



Numerical Methods

Questions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Pure Mathematics 3



Question 1

$$f(x) = x^3 - 2x - 5$$

Show that $f(x)$ has a root in the interval $[2, 3]$, justifying your answer.

(3 marks)

Total for question = 3 marks



Question 2

$$f(x) = e^x - 3x$$

(a) Show that $f(x)$ has a root in the interval $[0, 1]$.

(2 marks)

(b) Show that $f(x)$ has a root in the interval $[1, 2]$.

(2 marks)

Total for question = 4 marks



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Question 3

Show that the equation $\cos x = x$ has a root in the interval $[0.7, 0.8]$, where x is in radians.

(3 marks)

Total for question = 3 marks



Question 4

$$f(x) = x^3 - 3x + 1 = 0$$

(a) Show that $f(x)$ has a root in the interval $[1, 2]$.

(2 marks)

(b) Show that the equation can be rearranged to

$$x = (3x - 1)^{\frac{1}{3}}$$

(2 marks)

(c) Use the iteration formula

$$x_{n+1} = (3x_n - 1)^{\frac{1}{3}}$$

with $x_1 = 1.5$ to find the root in $[1, 2]$ correct to 3 decimal places.

(3 marks)

Total for question = 7 marks





Question 5

$$f(x) = x^2 - 5x + 2$$

(a) Show that $f(x)$ has a root in the interval $[4, 5]$.

(2 marks)

(b) Show that the equation $f(x) = 0$ can be rearranged to give

$$x = 5 - \frac{2}{x}$$

(2 marks)

(c) Use the iteration $x_{n+1} = 5 - 2/x_n$ with $x_1 = 4.5$ to find the root to 3 decimal places.

(3 marks)

Total for question = 7 marks





Question 6

$$f(x) = x^3 - x - 1$$

(a) Show that $f(x)$ has a root in the interval $[1, 2]$.

(2 marks)

(b) Show that the equation $f(x) = 0$ can be rearranged to give each of the following:

$$(i) \quad x = (x + 1)^{\frac{1}{3}} \qquad (ii) \quad x = x^3 - 1$$

(3 marks)

(c) Using $x_1 = 1.5$ for each, apply both iteration formulae and state which converges, giving the root to 3 decimal places.

(4 marks)

Total for question = 9 marks





Question 7

The equation $x^3 - 2x^2 - 1 = 0$ has a root near $x = 2$.

(a) Use the iteration formula

$$x_{n+1} = 2 + \frac{1}{x_n^2}$$

with $x_1 = 2$, to produce a table of values for x_1 to x_6 .

(3 marks)

(b) State the root to 2 decimal places.

(1 marks)

Total for question = 4 marks



Question 8

The iteration formula $x_{n+1} = \ln(x_n + 3)$ is used to find a root of the equation $f(x) = 0$.

(a) State the equation $f(x) = 0$ whose root is being found.

(1 marks)

(b) By considering $g(x) = \ln(x + 3)$ and showing that $|g'(x)| < 1$ near the root, explain why the iteration converges.

(3 marks)

(c) Starting with $x_1 = 1.5$, find the root to 3 decimal places.

(3 marks)

Total for question = 7 marks





Question 9

$$f(x) = 2 \sin x - x$$

(a) Show that $f(x)$ has a root in the interval $[1, 2]$.

(2 marks)

(b) Show that the equation can be rearranged to give $x = 2 \sin x$.

(1 marks)

(c) Use the iteration $x_{n+1} = 2 \sin x_n$ with $x_1 = 1.5$ to find the root to 3 decimal places.

(3 marks)

Total for question = 6 marks





Question 10

$$f(x) = e^x - 4x$$

(a) Show that $f(x)$ has a root in the interval $[2, 3]$.

(2 marks)

(b) Show that this root satisfies

$$x = \ln(4x)$$

(2 marks)

(c) Use the iteration $x_{n+1} = \ln(4x_n)$ with $x_1 = 2$ to find the root to 3 decimal places.

(3 marks)

Total for question = 7 marks





Question 11

$$f(x) = x + \ln x - 3$$

(a) Show that $f(x)$ has a root in the interval $[2, 3]$.

(2 marks)

(b) Show that the equation can be rearranged to

$$x = 3 - \ln x$$

(2 marks)

(c) Use the iteration $x_{n+1} = 3 - \ln x_n$ with $x_1 = 2.2$ to find the root to 3 decimal places.

(3 marks)

Total for question = 7 marks

