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INTERNATIONAL A LEVEL MATHEMATICS • PURE 2 (WMA12)

Pure 2 Practice Paper 1

Worked Solutions

AIMING FOR B

Written in the style of the Pearson Edexcel International Advanced Level paper (WMA12/01) • 75 marks

Every question solved in full on the paper itself, the way your teacher would write it out

How to read these solutions. Each page is the real question page, with a full answer written on the answer lines.

Red pen

M1 A1

boxed

the working a full-marks answer shows, with a short note on why each step is taken

the mark earned on that line: M = method, A = accuracy, B = independent mark

the final answer to each part

Sit the paper first. These solutions are most useful after you have tried every question yourself under timed conditions. Mark your paper with the mark scheme, then use this booklet to see exactly how each lost mark could have been earned.

Q	Topic	Marks	Page
1	Binomial expansion	7	2
2	Factor and remainder theorems	8	3
3	Circles and tangents	7	5
4	Exponential graphs, logarithms	7	7
5	Recurrence and sigma notation	7	8
6	Arithmetic and geometric series	8	9
7	Trigonometric equations	7	10
8	Optimisation	8	11
9	Area between two curves	7	13
10	Stationary points, trapezium rule	9	15

Question paper & mark scheme | Exam questions by topic | More practice papers

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1. The first three terms, in ascending powers of x, of the binomial expansion of

$$(a+bx)^6$$

where a and b are constants and $a > 0$, are

$$64 - 576x + px^2$$

where p is a constant.

(a) Find the value of a, the value of b and the value of p.

(5)

(b) Hence find the coefficient of x^2 in the expansion of

$$\left(3 - 2x + x^2\right)(a+bx)^6$$

(2)

(a) Compare the constant terms: $a^6 = 64$, and $a > 0$, so $a = 2$ B1

Term in x: $6a^5(bx) = -576x \Rightarrow 6 \times 32 \times b = -576$ M1

$192b = -576 \Rightarrow \boxed{b = -3}$ and $\boxed{a = 2}$ A1

Term in x^2: ${}^6C_2\, a^4(bx)^2 = 15 \times 2^4 \times (-3)^2 x^2$ (remember to square b) M1

$p = 15 \times 16 \times 9 = \boxed{2160}$ A1

(b) $(2 - 3x)^6 = 64 - 576x + 2160x^2 + \ldots$

An x^2 term comes from three products: number $\times x^2$, $x \times x$, and $x^2 \times$ number:

$3 \times 2160x^2 + (-2x) \times (-576x) + x^2 \times 64$ M1

$= 6480x^2 + 1152x^2 + 64x^2$, so the coefficient of x^2 is $\boxed{7696}$ A1

(Total for Question 1 is 7 marks)

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2. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = 3x^3 + ax^2 + bx + 4$$

where a and b are constants.

When $f(x)$ is divided by $(x-1)$ the remainder is -2

Given that $(3x-2)$ is a factor of $f(x)$,

(a) show that $2a + 3b = -22$

(2)

(b) Find the value of a and the value of b, and hence factorise $f(x)$ completely.

(4)

(c) Sketch the curve with equation $y = f(x)$, showing the coordinates of the points where the curve meets the coordinate axes.

(2)

(a) $(3x - 2)$ is a factor, so $f(\frac{2}{3}) = 0$ by the factor theorem:

$3(\frac{8}{27}) + a(\frac{4}{9}) + b(\frac{2}{3}) + 4 = 0$ M1

Multiply by 9: $8 + 4a + 6b + 36 = 0 \Rightarrow 4a + 6b = -44 \Rightarrow 2a + 3b = -22$ as required A1*

(b) Remainder theorem: $f(1) = -2$, so $3 + a + b + 4 = -2 \Rightarrow a + b = -9$ M1

Double it: $2a + 2b = -18$. Subtract it from $2a + 3b = -22$: $b = -4$, then $a = -5$

