractions as powers before differentiating, e.g. $\frac{8}{x^2} = 8x^{-2}$.

6 Forming the expression

Use the constraint (fixed volume, perimeter or area) to remove the second variable.

7 Maximum and minimum values

Solve $\frac{dV}{dx} = 0$ and substitute back into V , with units.

8 Practical domain

Reject values that make a length negative or zero, and say why.

9 Using stationary points

Stationary points fix the shape of a sketch and the values of k for which $f(x) = k$ has a given number of solutions.

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	6	4	5	4	5	8	8	8	9	10
1 Stationary points	•		•		•	•	•	•	•	•
2 Nature of a stationary point	•		•			•	•	•	•	•
3 Increasing and decreasing		•							•	
4 Increasing for all x				•						
5 Powers of x first			•				•			
6 Forming the expression						•	•	•		
7 Maximum and minimum values						•	•	•		
8 Practical domain						•				
9 Using stationary points					•					•
Interleaved: quadratics, completing the square, inequalities (P1)		•		•					•	•
Interleaved: circle measures and indices (P1)			•				•	•		
Interleaved: factor theorem, sketching cubics (P2 Ch 1, P1)					•				•	•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA12 paper, where one part leads into the next and skills from different topics are combined.

Must learn. “Justify” or “show that it is a maximum” needs the second derivative’s value (or sign) *and* a conclusion. In context, give the maximum or minimum value, not just the value of x .



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1. The curve C has equation $y = 2x^3 - 9x^2 + 12x + 1$

(a) Find the coordinates of the stationary points of C .

(4)

(b) Use $\frac{d^2y}{dx^2}$ to determine the nature of each stationary point.

(2)

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2.

$$f(x) = x^3 - 6x^2 - 15x + 2$$

Find the set of values of x for which $f(x)$ is decreasing.

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3. The curve C has equation $y = 2x^{\frac{3}{2}} - 12x + 7$, $x > 0$

(a) Find the coordinates of the stationary point of C .

(3)

(b) Determine the nature of this stationary point, giving a reason for your answer.

(2)

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4.

$$f(x) = x^3 + 3x^2 + 5x - 4$$

Show that $f(x)$ is an increasing function for all real values of x .

(4)

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5.

$$f(x) = x^3 - 3x^2 - 9x$$

(a) Find the coordinates of the stationary points of the curve with equation $y = f(x)$. (3)

(b) Sketch the curve with equation $y = f(x)$, showing the coordinates of the stationary points and of the point where the curve crosses the y -axis. (2)

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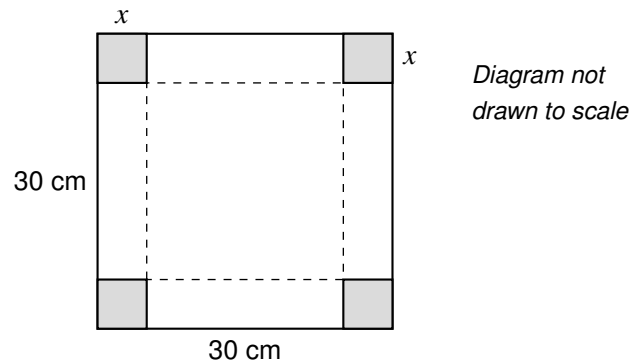


Figure 1

Figure 1 shows a square sheet of card of side 30 cm. A square of side x cm is cut from each corner and the sides are folded up along the dashed lines to make an open box.

- (a) Show that the volume, V cm³, of the box is given by

$$V = 4x^3 - 120x^2 + 900x$$

(2)

- (b) Use calculus to find the value of x for which V is stationary, explaining why any other value must be rejected.

(4)

- (c) Find the maximum volume of the box, and justify that it is a maximum.

