



eClassroom

A-LEVEL MATHEMATICS • PURE 2

Integration

Worksheet

Pearson Edexcel International A Level (WMA12) & A Level (9MA0) • Calculator allowed: show every step

Free Integration worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: definite integrals and areas • the trapezium rule • *Stretch:* regions between curves, below the axis, and proofs.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Areas between curves and lines, regions below the axis, equal areas and the trapezium rule.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

Full worked solutions

Exam questions by topic

Integration revision engine

Formula sheet builder

Name

Class

Date



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Evaluate $\int_1^3 (3x^2 + 2x) \, dx$

Integrate, then substitute the upper limit minus the lower limit. No +c

$$[x^3 + x^2]_1^3 = (27 + 9) - (1 + 1)$$

34

Your turn A1

Evaluate $\int_1^4 (4x^3 - 6\sqrt{x}) \, dx$

Worked example A2

Find the area of the region between $y = 4x - x^2$ and the x -axis.

The curve meets the axis at $x = 0$ and $x = 4$: these are the limits.

$$\left[2x^2 - \frac{1}{3}x^3 \right]_0^4 = 32 - \frac{64}{3}$$

$\frac{32}{3}$

Your turn A2

Find the area of the region between $y = 5x - x^2$ and the x -axis.

Worked example A3

Use the trapezium rule with 4 strips to estimate

$\int_1^3 \frac{1}{1+x^2} \, dx$, to 4 decimal places.

$$h = \frac{3-1}{4} = 0.5.$$

Ordinates: 0.5, 0.30769, 0.2, 0.13793, 0.1

$$\frac{0.5}{2} (0.5 + 0.1 + 2(0.64562))$$

0.4728

Your turn A3

Use the trapezium rule with 4 strips to estimate

$\int_0^2 3^x \, dx$, to 3 decimal places.



STRETCH

Worked example A4: below the axis

Find the area of the region between $y = x^2 - 4x$ and the x -axis.

$$\left[\frac{1}{3}x^3 - 2x^2 \right]_0^4 = \frac{64}{3} - 32 = -\frac{32}{3}$$

The region is below the axis, so the integral is negative: the area is its size.

$$\frac{32}{3}$$

Your turn A4

Find the area of the region between $y = x^3 - 9x$ and the x -axis for $0 \leq x \leq 3$

Worked example A5: between a curve and a line

Find the area between $y = x^2$ and $y = 2x$

They meet at $x = 0$ and $x = 2$; the line is on top. Integrate top minus bottom.

$$\int_0^2 (2x - x^2) dx = 4 - \frac{8}{3}$$

$$\frac{4}{3}$$

Your turn A5

Find the area between $y = x^2$ and $y = x + 6$

Worked example A6: an unknown limit

Given that $\int_1^k (2x + 1) dx = 18$ and $k > 1$, find k

$$[x^2 + x]_1^k = k^2 + k - 2 = 18, \text{ so } k^2 + k - 20 = 0$$

$(k + 5)(k - 4) = 0$; reject $k = -5$ because $k > 1$

$$k = 4$$

Your turn A6

Given that $\int_1^k (2x - 3) dx = 6$ and $k > 1$, find k

Worked example A7: over or under?

Will the trapezium rule overestimate or underestimate the area under $y = x^2$ for $1 \leq x \leq 3$?

The curve bends upwards (convex), so each trapezium lies above the curve.

an overestimate

Your turn A7

Will the trapezium rule overestimate or underestimate the area under $y = \sqrt{x}$ for $1 \leq x \leq 4$?



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Evaluate

(a) $\int_0^2 3x^2 \, dx$

(b) $\int_1^2 3x^2 \, dx$

(c) $\int_{-1}^2 3x^2 \, dx$

(d) $\int_2^0 3x^2 \, dx$

[4 marks]**B2** Evaluate

(a) $\int_1^4 \sqrt{x} \, dx$

(b) $\int_1^4 \frac{1}{\sqrt{x}} \, dx$

(c) $\int_1^2 \frac{2}{x^2} \, dx$

(d) $\int_1^4 \frac{x^2 + 2}{x^2} \, dx$

[4 marks]**B3** Find the area of the region between the x -axis and

(a) $y = 9 - x^2$

(b) $y = x^2 - 9$

(c) $y = 9 - x^2$, for $0 \leq x \leq 3$

(d) $y = (x - 1)(4 - x)$

[4 marks]**B4** The trapezium rule with 4 strips is used to estimate $\int_0^2 (x^2 + 1) \, dx$ (a) State the strip width h

(b) Find the estimate.