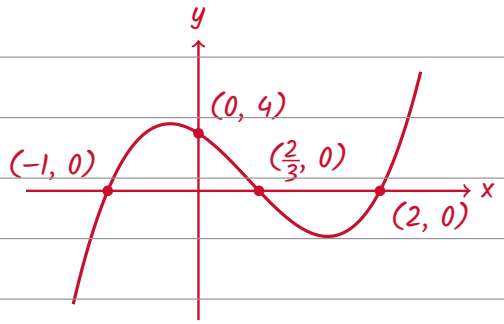
$a = -5$, $b = -4$ so $f(x) = 3x^3 - 5x^2 - 4x + 4$ A1

Divide by $(3x - 2)$: $3x^3 - 5x^2 - 4x + 4 = (3x - 2)(x^2 - x - 2)$ M1

Check: $(3x - 2)(x^2 - x - 2) = 3x^3 - 3x^2 - 6x - 2x^2 + 2x + 4$ ✓

$f(x) = (3x - 2)(x + 1)(x - 2)$ A1

(c) Roots $x = -1, \frac{2}{3}$ and 2; $f(0) = 4$. Positive x^3 term, so it rises on the right.



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Question 2 continued

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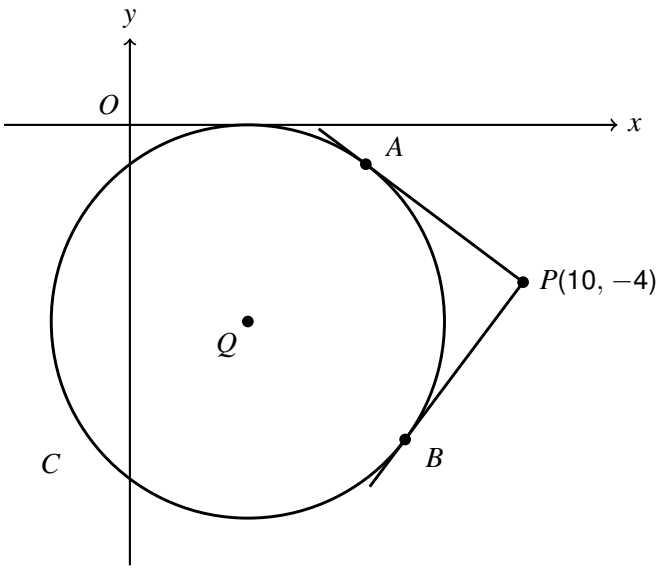

Figure 1

The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find the coordinates of the centre Q of C and the radius of C.

(3)

The point P has coordinates $(10, -4)$.

The two tangents to C from P touch C at the points A and B, as shown in Figure 1.

(b) Show that $PA = 5$

(2)

(c) Explain why $PAQB$ is a square, and write down its area.

(2)

(a) Complete the square for x and for y:

$(x - 3)^2 - 9 + (y + 5)^2 - 25 + 9 = 0 \Rightarrow (x - 3)^2 + (y + 5)^2 = 25$ M1

Centre $Q(3, -5)$ A1

Radius 5 ($\sqrt{25}$, not 25) A1

(b) A radius meets a tangent at 90°, so triangle QAP has a right angle at A.

$QP^2 = (10 - 3)^2 + (-4 - (-5))^2 = 49 + 1 = 50$ M1

Pythagoras: $PA^2 = QP^2 - QA^2 = 50 - 25 = 25$, so $PA = 5$ as required A1*

(c) $PA = PB = 5$ (tangents from P) and $QA = QB = 5$ (radii): all four sides are 5.

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Question 3 continued

Angles QAP and QBP are 90° (radius and tangent).

So PAQB is a rhombus with a right angle: a square. B1

Area = 5 × 5 = 25 B1

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(Total for Question 3 is 7 marks)

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4. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) On the same axes, sketch the curves with equations

$$y = 3^{x+1} \qquad \text{and} \qquad y = 5^{x-1}$$

showing the coordinates of the point where each curve meets the y-axis.

(3)

The curves meet at the point P.

(b) Find the exact x coordinate of P, giving your answer in the form $\log_m n$, where m and n are rational numbers.