(2)



Question 6 continued

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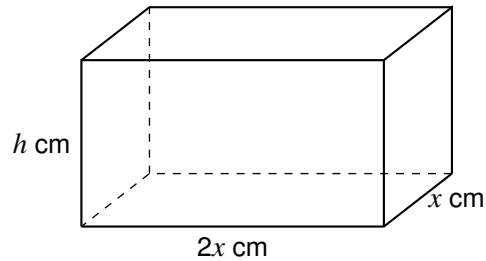


Diagram not
drawn to scale

Figure 2

A closed box in the shape of a cuboid has a base of length $2x \text{ cm}$ and width $x \text{ cm}$, and a height of $h \text{ cm}$, as shown in Figure 2. The volume of the box is 72 cm^3

- (a) Show that the total surface area, $S \text{ cm}^2$, of the box is given by

$$S = 4x^2 + \frac{216}{x}$$

(3)

- (b) Use calculus to find the minimum value of S .

(3)

- (c) Justify that the value of S found in part (b) is a minimum.

(2)

Question 7 continued

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(Total for Question 7 is 8 marks)



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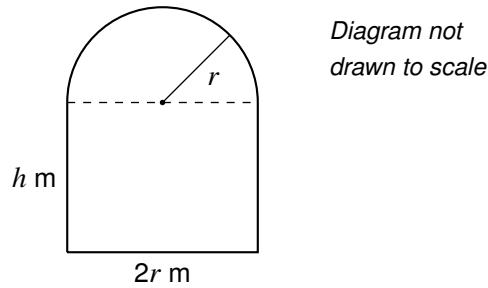
**Figure 3**

Figure 3 shows the design for a window. It is a rectangle of width $2r$ metres and height h metres, with a semicircle of radius r metres on top. The perimeter of the window is 10 m.

- (a) Show that the area, $A \text{ m}^2$, of the window is given by

$$A = 10r - \left(2 + \frac{\pi}{2}\right)r^2 \quad (3)$$

- (b) Use calculus to find the exact value of r for which A is a maximum, and show that the maximum area is $\frac{50}{4 + \pi} \text{ m}^2$

(4)

- (c) Justify that this value of A is a maximum.

(1)



Question 8 continued

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9.

$$f(x) = ax^3 + bx^2 - 12x + 5$$

where a and b are constants. The curve with equation $y = f(x)$ has a stationary point at $(1, -2)$.

(a) Find the value of a and the value of b . (4)

(b) Find the coordinates of the other stationary point, and determine the nature of both stationary points. (3)

(c) Find the set of values of x for which $f(x)$ is increasing. (2)

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Question 9 continued

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Question 9 continued

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(Total for Question 9 is 9 marks)



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10. In this question you must show all stages of your working.

The curve C has equation $y = x^3 - 6x^2 + 9x + 1$

- (a) Find the coordinates of the stationary points of C and determine their nature. (5)
- (b) The tangent to C at the maximum point meets C again at the point P . Using algebra, find the coordinates of P . (3)
- (c) Find the set of values of k for which the equation $x^3 - 6x^2 + 9x + 1 = k$ has three distinct real solutions. (2)

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Question 10 continued

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Question 10 continued

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(Total for Question 10 is 10 marks)

TOTAL FOR CHAPTER 7 IS 67 MARKS

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8

CHAPTER 8 • KEY SKILLS

Integration

Specification P2: 8.1, 8.2, 8.3 • 10 questions • 67 marks • about 80 minutes

1 Definite integrals

$$\int_a^b f'(x) dx = f(b) - f(a)$$

Show the substituted brackets; no +c.

2 Rewrite first

Split fractions into powers of x :

$$\frac{x-3}{\sqrt{x}} = x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$$

3 Area under a curve

Area = $\int_a^b y dx$ when the curve is above the x -axis. Find the limits from the roots.

4 Regions below the axis

Below the x -axis the integral is negative. Integrate each part separately and add the sizes.

5 Between a curve and a line

Find where they meet, then integrate (top – bottom), or subtract the area under the curve from a trapezium.

6 Unknown limits and constants

Evaluate in terms of k and solve the resulting equation. Check which roots are allowed.

7 The trapezium rule

$$\text{Area} \approx \frac{h}{2} (y_0 + y_n + 2(y_1 + \dots + y_{n-1}))$$

where $h = \frac{b-a}{n}$ and n strips need $n+1$ values of y .

8 Over or under?

If the curve bends upwards (convex) the trapezia lie above it: an overestimate. More strips improve the estimate.