(c) Find the exact value of the integral.

(d) Explain why the estimate is too large.

[4 marks]

CORE

continued: regions

B5 The curve $y = x^2$ and the line $y = 4$ enclose a region R

- (a) Find where they meet. (b) Find the area under the curve (c) Find the area of R between these points.

[3 marks]

B6 $y = x(x - 2)$

(a) Evaluate $\int_0^2 x(x - 2) dx$

(b) Evaluate $\int_2^3 x(x - 2) dx$

(c) Evaluate $\int_0^3 x(x - 2) dx$

- (d) Find the total area between the curve and the x -axis for $0 \leq x \leq 3$

[4 marks]

STRETCH

B7 Find the value of k in each case.

(a) $\int_0^k 2x dx = 25, k > 0$

(b) $\int_0^k 3x^2 dx = 64$

(c) $\int_1^k (2x + 3) dx = 14, k > 1$

(d) $\int_k^4 \sqrt{x} dx = \frac{14}{3}, 0 < k < 4$

[4 marks]

B8 Given that $\int_1^3 f(x) dx = 10$, find

(a) $\int_1^3 2f(x) dx$

(b) $\int_1^3 (f(x) + 1) dx$

(c) $\int_3^1 f(x) dx$

(d) $\int_1^3 (f(x) - 3x) dx$

[4 marks]

B9 The curve $y = x^2 - 2x$ and the line $y = x$ enclose a region R

- (a) Find the x coordinates of the points where they meet. (b) State which graph is on top between these points.
(c) Write down an integral for the area of R (d) Find the area of R

[4 marks]

B10 $\int_0^2 x^3 dx$ is estimated by the trapezium rule.

- (a) Find the estimate with 2 strips. (b) Find the estimate with 4 strips. (c) Find the exact value.

- (d) Explain what (a), (b) and (c) show about the trapezium rule for this curve.

[4 marks]



C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Sam evaluates $\int_1^3 2x \, dx$ as $[x^2]_1^3 = 1 - 9 = -8$
Explain Sam's mistake, and give the correct value. [2 marks]
- C2** Lily says the area between $y = x^2 - 4$ and the x -axis is $-\frac{32}{3}$
Explain Lily's mistake. [2 marks]
- C3** Kai finds the area between $y = x^2 - 1$ and the x -axis for $0 \leq x \leq 2$ by working out $\int_0^2 (x^2 - 1) \, dx = \frac{2}{3}$
Explain why Kai's answer is wrong, and find the correct area. [3 marks]
- C4** Jo uses the trapezium rule with 4 strips for $\int_1^3 x^2 \, dx$, taking the strip width as $\frac{3}{4}$
Explain Jo's mistake, and find the correct estimate. [3 marks]



STRETCH

- C5** Max says the trapezium rule underestimates $\int_1^2 x^3 \, dx$ because the curve $y = x^3$ is increasing.

Explain why Max's reason is wrong, and state whether the estimate is too large or too small. [3 marks]

- C6** The region R is bounded by the curve $y = 6 - x^2$, the line $y = 2$ and the y -axis, with $x \geq 0$. Ali gives the area of R as $\int_0^2 (6 - x^2) \, dx = \frac{28}{3}$

Explain what Ali has forgotten, and find the area of R [3 marks]

- C7** Jo finds that $\int_{-2}^2 x^3 \, dx = 0$ and says the curve $y = x^3$ encloses no area with the x -axis for $-2 \leq x \leq 2$

Explain why Jo is wrong, and find the total area. [3 marks]

- C8** (a) Show that the area under $y = x^2$ from $x = 0$ to $x = a$, where $a > 0$, is one third of the area of the rectangle with corners $(0, 0)$ and (a, a^2)

(2)

(b) Prove that, for any positive integer n , the area under $y = x^n$ from $x = 0$ to $x = a$ is $\frac{1}{n+1}$ of the area of the rectangle with corners $(0, 0)$ and (a, a^n)

(3)

[5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE

Secure the method

- D1** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Use algebraic integration to find the exact value of the definite integral

$$\int_1^4 \left(6\sqrt{x} - \frac{2}{x^2} \right) dx$$

You must write each term as a power of x first, and show the substituted limits clearly.

