



eClassroom

A-LEVEL MATHEMATICS • PURE 3

Trigonometric Formulae

Worked Solutions

Pearson Edexcel International A Level (WMA13) & A Level (9MA0) • Calculator allowed: show every step

Full worked solutions for the free Trigonometric Formulae worksheet, A-Level and International A-Level Maths



Tap a section to jump straight to it

Full worked solutions for every question on the Pure 3 Trigonometric Formulae worksheet, in the same order and with the same question numbers.

How to mark your work. Tick each step you got right, not just the final answer. At A-level most marks are for method, so a wrong final answer can still earn most of the marks. If you lost marks, find the *first* line where your working is different from ours. That is the thing to fix.

In Section E: **M** = method mark, **dM** = method mark that depends on the previous M mark, **A** = accuracy mark (needs the M mark before it), **B** = independent mark.

Question sheet

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A**Your turn answers**

Compare each line of your working with the solution.

CORE

$$\text{A1} \quad \cos(60^\circ + 45^\circ) = \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$$

$$\text{A2} \quad \sin A = \frac{12}{13}: \sin 2A = 2 \cdot \frac{12}{13} \cdot \left(-\frac{5}{13}\right) = \boxed{-\frac{120}{169}}; \cos 2A = \frac{25 - 144}{169} = \boxed{-\frac{119}{169}}$$

$$\text{A3} \quad 2\cos^2 x - \cos x - 1 = 0, (2\cos x + 1)(\cos x - 1) = 0: \boxed{0^\circ, 120^\circ, 240^\circ}$$

STRETCH

$$\text{A4} \quad R\cos\alpha = 5, R\sin\alpha = 12: R = 13, \tan\alpha = \frac{12}{5}: \boxed{13\cos(x + 67.38^\circ)}$$

$$\text{A5} \quad \cos(x + 67.38^\circ) = 0.5, 67.38^\circ \leq x + 67.38^\circ < 427.38^\circ: x + 67.38^\circ = 300^\circ \text{ or } 420^\circ: \boxed{232.6^\circ, 352.6^\circ}$$

$$\text{A6} \quad \text{Least when } \cos(x + 67.38^\circ) = -1, x + 67.38^\circ = 180^\circ: \boxed{-13 \text{ at } x = 112.6^\circ}$$

$$\text{A7} \quad \frac{2\sin x \cos x}{1 + 2\cos^2 x - 1} = \frac{2\sin x \cos x}{2\cos^2 x} = \frac{\sin x}{\cos x} = \tan x \quad \boxed{\text{as required}}$$



B**Fluency solutions**

One mark for each correct part. The notes in *italics* explain why each sequence was chosen.

CORE

B1 (a) $\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x$ (b) $\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x$ (c) $\frac{\tan x + 1}{1 - \tan x}$ (d) $\frac{\sqrt{2}}{2} (\sin x - \cos x)$

The sign in the cos formula is the opposite of the sign in the angle.

B2 (a) $\frac{\sqrt{6} - \sqrt{2}}{4}$ (b) $-\frac{\sqrt{6} + \sqrt{2}}{4}$ (c) $2 - \sqrt{3}$ (d) $\frac{\sqrt{6} + \sqrt{2}}{4}$

Split each angle into 30° , 45° , 60° and their combinations.

B3 (a) $\sin 6x$ (b) $\cos 4x$ (c) $\cos x$ (d) $\tan 4x$

The double angle formulae work for any angle: $2 \sin \theta \cos \theta = \sin 2\theta$ with $\theta = 3x$

B4 $\sin A = \frac{3}{5}$, $\cos A = \frac{4}{5}$, $\sin B = \frac{12}{13}$

(a) $\frac{63}{65}$ (b) $\frac{56}{65}$ (c) $\frac{24}{25}$ (d) $-\frac{119}{169}$

Find all four ratios from the triangles first, then substitute.

B5 (a) $2 \sin^2 x + \sin x - 1 = 0$: $30^\circ, 150^\circ, 270^\circ$ (b) $\cos x(2 \sin x - 1) = 0$: $30^\circ, 90^\circ, 150^\circ, 270^\circ$
(c) $2 \cos^2 x + 3 \cos x + 1 = 0$: $120^\circ, 180^\circ, 240^\circ$

Choose the form of $\cos 2x$ that leaves only one function.