(3)

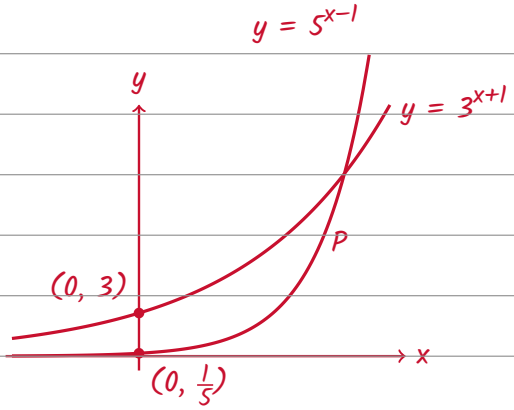
(c) Hence solve the inequality $3^{x+1} > 5^{x-1}$

(1)

(a) Both are exponential growth curves above the x-axis, flattening towards it on the left.

At $x = 0$: $3^1 = 3$ and $5^{-1} = \frac{1}{5}$. 5^{x-1} grows faster, so it overtakes 3^{x+1} once.

B1 B1 B1



(b) At P: $3^{x+1} = 5^{x-1}$. Take logs of both sides:

$(x + 1)\log 3 = (x - 1)\log 5$ M1

$x \log 3 + \log 3 = x \log 5 - \log 5 \Rightarrow x(\log 5 - \log 3) = \log 5 + \log 3$

$x \log \frac{5}{3} = \log 15 \Rightarrow x = \frac{\log 15}{\log (5/3)}$ dM1

$x = \log_{5/3} 15$ $(= 5.30\ldots)$ A1

(c) To the left of P, the curve $y = 3^{x+1}$ is above $y = 5^{x-1}$ (see the sketch):

$x < \log_{5/3} 15$ B1

(Total for Question 4 is 7 marks)

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5. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 2 \qquad u_{n+1} = \frac{u_n - 1}{u_n + 1} \qquad n \geqslant 1$$

(a) Find the value of u_2, u_3, u_4 and u_5

(2)

(b) Show that, for all $n \geqslant 1$,

$$u_{n+2} = -\frac{1}{u_n}$$

(2)

(c) Find the least value of n for which

$$\sum_{r=1}^{n} u_r < -20$$

(3)

(a) $u_2 = \frac{2-1}{2+1} = \frac{1}{3}$, $u_3 = \frac{1/3 - 1}{1/3 + 1} = \frac{-2/3}{4/3} = -\frac{1}{2}$ B1

$u_4 = \frac{-1/2 - 1}{-1/2 + 1} = \frac{-3/2}{1/2} = -3$, $u_5 = \frac{-3-1}{-3+1} = \frac{-4}{-2} = 2$ B1

(b) Write u for u_n. Then $u_{n+1} = \frac{u-1}{u+1}$, and apply the rule again:

$u_{n+2} = \frac{(u-1)/(u+1) - 1}{(u-1)/(u+1) + 1}$ then multiply the top and bottom by $(u+1)$: M1

$= \frac{u - 1 - (u+1)}{u - 1 + (u+1)} = \frac{-2}{2u} = -\frac{1}{u}$, so $u_{n+2} = -\frac{1}{u_n}$ as required A1*

(c) Using (b) twice: $u_{n+4} = -1 \div u_{n+2} = u_n$, so the terms repeat every 4.

One block of 4 terms: $2 + \frac{1}{3} - \frac{1}{2} - 3 = -\frac{7}{6}$ M1

17 blocks: $S_{68} = 17 \times (-\frac{7}{6}) = -\frac{119}{6} = -19.83\ldots$ (not yet below -20)

Next terms $2, \frac{1}{3}, -\frac{1}{2}, -3$: $S_{69} = -17.83\ldots$, $S_{70} = -17.5$, $S_{71} = -18$, $S_{72} = -21$ M1

The sums go back up inside a block, so the first sum below -20 is S_{72}: $n = 72$ A1

(Total for Question 5 is 7 marks)

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6. Ali and Bina are training for a long-distance run.

Ali runs 5 km on day 1. On each day after that, he runs 0.4 km more than on the day before.

Bina runs 3 km on day 1. On each day after that, she runs 6% further than on the day before.

(a) Show that Ali runs a total of 176 km over the first 20 days.

(2)

(b) Find the day on which Bina's daily distance first exceeds 15 km.

(3)

(c) Find the day on which Ali's total distance first reaches 500 km.