[3 marks]

- D2** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 1 shows the curve C with equation $y = 3 + 2x - x^2$. The curve meets the x -axis at the points A and B . The region R , shown shaded, is bounded by C and the x -axis.

- (a) Find the x coordinates of A and B

(1)

- (b) Use algebraic integration to find the exact area of R , giving your answer as a fraction in its simplest form.

(3)

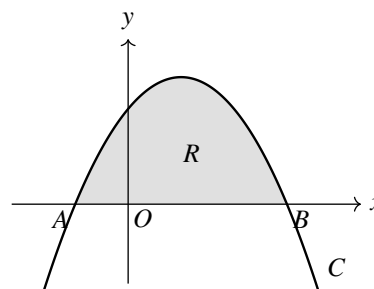


Figure 1

[4 marks]

D3 $y = \frac{4}{1+x^2}$

- (a) Complete the table, giving each value to 4 decimal places.

(1)

x	0	0.25	0.5	0.75	1
y	4		3.2		2

- (b) Use the trapezium rule, with all the values of y in the table, to find an estimate for $\int_0^1 \frac{4}{1+x^2} dx$, giving your answer to 4 decimal places. (The exact value of this integral is π , which you are not expected to show.)

(2)

[3 marks]



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

- D4** Figure 2 shows the curve C with equation $y = 8 - x^2$ and the line l with equation $y = 2x$. The curve and the line meet at the points P and Q . The region R , shown shaded, is bounded by C and l .

Find the x coordinates of P and Q , and then use algebraic integration to find the exact area of R , showing clearly which function you subtract from which.

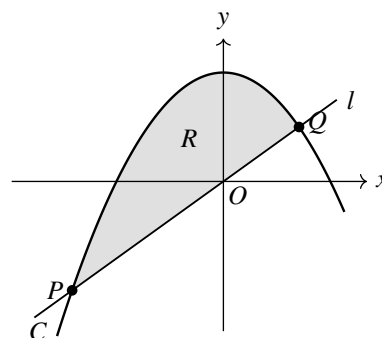


Diagram NOT accurately drawn

Figure 2

[3 marks]

STRETCH

- D5** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 3 shows the curve with equation $y = \sqrt{x}(4 - x)$, $x \geq 0$, which meets the x -axis at the origin and at $x = 4$.

The region R , shown shaded, is bounded by the curve and the x -axis.

Use algebraic integration to find the exact area of R , writing the equation of the curve as a sum of powers of x first and giving your answer as a single fraction.

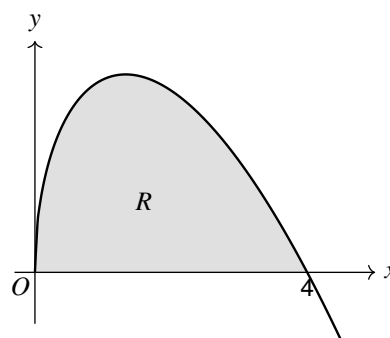


Figure 3

[4 marks]

- D6** Figure 4 shows the curve with equation $y = x^2 - 6x + 5$ for $0 \leq x \leq 5$. The region R_1 lies above the x -axis and the region R_2 lies below it.

(a) Find the values of x where the curve meets the x -axis.

(1)

(b) Use algebraic integration to find the total area of R_1 and R_2 , explaining why the two regions must be found separately.

(3)

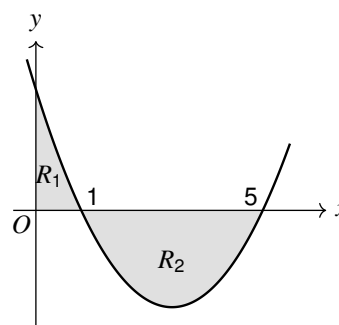


Figure 4

[4 marks]



- D7** Figure 5 shows the curve $y = x^2 - 3x$. The region R_1 is bounded by the curve and the x -axis for $0 \leq x \leq 3$, and the region R_2 is bounded by the curve, the x -axis and the line $x = k$, where $k > 3$. The areas of R_1 and R_2 are equal.

- (a) Explain why $\int_0^k (x^2 - 3x) \, dx = 0$
- (b) Hence use algebraic integration to find the exact value of k , explaining why any other solution is rejected.

