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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

Pure Mathematics 4

Exam Questions by Topic

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Multi-step exam questions at the difficulty of the real WMA14 paper, with a full mark scheme

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	own PDF	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	own PDF	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	own PDF	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	2	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	own PDF	
6 Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	own PDF	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	own PDF	

[Full mark scheme](#)

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CHAPTER 4 • KEY SKILLS

Binomial Expansion

Specification P4: Sequences and series • 10 questions • 81 marks • about 97 minutes

1 The expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots \text{ for any rational } n, \text{ valid for } |x| < 1$$

2 Replacing x

In $(1-4x)^{-3}$ use $(-4x)$ in every term, with brackets: $(-4x)^2 = 16x^2$ and $(-4x)^3 = -64x^3$

3 Taking out a^n

$$(9+4x)^{-\frac{1}{2}} = 9^{-\frac{1}{2}} \left(1 + \frac{4}{9}x\right)^{-\frac{1}{2}} = \frac{1}{3} \left(1 + \frac{4}{9}x\right)^{-\frac{1}{2}}$$

Not $9 \left(1 + \frac{4}{9}x\right)^{-\frac{1}{2}}$

4 Validity

$(a+bx)^n$ is valid for $|x| < \left|\frac{a}{b}\right|$. Use a strict inequality, no $\pm$. For two series, take the narrower interval.

5 Unknown constants

Write each coefficient in terms of the unknowns and compare, e.g. $nk = -2$ and $\frac{n(n-1)}{2}k^2 = 6$

6 Products

$(a+bx)(1+px+qx^2)$: collect each power of x carefully. Only keep the terms asked for.

7 Partial fractions first

Split, then expand each part:

$$\frac{3}{x-2} = -\frac{3}{2} \left(1 - \frac{x}{2}\right)^{-1}$$

8 Approximations

Choose x inside the valid interval, find what the original expression equals (it may be a reciprocal), then substitute into the series.

9 Accuracy

Percentage error

$$= \frac{|\text{approx} - \text{exact}|}{\text{exact}} \times 100. \text{ The smaller } |x|, \text{ the better the approximation.}$$

Skills map every skill above is tested at least once; interleaved skills come from GCSE or other chapters

Skill	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10
Marks	5	6	6	6	6	11	11	10	10	10
1 The expansion	•	•	•	•	•	•	•	•	•	•
2 Replacing x	•	•		•	•	•				•
3 Taking out a^n	•			•	•			•	•	
4 Validity	•		•	•		•	•	•	•	•
5 Unknown constants		•	•					•		
6 Products		•				•				
7 Partial fractions first				•			•			
8 Approximations					•	•		•	•	
9 Accuracy						•			•	•
Interleaved: partial fractions (Chapter 2)				•			•			
Interleaved: surds, indices and simultaneous equations (P1)			•		•	•		•		
Interleaved: circles: tangent and radius (P2)									•	
Interleaved: proof by contradiction, completing the square (Chapter 1, P1)										•

Exam difficulty. Questions 1 to 5 check each skill on its own. Questions 6 to 10 are multi-step questions built like the last questions on a real WMA14 paper, where one part leads into the next and skills from different topics are combined.

Must learn. Show the unsimplified binomial coefficients, then simplify each term. Take out a^n , not a . When approximating, check what the original expression is equal to at your value of x , and give a fraction when a rational answer is asked for.



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1. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of

$$\frac{1}{\sqrt{9+4x}}$$

giving each coefficient as a fraction in its simplest form.

(4)

- (b) State the range of values of x for which this expansion is valid.

(1)

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Question 1 continued

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(Total for Question 1 is 5 marks)



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2. (a) Find the binomial expansion of $(1 - 6x)^{\frac{1}{3}}$ in ascending powers of x , up to and including the term in x^2 , simplifying each term.

(3)

Given that

$$(a + bx)(1 - 6x)^{\frac{1}{3}} = 3 - 7x + cx^2 + \dots$$

where a , b and c are constants,

- (b) find the values of a , b and c .

(3)

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Question 2 continued

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(Total for Question 2 is 6 marks)



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3. In the binomial expansion of $(1 + kx)^n$, where k and n are constants and n is not a positive integer, the coefficient of x is -2 and the coefficient of x^2 is 6

(a) Find the value of k and the value of n .

(4)

(b) Find the coefficient of x^3

(1)

(c) State the range of values of x for which the expansion is valid.