9 Related integrals

Split the integral up:

$$\int_a^b k f(x) dx = k \int_a^b f(x) dx$$

$$\text{and } \int_a^b c dx = c(b-a)$$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	4	5	5	5	5	8	8	8	9	10
1 Definite integrals	•	•	•		•	•	•		•	•
2 Rewrite first	•	•								•
3 Area under a curve			•						•	•
4 Regions below the axis					•					
5 Between a curve and a line						•				
6 Unknown limits and constants		•					•			
7 The trapezium rule				•				•		•
8 Over or under?				•				•		•
9 Related integrals							•	•		
Interleaved: stationary points (Chapter 7)									•	•
Interleaved: factor theorem, exponentials (Chapters 1, 3)							•	•		
Interleaved: quadratics, transformations (P1)			•			•	•			

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA12 paper, where one part leads into the next and skills from different topics are combined.

Must learn. The trapezium rule *is* in the formula booklet, but you must know that n strips need $n+1$ values. An area is always positive: never let a negative integral cancel a positive one.



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1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Find the exact value of

$$\int_1^4 \left(3x^2 - \frac{4}{\sqrt{x}} \right) dx$$

(4)

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(Total for Question 1 is 4 marks)



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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given that

$$\int_1^9 \frac{kx - 3}{\sqrt{x}} dx = 40$$

where k is a constant, find the value of k .

(5)

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(Total for Question 2 is 5 marks)



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3. The curve C has equation $y = 6x - x^2$

(a) Find the x coordinates of the points where C meets the x -axis.

(1)

(b) Use integration to find the area of the region bounded by C and the x -axis.

(4)

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Question 3 continued

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(Total for Question 3 is 5 marks)



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4. The table shows values of $y = \sqrt{x^2 + 1}$ for $0 \leq x \leq 2$, rounded to 4 decimal places.

x	0	0.5	1	1.5	2
y	1		1.4142		2.2361

(a) Complete the table.

(1)

(b) Use the trapezium rule, with all the values in the table, to find an estimate for

$\int_0^2 \sqrt{x^2 + 1} \, dx$, giving your answer to 3 decimal places.

(3)

(c) State, with a reason, whether your answer is an overestimate or an underestimate.

(1)

Question 4 continued

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(Total for Question 4 is 5 marks)



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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation $y = x^3 - 6x^2 + 8x = x(x - 2)(x - 4)$

- (a) Show that $\int_0^4 (x^3 - 6x^2 + 8x) \, dx = 0$ (2)
- (b) Explain why this integral does not give the area enclosed between C and the x -axis, and find this area. (3)

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Question 5 continued

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(Total for Question 5 is 5 marks)



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6. In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

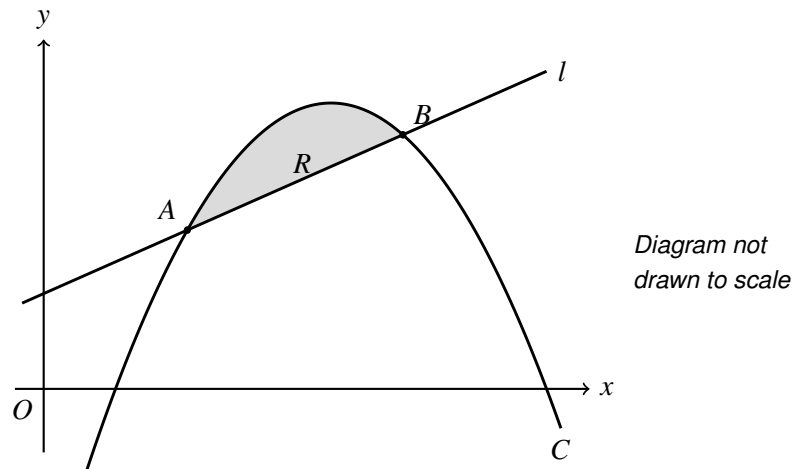
**Figure 1**

Figure 1 shows part of the curve C with equation $y = -x^2 + 8x - 7$ and the line l with equation $y = x + 3$

The line meets C at the points A and B . The region R , shown shaded, is bounded by C and l .

- (a) Find the coordinates of A and B .

(3)

- (b) Use calculus to find the exact area of R .

(5)



Question 6 continued

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(Total for Question 6 is 8 marks)



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7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given that

$$\int_1^k (3x^2 - 2x) \, dx = 48$$

where k is a real constant,

- (a)** show that $k^3 - k^2 - 48 = 0$

(3)

- (b)** show that $k = 4$ is the only real solution of this equation.

(3)

- (c) Hence, giving a reason, write down the value of $\int_{-4}^{-1} (3x^2 + 2x) \, dx$

(2)

Question 7 continued

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(Total for Question 7 is 8 marks)



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8. The curve C has equation $y = 2^x$

- (a) Use the trapezium rule with 4 strips of equal width to find an estimate for $\int_0^2 2^x dx$, giving your answer to 3 decimal places.

(4)

- (b) Explain why your answer to part (a) is an overestimate.

(1)

- (c) Using your answer to part (a), find an estimate for $\int_0^2 (2^{x+1} + 3) dx$

(3)

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9. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

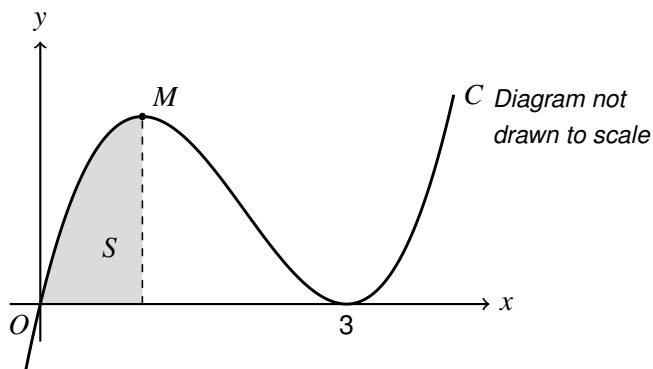


Figure 2

Figure 2 shows part of the curve C with equation $y = x^3 - 6x^2 + 9x$

The point M is the maximum point of C . The region S , shown shaded, is bounded by C , the x -axis and the line through M parallel to the y -axis.

- (a) Use calculus to find the coordinates of M . (3)
- (b) Find the exact area of S . (3)
- (c) Find the exact area of the region bounded by C and the x -axis for $0 \leq x \leq 3$ (2)
- (d) Hence write down the area of the region bounded by C , the x -axis and the line through M parallel to the y -axis, for $1 \leq x \leq 3$ (1)



Question 9 continued

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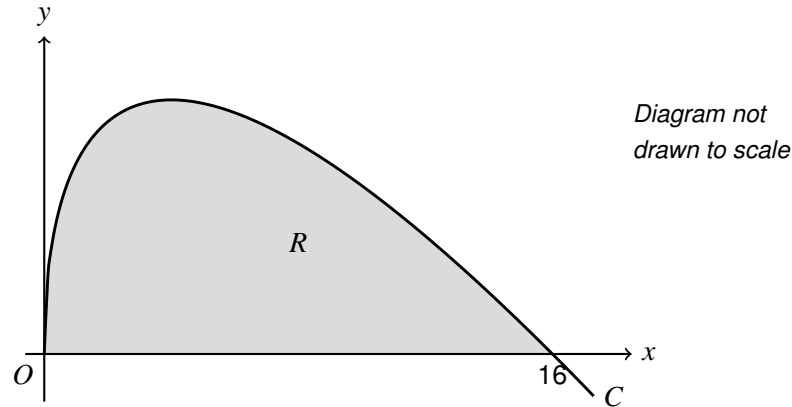
**Figure 3**

Figure 3 shows part of the curve C with equation $y = 8\sqrt{x} - 2x$, $x \geq 0$

The curve meets the x -axis at the origin and at the point where $x = 16$. The region R , shown shaded, is bounded by C and the x -axis.

- (a) Use calculus to find the coordinates of the maximum point of C . (3)
- (b) Find the exact area of R . (4)
- (c) Use the trapezium rule with 4 strips of equal width to find an estimate for the area of R , giving your answer to 2 decimal places. (2)
- (d) Find the percentage error in your estimate, to 2 significant figures. (1)



Question 10 continued

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TOTAL FOR CHAPTER 8 IS 67 MARKS

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