Straight Line Graphs

Worked Solutions

Suitable for:

Pearson Edexcel International A-Level (iAL) — Pure Mathematics 1

Question 1 — Worked Solution

(a) Equation of line LM

Gradient of LM:

$$m = \frac{-3-5}{8-2} = \frac{-8}{6} = -\frac{4}{3}$$

Using point M(8, –3):

$$y-(-3) = -\frac{4}{3}(x-8) \;\Rightarrow\; 3y+9 = -4x+32$$

$$4x+3y-23=0$$

$$\therefore 4x+3y-23=0$$

(b) Finding p

Angle LMN = 90° so MN ⊥ LM. Gradient of MN = 3/4 (negative reciprocal of –4/3).

N = (16, p). Using M(8, –3):

$$\frac{p-(-3)}{16-8} = \frac{3}{4} \;\Rightarrow\; \frac{p+3}{8} = \frac{3}{4}$$

$$p+3=6$$

$$\therefore p=3$$

(c) y-coordinate of K

In rectangle LMNK, LK is parallel and equal to MN, and KN is parallel to LM.

Vector MN = (16–8, 3–(–3)) = (8, 6). So K = L + (8, 6) = (2+8, 5+6):

$$\therefore y\text{-coordinate of } K = 11$$

Question 2 — Worked Solution

(a) Equation of *l*2

*l*1: 3x + 4y = 24, so gradient of *l*1 = –3/4.

*l*2 is perpendicular: gradient = 4/3. *l*2 passes through O(0,0):

$$\therefore y = \frac{4}{3}x \quad (\text{equivalently } 4x - 3y = 0)$$

(b) Area of triangle OBC

B: *l*1 crosses y-axis (x = 0): 4y = 24 → y = 6. So B = (0, 6).

C: solve *l*1 and *l*2 simultaneously:

$$3x + 4 \cdot \frac{4x}{3} = 24 \;\Rightarrow\; \frac{9x + 16x}{3} = 24 \;\Rightarrow\; 25x = 72$$

$$x = \frac{72}{25}, \quad y = \frac{4}{3} \cdot \frac{72}{25} = \frac{96}{25}$$

Area of triangle OBC: OB is along the y-axis (length 6). Perpendicular height = x-coordinate of C = 72/25.

$$\text{Area} = \frac{1}{2} \times 6 \times \frac{72}{25} = \frac{216}{25}$$

$$\therefore \text{Area} = \frac{216}{25}$$

Question 3 — Worked Solution

(a) Equation of *l*2

Gradient of *l*1 through P(0,3) and Q(4,11):

$$m_1 = \frac{11-3}{4-0} = \frac{8}{4} = 2$$

Gradient of *l*2 = –1/2 (perpendicular). Through Q(4, 11):

$$y - 11 = -\frac{1}{2}(x-4) \;\Rightarrow\; 2y - 22 = -(x-4)$$

$$x + 2y - 26 = 0$$

$$\therefore x + 2y - 26 = 0$$

(b) Coordinates of R

R is where *l*2 meets x-axis (y = 0):

$$x + 0 - 26 = 0 \;\Rightarrow\; x = 26$$

$$\therefore R = (26,\ 0)$$

(c) Area of quadrilateral ORQP

Use the shoelace formula with O(0,0), R(26,0), Q(4,11), P(0,3):

$$\text{Area} = \frac{1}{2}|(x_O(y_R - y_P) + x_R(y_Q - y_O) + x_Q(y_P - y_R) + x_P(y_O - y_Q))|$$

$$= \frac{1}{2}|0(0-3) + 26(11-0) + 4(3-0) + 0(0-11)|$$

$$= \frac{1}{2}|0 + 286 + 12 + 0| = \frac{298}{2}$$

$$\therefore \text{Area of } ORQP = 149$$

Question 4 — Worked Solution

(a) Equation of *l*2

*l*1: 2y = 3x – 6, so gradient = 3/2.

P: x = 6 → 2y = 18 – 6 = 12 → y = 6. So P = (6, 6).

Gradient of *l*2 = –2/3 (perpendicular to *l*1). Through P(6, 6):

$$y - 6 = -\frac{2}{3}(x - 6) \ \Rightarrow \ 3y - 18 = -2x + 12$$

$$2x + 3y - 30 = 0$$

$$\therefore 2x + 3y - 30 = 0$$

(b) Area of triangle SPT

S: *l*1 meets x-axis (y = 0): 0 = 3x – 6 → x = 2. S = (2, 0).

T: *l*2 meets x-axis (y = 0): 2x – 30 = 0 → x = 15. T = (15, 0).

ST = 15 – 2 = 13 (base along x-axis).

Height = y-coordinate of P = 6.

$$\text{Area} = \frac{1}{2} \times 13 \times 6 = 39$$

$$\therefore \text{Area of triangle } SPT = 39$$

Question 5 — Worked Solution

(a) Gradient of *l*1

Rearrange 4x – 3y + 24 = 0:

$$3y = 4x + 24 \;\Rightarrow\; y = \frac{4}{3}x + 8$$

$$\therefore \text{Gradient of } l_1 = \frac{4}{3}$$

(b) Equation of *l*2

B: *l*1 crosses y-axis (x = 0): y = 8. So B = (0, 8).

Gradient of *l*2 = –3/4. Through B(0, 8):

$$y = -\frac{3}{4}x + 8 \;\Rightarrow\; 3x + 4y - 32 = 0$$

$$\therefore 3x + 4y - 32 = 0$$

(c) Area of triangle ABC

A: *l*1 crosses x-axis (y = 0): 4x + 24 = 0 → x = –6. A = (–6, 0).

C: *l*2 crosses x-axis (y = 0): 3x = 32 → x = 32/3. C = (32/3, 0).

AC = 32/3 – (–6) = 32/3 + 6 = 50/3 (base along x-axis).

Height = y-coordinate of B = 8.

$$\text{Area} = \frac{1}{2} \times \frac{50}{3} \times 8 = \frac{200}{3}$$

$$\therefore \text{Area of triangle } ABC = \frac{200}{3}$$

Question 6 — Worked Solution

(a) Equation of *l*2

*l*1: y = –3x + 5, so gradient = –3.

Gradient of *l*2 = 1/3 (perpendicular). Through (6, 4):

$$y - 4 = \frac{1}{3}(x - 6) \Rightarrow 3y - 12 = x - 6$$

$$x - 3y + 6 = 0$$

$$\therefore x - 3y + 6 = 0$$

(b) Coordinates of A and B

A: *l*2 meets x-axis (y = 0): x + 6 = 0 → x = –6.

B: *l*2 meets y-axis (x = 0): –3y + 6 = 0 → y = 2.

$$\therefore A = (-6,\ 0), \quad B = (0,\ 2)$$

(c) Area of triangle OAB

O is the origin. OA lies along x-axis (length 6). Height = y-coordinate of B = 2.

$$\text{Area} = \frac{1}{2} \times 6 \times 2 = 6$$

$$\therefore \text{Area of triangle } OAB = 6$$