(1)

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Question 3 continued

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(Total for Question 3 is 6 marks)



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4.

$$f(x) = \frac{x+7}{(1+x)(x-2)}$$

- (a) Express $f(x)$ in partial fractions. (2)
- (b) Hence find the series expansion of $f(x)$, in ascending powers of x , up to and including the term in x^2 . Give each coefficient as a simplified fraction. (3)
- (c) State the range of values of x for which your expansion is valid. (1)

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Question 4 continued

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(Total for Question 4 is 6 marks)



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5. (a) Find the first four terms, in ascending powers of x , of the binomial expansion of $(4 - x)^{-\frac{1}{2}}$, giving each coefficient in its simplest form. (4)
- (b) By substituting $x = 1$ into your expansion, find an approximation to $\sqrt{3}$, giving your answer as a fraction. (2)

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Question 5 continued

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Question 6 continued

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Question 6 continued

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(Total for Question 6 is 11 marks)



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7.

$$f(x) = \frac{6x^2 + 3}{(1+x)(1-2x)^2} \quad |x| < \frac{1}{2}$$

- (a) Find the values of the constants A , B and C such that

$$f(x) \equiv \frac{A}{1+x} + \frac{B}{1-2x} + \frac{C}{(1-2x)^2}$$

(4)

- (b) Hence find the series expansion of $f(x)$, in ascending powers of x , up to and including the term in x^2

(4)

- (c) Explain why the expansion is valid only for $|x| < \frac{1}{2}$

(1)

- (d) Show that the coefficient of x^n in the expansion of $f(x)$ is $(-1)^n + (3n+2)2^n$

(2)

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Question 7 continued

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(Total for Question 7 is 11 marks)



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8. The binomial expansion of

$$(2 - kx)^{-3} \quad |kx| < 2$$

where k is a positive constant, may be written as $A + Bx + Cx^2 + Dx^3 + \dots$

- (a) Find the value of A .

(1)

Given that $C = 2B$,

- (b) show that $k^2 - 2k = 0$

(3)

- (c) Hence find the value of k and the value of D .

(3)

- (d) State the range of values of x for which the expansion is valid.

(1)

- (e) Use the expansion, with a suitable value of x , to find an approximation to $\frac{1}{1.9^3}$, giving your answer to 4 decimal places.

(2)



Question 8 continued

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Question 8 continued

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9.

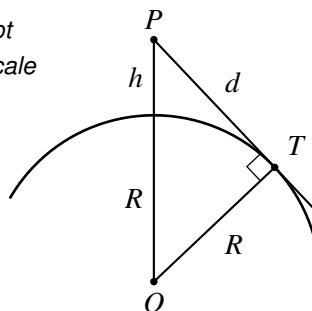
Diagram not
drawn to scale**Figure 1**

Figure 1 shows the Earth, modelled as a sphere with centre O and radius R km. An observer at P is h km above the surface. The line PT touches the surface at T , so the distance to the horizon is $d = PT$ km.

(a) Show that $d = \sqrt{2Rh} \left(1 + \frac{h}{2R} \right)^{\frac{1}{2}}$ (2)

(b) Hence show that, for small values of h ,

$$d \approx \sqrt{2Rh} \left(1 + \frac{h}{4R} - \frac{h^2}{32R^2} \right)$$

(3)

(c) State, in terms of R , the values of h for which the expansion is valid. (1)

A simpler model uses $d \approx \sqrt{2Rh}$. Take $R = 6400$

(d) Find the percentage error in the simpler model for an observer at a height of 400 km, giving your answer to 2 significant figures. (3)

(e) Explain why the simpler model always underestimates d . (1)



Question 9 continued

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Question 9 continued

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10. (a) Find the binomial expansion of $(1 + 2x)^{\frac{3}{2}}$ in ascending powers of x , up to and including the term in x^3 , simplifying each term. (4)
- (b) State the range of values of x for which this expansion is valid. (1)

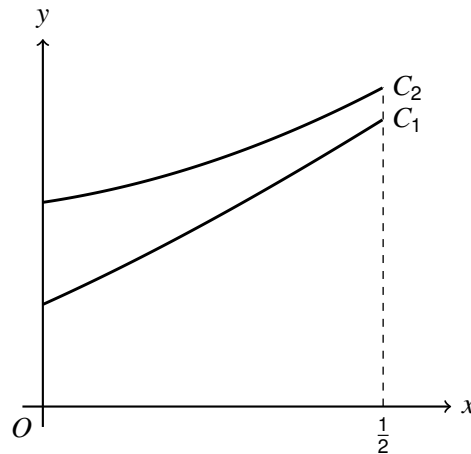


Figure 2

Figure 2 shows the curves C_1 and C_2 , for $0 \leq x \leq \frac{1}{2}$, with equations

$$C_1: y = 1 + 3x + \frac{3}{2}x^2 - \frac{1}{2}x^3 \quad C_2: y = 2 + x + \frac{5}{2}x^2$$

The curve C_1 is the approximation to $y = (1 + 2x)^{\frac{3}{2}}$ given by part (a).

- (c) Use algebra and proof by contradiction to prove that C_1 and C_2 do not meet for $0 < x < \frac{1}{2}$. (4)
- (d) Explain why part (c) does not prove that the curve with equation $y = (1 + 2x)^{\frac{3}{2}}$ does not meet C_2 for $0 < x < \frac{1}{2}$. (1)



Question 10 continued

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(Total for Question 10 is 10 marks)**TOTAL FOR CHAPTER 4 IS 81 MARKS****Join students already acing their exams.**Book your FREE lesson at www.eclassroom.org.uk