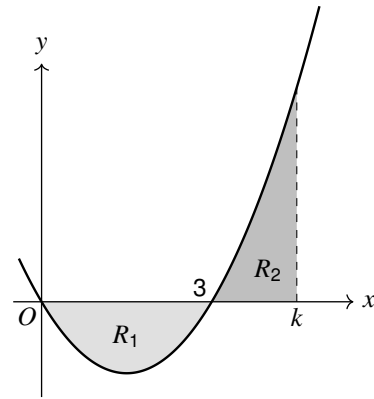


Figure 5

[4 marks]

- D8** Figure 6 shows the curve C with equation $y = x^2$ and the line l , which is the tangent to C at the point $P(2, 4)$. The line l meets the x -axis at T . The region R , shown shaded, is bounded by C , l and the x -axis.

- (a) Find an equation of l and the coordinates of T
- (b) Find the exact area of R , showing clearly how you use the area of a triangle.

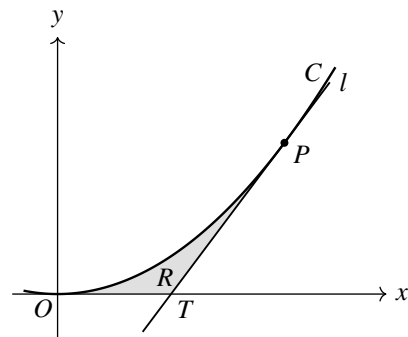


Figure 6

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA12. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

- E1** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

A student needs the area of the region under the curve $y = \frac{(x+2)^2}{\sqrt{x}}$ between the lines $x = 1$ and $x = 4$

- (a) Write $\frac{(x+2)^2}{\sqrt{x}}$ in the form $x^p + 4x^q + 4x^r$, where p , q and r are constants.

.....
(2)

- (b) Hence use algebraic integration to find the exact value of

$$\int_1^4 \frac{(x+2)^2}{\sqrt{x}} dx$$

giving your answer as a fraction in its simplest form.

.....
(3)

(Total for Question E1 is 5 marks)



E2 The function $\sqrt{x^3+1}$ cannot be integrated by the methods of this course, so its integral is to be estimated.

$$y = \sqrt{x^3+1}$$

(a) Complete the table below, giving each value of y to 4 decimal places.

x	0	0.5	1	1.5	2
y	1	1.0607		2.0917	3

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an estimate for

$$\int_0^2 \sqrt{x^3+1} \, dx, \text{ giving your answer to 3 decimal places.}$$

.....

(3)

(c) Using your answer to part (b), and making your method clear, estimate

$$(i) \int_0^2 (5 - \sqrt{x^3+1}) \, dx \quad (ii) \int_1^3 \sqrt{(x-1)^3+1} \, dx$$

(2)

(Total for Question E2 is 6 marks)



STRETCH

- E3** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 7 shows the curve C with equation

$$y = x(x-2)^2 \quad \text{and the line } l \text{ with equation } y = x$$

The line l meets C at the origin O and at the points A and B . The region R , shown shaded, is bounded by C and l between O and A

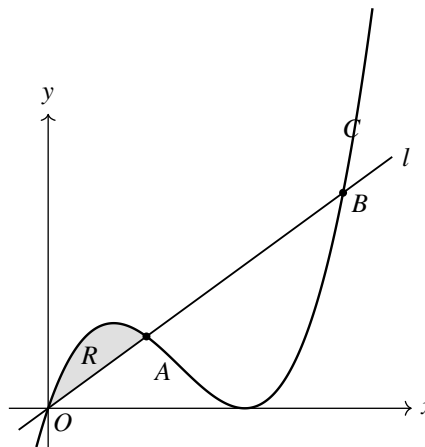


Figure 7

- (a) Use algebra to find the coordinates of A and B

.....
(3)

- (b) Use algebraic integration to find the exact area of R , showing clearly which function you subtract from which.

.....
(4)

- (c) Explain, with a reason, which of C and l is higher between A and B

(1)

(Total for Question E3 is 8 marks)

Want more exam practice? The **Exam Questions by Topic** booklets have 10 more full exam questions on Pure 2 Chapter 8, with a full mark scheme.

[Open the booklet](#)

Finished? Mark your work with the **full worked solutions**, fill in your scores on the front page, then keep the skill sharp with unlimited questions on the **Integration revision engine**.



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk