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INTERNATIONAL A LEVEL MATHEMATICS • FURTHER PURE 3 (WFM03)

Further Pure 3 Practice Paper 3

Edexcel IAL Further Pure Mathematics 3

AIMING FOR A/A*

Written in the style of the Pearson Edexcel International Advanced Level paper (WFM03/01) • Calculator allowed

A full 75-mark paper set above the hardest recent series: long questions, one printed result and a trap in every question

Time	Total	You must have
1 hour 30 minutes	**75 marks**	**Mathematical Formulae and Statistical Tables, calculator**

Instructions

- Use black ink or ball-point pen. Pencil may be used for diagrams and sketches.
- Answer **all** questions, in the spaces provided.
- Show enough working to make your method clear. Answers without working may not gain full credit.
- Give inexact answers to **three significant figures** unless the question says otherwise.

Information

- There are 7 questions in this paper. The total mark is 75.
- The marks for each question are shown in brackets.

Advice

- Sit the paper in one go, in 1 hour 30 minutes, with the mark scheme closed.
- Read each question carefully and try every question. Check your answers if you have time.
- Then mark it line by line with the mark scheme and fill in your score.

Track your score

Q	Topic	Marks	Score
1	Hyperbolic expressions in the form $R\sinh(x+\alpha)$	10	
2	An inverse hyperbolic function and its domain	10	
3	Diagonalising with a repeated eigenvalue	11	
4	A reflection in a plane and a parallel plane	11	
5	A reduction formula for $\tanh^n x$	11	
6	A tangent to an ellipse, an area and a locus	11	
7	Arc length and surface area by substitution	11	
	Total	**75**	

Rough grade guide (out of 75): A* **57** • A **51** • B **44** • C **37**

Based on recent WFM03 grade boundaries. Real boundaries change every series, so treat this as a guide, not a grade.

Mark scheme | Exam questions by topic | More practice papers

Name ..... Date .....

1. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

$$f(x) = 3\cosh x + 5\sinh x$$

(a) Explain why $f(x)$ cannot be written in the form $R\cosh(x+\alpha)$, where $R > 0$, and express $f(x)$ in the form $R\sinh(x+\alpha)$, where $R > 0$ and α is an exact natural logarithm.

(4)

(b) Hence, or otherwise, solve the equation $f(x) = 4$, giving your answer as a single natural logarithm.

(3)

(c) Solve the equation

$$5\cosh x + 3\sinh x = 4$$

and explain why it has exactly one solution.

(3)

(Total for Question 1 is 10 marks)

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2.

$$f(x) = \text{artanh}\left(\frac{x-1}{x+1}\right)$$

(a) Find the set of values of x for which $f(x)$ is defined.

(2)

(b) Find $f'(x)$ in its simplest form, and hence express $f(x)$ in terms of $\ln x$

(4)

(c) Use integration by parts to find the exact value of

$$\int_1^e x\,\text{artanh}\left(\frac{x-1}{x+1}\right)\mathrm{d}x$$

(4)

(Total for Question 2 is 10 marks)

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3.

$$\mathbf{A} = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$

(a) Show that $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ is an eigenvector of $\mathbf{A}$ and find the corresponding eigenvalue.

(2)

(b) Find the other eigenvalues of $\mathbf{A}$, showing that one of them is repeated.

(4)

(c) Find an orthogonal matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^{\mathrm{T}}\mathbf{AP} = \mathbf{D}$

(5)

(Total for Question 3 is 11 marks)

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4. The plane Π_1 has equation $2x - y + 2z = 3$ and the point A has coordinates $(5, -1, 5)$

(a) Find the coordinates of A', the reflection of A in Π_1

(5)

The plane Π_2 is parallel to Π_1, and the perpendicular distance between Π_1 and Π_2 is 2. Π_2 lies between A and Π_1

(b) Find a Cartesian equation of Π_2

(3)

(c) Find the shortest distance from A' to Π_2

(3)

(Total for Question 4 is 11 marks)

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5.

$$I_n = \int_0^{\ln 3} \tanh^n x \, \mathrm{d}x, \qquad n \geqslant 0$$

(a) Show that, for $n \geqslant 2$,

$$I_n = I_{n-2} - \frac{1}{n-1}\left(\frac{4}{5}\right)^{n-1}$$

(5)

(b) Find the exact value of I_1

(2)

(c) Find the exact value of I_5

(3)

(d) Without evaluating any more integrals, explain why $I_6 < I_5$

(1)

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Question 5 continued

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(Total for Question 5 is 11 marks)

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6.

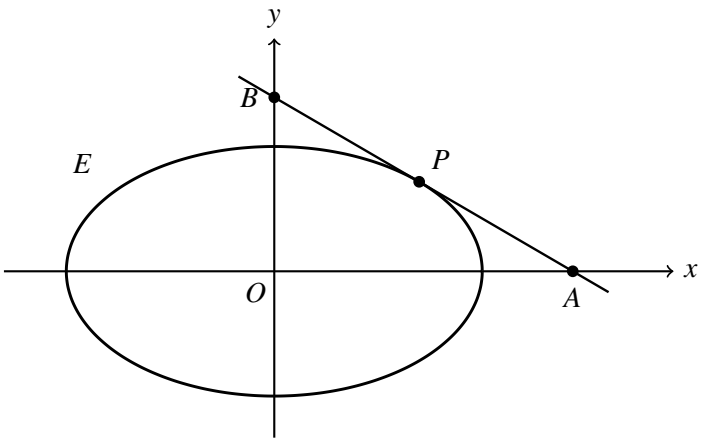

Figure 1

Figure 1 shows the ellipse E with equation $\frac{x^2}{25} + \frac{y^2}{9} = 1$. The point $P(5\cos\theta, 3\sin\theta)$, $0 < \theta < \frac{\pi}{2}$, lies on E. The tangent to E at P meets the x-axis at A and the y-axis at B

(a) Find an equation of the tangent to E at P, giving your answer in the form $px\cos\theta + qy\sin\theta = r$, where p, q and r are integers.

(4)

(b) Find the least possible area of triangle OAB, where O is the origin, and the exact coordinates of P when it occurs.

(4)

(c) The point M is the midpoint of AB. Find a Cartesian equation of the locus of M as θ varies, stating any restrictions on x and y

(3)

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Question 6 continued

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(Total for Question 6 is 11 marks)

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7. In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{1}{2}x^2, \qquad 0 \leqslant x \leqslant \sqrt{3}$$

(a) Use the substitution $x = \sinh u$ to find the exact length of C, giving your answer in the form $a\sqrt{3} + b\ln\left(2+\sqrt{3}\right)$, where a and b are rational numbers.

(5)

(b) The curve C is rotated through 2π radians about the x-axis. Find the exact area of the surface generated, giving your answer in the form $\pi\left(p\sqrt{3} - q\ln\left(2+\sqrt{3}\right)\right)$, where p and q are rational numbers.

(6)

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Question 7 continued

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(Total for Question 7 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS

END

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