(3)

(a) Ali: arithmetic with $a = 5$, $d = 0.4$. $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{20} = \frac{20}{2}[2 \times 5 + 19 \times 0.4]$ M1

$= 10 \times (10 + 7.6) = 10 \times 17.6 = 176$ km as required A1*

(b) Bina: geometric with $a = 3$, $r = 1.06$, so on day n she runs $3 \times 1.06^{n-1}$ km

$3 \times 1.06^{n-1} > 15$ M1

$1.06^{n-1} > 5 \Rightarrow (n-1)\log 1.06 > \log 5 \Rightarrow n - 1 > \frac{\log 5}{\log 1.06} = 27.62...$ M1

$n > 28.62...$, so the first whole day is $\boxed{\text{day 29}}$ (day 28: 14.5 km, day 29: 15.3 km) A1

(c) $\frac{n}{2}[10 + 0.4(n-1)] \geqslant 500 \Rightarrow \frac{n}{2}(9.6 + 0.4n) \geqslant 500 \Rightarrow 0.2n^2 + 4.8n \geqslant 500$ M1

Multiply by 5: $n^2 + 24n - 2500 \geqslant 0$. Solving: $n = (-24 + \sqrt{10576}) \div 2$ dM1

$n = 39.4...$, so the total first reaches 500 km on $\boxed{\text{day 40}}$ ($S_{39} = 491.4$, $S_{40} = 512$) A1

(Total for Question 6 is 8 marks)

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7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(a) Show that the equation

$$3\sin\theta\tan\theta = 2 + 5\cos\theta$$

can be written in the form

$$8\cos^2\theta + 2\cos\theta - 3 = 0$$

(2)

(b) Hence solve, for $0 \leqslant x < 180^\circ$,

$$3\sin 2x\tan 2x = 2 + 5\cos 2x$$

giving your answers to one decimal place where appropriate.

(5)

(a) Use $\tan\theta = \frac{\sin\theta}{\cos\theta}$: $3\sin\theta \times \frac{\sin\theta}{\cos\theta} = 2 + 5\cos\theta$

Multiply by $\cos\theta$: $3\sin^2\theta = 2\cos\theta + 5\cos^2\theta$ M1

Use $\sin^2\theta = 1 - \cos^2\theta$: $3 - 3\cos^2\theta = 2\cos\theta + 5\cos^2\theta \Rightarrow 8\cos^2\theta + 2\cos\theta - 3 = 0$ A1*

(b) This is the equation in (a) with $\theta = 2x$, so $8\cos^2 2x + 2\cos 2x - 3 = 0$

Factorise: $(4\cos 2x + 3)(2\cos 2x - 1) = 0$ M1

$\cos 2x = \frac{1}{2}$ or $\cos 2x = -\frac{3}{4}$ A1

$0 \leqslant x < 180^\circ$ means $0 \leqslant 2x < 360^\circ$, so find every value of $2x$ in that range first:

$\cos 2x = \frac{1}{2}$: $2x = 60^\circ, 300^\circ$ $\cos 2x = -\frac{3}{4}$: $2x = 138.59^\circ$, $360^\circ - 138.59^\circ = 221.41^\circ$ M1

Halve each one: $x = 30^\circ, 150^\circ, 69.30^\circ, 110.70^\circ$ dM1

$x = 30^\circ, 69.3^\circ, 110.7^\circ, 150^\circ$ (none has $\cos 2x = 0$, so $\tan 2x$ is defined) A1

(Total for Question 7 is 7 marks)

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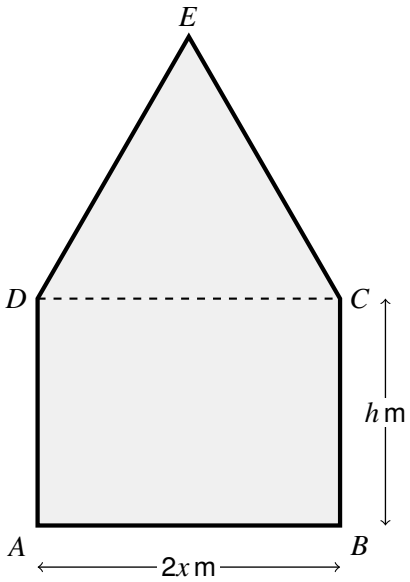

Figure 2

Figure 2 shows the design for a window $ABCDE$.

The window is a rectangle $ABCD$ with an equilateral triangle DCE on top.

$AB = 2x$ m and $BC = h$ m.

The total area of the window is 12 m^2.

A frame is fitted around the outside edge $ABCED$ of the window. The length of the frame is P metres.

(a) Show that

$$P = \frac{12}{x} + \left(6 - \sqrt{3}\right)x$$

(3)

(b) Use calculus to find the minimum value of P, giving your answer to 3 significant figures.

(3)

(c) Justify that the value you found in part (b) is a minimum.

(2)

(a) Rectangle: $2x \times h = 2xh$. Equilateral triangle, side $2x$: $\frac{1}{2}(2x)^2 \sin 60° = \sqrt{3}x^2$

Total area: $2xh + \sqrt{3}x^2 = 12 \Rightarrow h = \frac{12 - \sqrt{3}x^2}{2x}$ M1

Frame: $AB + BC + CE + ED + DA = 2x + h + 2x + 2x + h = 6x + 2h$ (DC is inside) M1

$P = 6x + \frac{12 - \sqrt{3}x^2}{x} = 6x + \frac{12}{x} - \sqrt{3}x = \frac{12}{x} + (6 - \sqrt{3})x$ as required A1*

(b) $P = 12x^{-1} + (6 - \sqrt{3})x \Rightarrow \frac{dP}{dx} = -12x^{-2} + 6 - \sqrt{3}$ M1

$\frac{dP}{dx} = 0$: $\frac{12}{x^2} = 6 - \sqrt{3} \Rightarrow x^2 = \frac{12}{6 - \sqrt{3}} = 2.811\ldots \Rightarrow x = 1.676\ldots$ dM1

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Question 8 continued

$P = \frac{12}{1.676...} + (6 - \sqrt{3}) \times 1.676... = 7.156... + 7.156... = \boxed{14.3 \text{ m}}$ A1

(c) $\frac{d^2P}{dx^2} = 24x^{-3} = \frac{24}{x^3}$ M1

At $x = 1.676...$, $\frac{24}{x^3} > 0$ (x is positive), so P is a minimum A1

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(Total for Question 8 is 8 marks)

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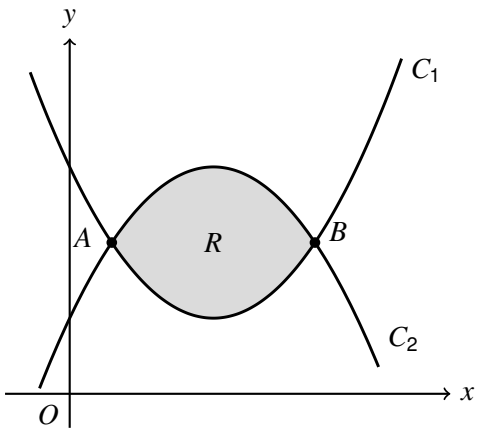

Figure 3

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Figure 3 shows the curves C_1 and C_2 with equations

$$C_1 : y = x^2 - 4x + 6 \qquad C_2 : y = 2 + 4x - x^2$$

The curves meet at the points A and B.

(a) Show that the x coordinates of A and B are $2 - \sqrt{2}$ and $2 + \sqrt{2}$

(3)

The region R, shown shaded in Figure 3, is bounded by C_1 and C_2.

(b) Use algebraic integration to find the exact area of R.

