GCSE Mathematics

Algebraic Proof

Questions

Pearson Edexcel GCSE & iGCSE Mathematics eClassroom

Section A — Foundation

Worked Examples

[Fluency]

Show that $(n+1)^2 - n^2 = 2n + 1$ for all integer values of n.

$$(n+1)^2 - n^2 = n^2 + 2n + 1 - n^2 = 2n + 1 \checkmark$$

[Reasoning]

Show, by counter-example, that $n^2 + n + 41$ is not always prime.

$$n = 40 :\ 40^2 + 40 + 41 = 1681 = 41 \times 41 \text{ — not prime} \checkmark$$

[Problem Solving]

Verify that $x = 3$ is a solution of $x^2 - 2x - 3 = 0$.

$$3^2 - 2(3) - 3 = 9 - 6 - 3 = 0 \checkmark$$

[Fluency]

1.

Show that $(n+1)^2 - n^2 = 2n + 1$ for all integer values of n.

(2 marks)

[Fluency]

2.

Verify that $x = -3$ is a solution of $x^2 + x - 6 = 0$.

(2 marks)

[Fluency]

3.

Show that the sum of any two consecutive even numbers is divisible by 4.

Hint: let the two numbers be 2n and 2n+2.

(3 marks)

[Reasoning]

4.

Show that $(2n + 1)^2 - 1$ is always a multiple of 4.

(3 marks)

[Reasoning]

5.

Prove that the sum of three consecutive integers is always a multiple of 3.

(3 marks)

[Reasoning]

6.

Find a counter-example to disprove the statement:
'For all positive integers n, $n^2 + n + 1$ is prime.'

(3 marks)

[Reasoning]

7.

A student claims: 'The product of two prime numbers is always odd.'
Disprove this statement with a counter-example.

(2 marks)

[Problem Solving]

8.

Show that $(n + 3)^2 – (n + 1)^2 \neq 4$ for all integer values of n.

(3 marks)

[Problem Solving]

9.

Show that the sum of the first n positive integers is $n(n+1)/2$.
Hint: write the sum forwards and backwards, then add.

(4 marks)

[Problem Solving]

10.

Show that $(x + y)^2 \neq x^2 + y^2$ in general.
For what condition on x and y does equality hold?

(3 marks)

Section B — Higher

Worked Examples

[Fluency]

Prove that the product of two consecutive odd numbers is always odd.

Let the two consecutive odd numbers be (2n+1) and (2n+3).

$$(2n+1)(2n+3) = 4n^2 + 8n + 3 = 2(2n^2 + 4n + 1) + 1$$

This is of the form 2k+1, so is always **odd**. ✓

[Reasoning]

Prove that the sum of three consecutive even integers is divisible by 6.

Let the integers be 2n, 2n+2, 2n+4.

$$2n + 2n + 2 + 2n + 4 = 6n + 6 = 6(n+1) \checkmark$$

[Problem Solving]

Prove that if n is odd, n^2 is odd.

Let n = 2k+1.

$$n^2 = (2k+1)^2 = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1 \checkmark$$

[Fluency]

1.

Prove that the product of two consecutive odd numbers is always odd.

(3 marks)

[Fluency]

2.

Prove that the sum of three consecutive even integers is divisible by 6.

(3 marks)

[Fluency]

3.

Prove that if n is an odd integer, then n^2 is also odd.

(3 marks)

[Reasoning]

4.

Prove that the difference between the squares of any two consecutive integers is always odd.

(3 marks)

[Reasoning]

5.

Prove that $(n + 1)^2 – (n – 1)^2 = 4n$ for all integers n.

(3 marks)

[Reasoning]

6.

Prove that the sum of the squares of two consecutive odd numbers leaves a remainder of 2 when divided by 4.

(4 marks)

[Reasoning]

7.

n is a positive integer. Prove that $n(n+1)(n+2)$ is always divisible by 6.

(4 marks)

[Problem Solving]

8.

Prove that for all real x: $x^2 + 6x + 10 > 0$.

(4 marks)

[Problem Solving]

9.

Prove that the sum of the first n odd positive integers equals n^2.

(4 marks)

[Problem Solving]

10.

a and b are real numbers with $a > b > 0$.
Prove that $a^2 > b^2$.

(4 marks)