B6 (a) $2 \sin(x + 60^\circ)$ (b) $2 \cos(x + 30^\circ)$ (c) $2\sqrt{2} \sin(x - 45^\circ)$ (d) $17 \cos(x - 61.93^\circ)$

R comes from Pythagoras; $\tan \alpha$ is the ratio of the coefficients in the order the expansion gives.

STRETCH

B7 (a) 13 (b) -13 (c) 33 (d) least denominator $20 - 13 = 7$: $\frac{26}{7}$

A fraction is largest when its positive denominator is smallest.



B8 (a) $\sin^2 x + 2 \sin x \cos x + \cos^2 x$ (b) $(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x)$ (c) $\frac{\cos^2 x - \sin^2 x}{\sin x \cos x} = \frac{2 \cos 2x}{\sin 2x}$
 (d) $\sin(2x + x)$ expanded with $\cos^2 x = 1 - \sin^2 x$

each identity holds

Pick the form of $\cos 2x$ that makes the target appear.

B9 (a) $\tan x = \frac{1}{2}$: $26.6^\circ, 206.6^\circ$ (b) $\tan x = \sqrt{3}$: $60^\circ, 240^\circ$
 (c) $\tan x = \frac{1}{2 - \sqrt{3}} = 2 + \sqrt{3}$: $75^\circ, 255^\circ$ (d) $\tan x = -\frac{\sqrt{3}}{3}$: $150^\circ, 330^\circ$

Expand, collect $\sin x$ and $\cos x$, then divide to reach $\tan x$

B10 (a) $\frac{1 + \cos 2x}{2}$ (b) $\frac{1 - \cos 2x}{2}$ (c) $\frac{1 + \frac{\sqrt{3}}{2}}{2} = \frac{2 + \sqrt{3}}{4}$ (d) $\frac{2 - \sqrt{2}}{4}$

These forms are also the key to integrating $\sin^2 x$ and $\cos^2 x$ later.

C

Reasoning solutions

Your wording can be different. Check that your reason makes the same point.

CORE

C1 sin of a sum is not the sum of the sines: $\sin(45^\circ + 30^\circ) = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$

C2 The 2 is missing: $\cos^2 x - \sin^2 x, 2 \cos^2 x - 1, 1 - 2 \sin^2 x$

C3 $R \cos \alpha = 7$ and $R \sin \alpha = 24$, so $\tan \alpha = \frac{24}{7}$, not $\frac{7}{24}$: $R = 25, \alpha = 1.287$

C4 A positive fraction is smallest when its denominator is largest, at $\cos x = 1$: **minimum** $\frac{5}{43}$

STRETCH

C5 Dividing by $\sin x$ loses $\sin x = 0$. $\sin x(4 \cos x - 3) = 0$: $0^\circ, 41.4^\circ, 180^\circ, 318.6^\circ$

C6 $30^\circ \leq 2x + 30^\circ < 390^\circ$, so $2x + 30^\circ = 60^\circ$ or 300° : $x = 15^\circ, x = 135^\circ$



C7 Ali assumes the result. $\cos(A + A) = \cos^2 A - \sin^2 A = (1 - \sin^2 A) - \sin^2 A = \boxed{1 - 2\sin^2 A}$

C8 (a) $\tan(A + B) = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}$; divide top and bottom by $\cos A \cos B$ **as required**

(b) $\tan 2A \equiv \frac{2 \tan A}{1 - \tan^2 A}$ (c) $\frac{\sqrt{3} - 1}{1 + \sqrt{3}} = \boxed{2 - \sqrt{3}}$

D**Problem solving solutions**

There is often more than one correct method. Any complete, correct method scores full marks.