(4)

(a) At A and B the curves meet: $x^2 - 4x + 6 = 2 + 4x - x^2$ M1

$2x^2 - 8x + 4 = 0 \Rightarrow x^2 - 4x + 2 = 0$ A1

Complete the square: $(x-2)^2 - 4 + 2 = 0 \Rightarrow (x-2)^2 = 2 \Rightarrow x = 2 \pm \sqrt{2}$ as required A1*

(b) C_2 is on top between A and B: $(2 + 4x - x^2) - (x^2 - 4x + 6) = -2x^2 + 8x - 4$

Area $= \int (-2x^2 + 8x - 4)\,dx$ from $x = 2 - \sqrt{2}$ to $x = 2 + \sqrt{2}$ M1

$= \left[-\frac{2}{3}x^3 + 4x^2 - 4x \right]$ A1

Useful: $(2 \pm \sqrt{2})^2 = 6 \pm 4\sqrt{2}$, $(2 \pm \sqrt{2})^3 = 20 \pm 14\sqrt{2}$

$x = 2 + \sqrt{2}$: $-\frac{2}{3}(20 + 14\sqrt{2}) + 4(6 + 4\sqrt{2}) - 4(2 + \sqrt{2}) = \frac{8}{3} + \frac{8}{3}\sqrt{2}$

$x = 2 - \sqrt{2}$ gives $\frac{8}{3} - \frac{8}{3}\sqrt{2}$. Subtract: $\left(\frac{8}{3} + \frac{8}{3}\sqrt{2}\right) - \left(\frac{8}{3} - \frac{8}{3}\sqrt{2}\right)$ dM1

Area $= \frac{16}{3}\sqrt{2}$ A1

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Question 9 continued

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(Total for Question 9 is 7 marks)

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10. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = 2x + \frac{8}{x^2} \qquad x > 0$$

(a) Using calculus, find the coordinates of the stationary point of C and determine its nature.

(4)

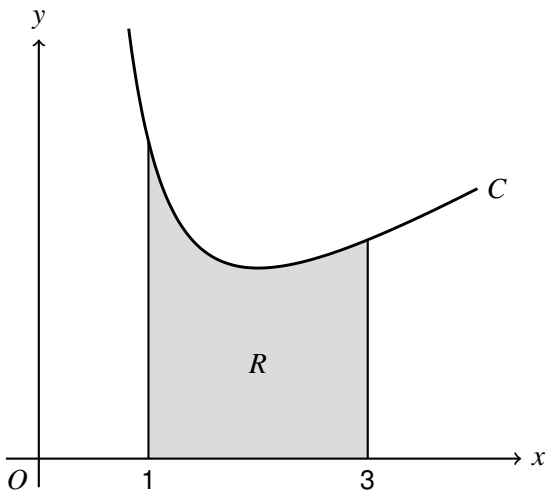

Figure 4

Figure 4 shows part of the curve C.

The region R, shown shaded in Figure 4, is bounded by C, the x-axis and the lines $x = 1$ and $x = 3$

(b) Use the trapezium rule with four strips of equal width to find an estimate for the area of R, giving your answer to 3 significant figures.

(3)

(c) Use algebraic integration to find the exact area of R.

(2)

(a) Write $y = 2x + 8x^{-2}$, so $\frac{dy}{dx} = 2 - 16x^{-3}$ M1

$\frac{dy}{dx} = 0$: $\frac{16}{x^3} = 2 \Rightarrow x^3 = 8 \Rightarrow x = 2$, $y = 4 + \frac{8}{4} = 6$ (2, 6) A1

$\frac{d^2y}{dx^2} = 48x^{-4} = \frac{48}{x^4}$ M1

At $x = 2$: $\frac{48}{16} = 3 > 0$, so (2, 6) is a minimum A1

(b) 4 strips from 1 to 3: $h = \frac{3-1}{4} = 0.5$. y values at $x = 1, 1.5, 2, 2.5, 3$:

10, 6.5556, 6, 6.28, 6.8889 B1

Area $\approx \frac{1}{2} \times 0.5 \times [10 + 6.8889 + 2(6.5556 + 6 + 6.28)]$ M1

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Question 10 continued

$= 0.25 \times [16.8889 + 37.6711] = 0.25 \times 54.56 = \boxed{13.6}$ (13.64) A1

(c) $\int (2x + 8x^{-2})\,dx = [\,x^2 - 8x^{-1}\,]$ between $x = 1$ and $x = 3$ M1

$= (9 - \frac{8}{3}) - (1 - 8) = \frac{19}{3} + 7 = \boxed{\frac{40}{3}}$ A1

(The trapezium rule gave too much: the curve bends upwards between $x = 1$ and 3.)

(Total for Question 10 is 9 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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