CORE

D1 $\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x = 2 \left(\frac{1}{2} \cos x - \frac{\sqrt{3}}{2} \sin x \right) = \cos x - \sqrt{3} \sin x$

$\frac{3\sqrt{3}}{2} \sin x = \frac{1}{2} \cos x$, so $\tan x = \frac{1}{3\sqrt{3}} = \boxed{\frac{\sqrt{3}}{9}}$

D2 $10 \sin \theta \cos \theta = \frac{3 \sin \theta}{\cos \theta}$, so $\sin \theta (10 \cos^2 \theta - 3) = 0$

$\sin \theta = 0$ gives 0 or π , not in $0 < \theta < \pi$; $\cos \theta = \pm \sqrt{0.3}$: $\theta = 0.991, \theta = 2.15$

D3 $1 - 2 \sin^2 x = 5 \sin x - 2$, so $2 \sin^2 x + 5 \sin x - 3 = 0$, $(2 \sin x - 1)(\sin x + 3) = 0$; $\sin x = -3$ is impossible

$30^\circ, 150^\circ$

D4 $\tan(A + B) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}} = 1$. Both angles are acute, so $0 < A + B < 180^\circ$: $A + B = 45^\circ$

STRETCH

D5 (a) $\frac{2+t}{1-2t} = -1$, so $2+t = 2t-1$: $\tan y = 3$

(b) $\tan 2x = \frac{4}{1-4} = \boxed{-\frac{4}{3}}$; $\tan(2x+y) = \frac{-\frac{4}{3} + 3}{1 + 4} = \boxed{\frac{1}{3}}$



D6 (a) $\frac{\sin 3x \cos x - \cos 3x \sin x}{\sin x \cos x} = \frac{\sin(3x - x)}{\sin x \cos x} = \frac{2 \sin x \cos x}{\sin x \cos x} = 2$ as required

(b) where $\sin x = 0$ or $\cos x = 0$: $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

D7 (a) $3 \sin 2x + 4(1 + \cos 2x) + 1$: $a = 3, b = 4, c = 5$ (b) $5 \sin(2x + 0.9273) + 5$: maximum 10

(c) $2x + 0.9273 = \frac{\pi}{2}$: $x = 0.322$

D8 (a) $\cos 4x = 2 \cos^2 2x - 1 = 2(2 \cos^2 x - 1)^2 - 1 = 8 \cos^4 x - 8 \cos^2 x + 1$ as required

(b) $\cos \frac{\pi}{2} = 0$: $8c^4 - 8c^2 + 1 = 0$, $c^2 = \frac{2 \pm \sqrt{2}}{4}$; $\cos \frac{\pi}{8} > \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$, so take the larger, positive root:

$\frac{\sqrt{2 + \sqrt{2}}}{2}$

E**Exam practice solutions**

Mark-scheme notes are shown on the right of each line.

CORE

Exam questions for everyone

E1 (a) $3(2 \cos^2 x - 1) + 7 \cos x + 5 = 0$ M1

$6 \cos^2 x + 7 \cos x + 2 = 0$ as required A1*

(b) $(2 \cos x + 1)(3 \cos x + 2) = 0$ M1

$\cos x = -\frac{1}{2}$: $120^\circ, 240^\circ$ A1

$\cos x = -\frac{2}{3}$: $131.8^\circ, 228.2^\circ$ A1

E2 (a) $R \cos \alpha = 2, R \sin \alpha = 5$ M1

$R = \sqrt{29}$, $\tan \alpha = \frac{5}{2}$: $\alpha = 68.2^\circ$ A1 A1

(b) $\sin(x - 68.2^\circ) = \frac{3}{\sqrt{29}}$, with $-68.2^\circ \leq x - 68.2^\circ < 291.8^\circ$ M1

$x - 68.2^\circ = 33.85^\circ$ or 146.15° dM1

$x = 102.1^\circ, x = 214.3^\circ$ A1



STRETCH

- E3** (a) $4 \cos^2 \theta = 2(1 + \cos 2\theta)$, so $f(\theta) = 2 + 2 \cos 2\theta - 3 \sin 2\theta$ as required M1 A1*
- (b) $R \cos \alpha = 2$, $R \sin \alpha = 3$: $\sqrt{13} \cos(2\theta + 0.9828)$ M1 A1
- (c) maximum $2 + \sqrt{13}$ when $2\theta + 0.9828 = 2\pi$: $\theta = 2.650$ B1 B1
- (d) $\cos(2\theta + \alpha) = \frac{3}{\sqrt{13}}$, and $-1 < \frac{3}{\sqrt{13}} < 1$; $2\theta + \alpha$ covers one full period: 2 solutions M1 A1

