



eClassroom

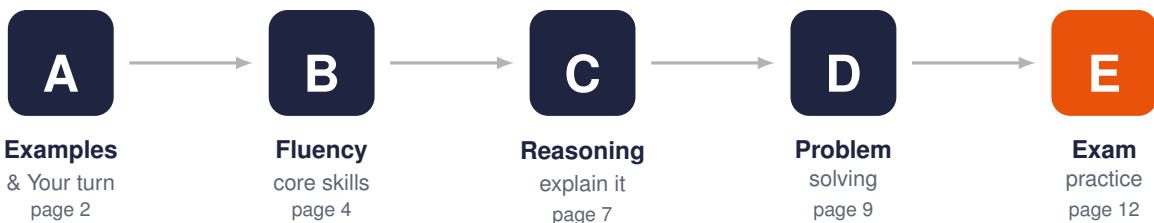
A-LEVEL MATHEMATICS • PURE 1

Quadratics

Worksheet

Pearson Edexcel International A Level (WMA11) & A Level (9MA0) • Calculator allowed: show every step

Free Quadratics worksheet for A-Level and International A-Level Maths, with full worked solutions



Tap a section to jump straight to it

By the end of this worksheet you can: solve quadratic equations exactly, by factorising and by the formula • complete the square and read off the turning point • find the equation of a quadratic curve from its key features • use the discriminant to decide how many real roots an equation has • *Stretch:* find the values of a constant from the discriminant, solve disguised quadratics and prove results.

Which questions are for me?

CORE The methods every A-level student must secure. Start here.

STRETCH Parameters in the discriminant, disguised quadratics in unfamiliar forms, and proof.

Every section has both, clearly labelled. Questions get harder as you go, and every skill in Sections A to D is needed in Section E.

Track your progress

Section	B	C	D	E
Core marks	23	10	13	11
Stretch marks	16	14	17	8
My score				

Full worked solutions

Exam questions by topic

Algebra revision engine

Formula sheet builder

Name

Class

Date



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

A

Worked examples & Your turn

Read each example carefully. Cover it up, then answer the question beside it using the same method.

CORE

Secure the method

Worked example A1

Solve $3x^2 - 4x - 2 = 0$, giving your answers in the

form $\frac{a \pm \sqrt{b}}{c}$

It does not factorise, so use the formula with $a = 3$, $b = -4$ and $c = -2$

$$x = \frac{4 \pm \sqrt{16 + 24}}{6} = \frac{4 \pm \sqrt{40}}{6}$$

Simplify the surd, then cancel: $\sqrt{40} = 2\sqrt{10}$

$$x = \frac{2 \pm \sqrt{10}}{3}$$

Your turn A1

Solve $2x^2 + 6x - 1 = 0$, giving your answers in the

form $\frac{a \pm \sqrt{b}}{c}$

Worked example A2

Write $2x^2 + 12x + 7$ in the form $a(x+b)^2 + c$, and state the coordinates of the turning point.

Take the 2 out of the x terms only, then halve the 6.

$$2(x^2 + 6x) + 7 = 2[(x+3)^2 - 9] + 7$$

$$2(x+3)^2 - 11, \text{ minimum point } (-3, -11)$$

Your turn A2

Write $3x^2 - 18x + 20$ in the form $a(x+b)^2 + c$, and state the coordinates of the turning point.

Worked example A3

Find the values of k for which $(k+1)x^2 + 4x + k - 2 = 0$ has equal roots.

Equal roots: $b^2 - 4ac = 0$, with $a = k+1$, $b = 4$ and $c = k-2$

$$16 - 4(k+1)(k-2) = 0, \text{ so } (k+1)(k-2) = 4$$

$$k^2 - k - 6 = 0, \text{ so } (k-3)(k+2) = 0$$

$$k = 3 \text{ or } k = -2$$

Your turn A3

Find the values of k for which $kx^2 - 6x + k - 8 = 0$ has equal roots.



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk

STRETCH

Worked example A4: a negative x^2 term

Write $4 - 3x - 2x^2$ in the form $p - q(x+r)^2$. Find its maximum.

Take -2 out of the x terms, then halve $\frac{3}{2}$

$$-2\left(x^2 + \frac{3}{2}x\right) + 4 = -2\left[\left(x + \frac{3}{4}\right)^2 - \frac{9}{16}\right] + 4$$

$$\frac{41}{8} - 2\left(x + \frac{3}{4}\right)^2, \quad \max \frac{41}{8} \text{ at } x = -\frac{3}{4}$$

Your turn A4

Write $3 + 5x - 2x^2$ in the form $p - q(x-r)^2$. Find its maximum.

Worked example A5: a curve from its turning point

A quadratic curve has a maximum point at $(3, 8)$ and passes through the point $(5, 0)$. Find its equation in the form $y = ax^2 + bx + c$

Use the turning-point form, then substitute $(5, 0)$

$$y = a(x-3)^2 + 8, \text{ so } 0 = 4a + 8 \text{ and } a = -2$$

$$y = -2x^2 + 12x - 10$$

Your turn A5

A quadratic curve has a minimum point at $(-2, -9)$ and passes through the point $(1, 9)$. Find its equation in the form $y = ax^2 + bx + c$

Worked example A6: a range of values of k

Find the set of values of k for which $x^2 + 2kx + 3k + 4 = 0$ has no real roots.

No real roots: $b^2 - 4ac < 0$

$$4k^2 - 4(3k+4) < 0, \text{ so } k^2 - 3k - 4 < 0$$

$(k-4)(k+1) < 0$, so k lies between the roots.

$$-1 < k < 4$$

Your turn A6

Find the set of values of k for which $x^2 + (k+3)x + 4k = 0$ has two distinct real roots.

Worked example A7: a disguised quadratic

$$\text{Solve } x^{\frac{2}{3}} - 3x^{\frac{1}{3}} - 4 = 0$$

Let $u = x^{\frac{1}{3}}$, so that $x^{\frac{2}{3}} = u^2$

$$u^2 - 3u - 4 = 0, \text{ so } (u-4)(u+1) = 0$$

$x^{\frac{1}{3}} = 4$ or -1 , so cube both sides.

$$x = 64 \text{ or } x = -1$$

Your turn A7

$$\text{Solve } x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 8 = 0$$



B**Fluency**

Each question is a designed sequence. From one part to the next, ask yourself: what changed, and what did it do to the answer?

CORE*Secure the method***B1** Solve

(a) $x^2 - 3x - 28 = 0$

(b) $4x^2 + 4x - 15 = 0$

(c) $x(x+2) = 15$

(d) $9x^2 - 16 = 0$

[4 marks]**B2** Use the quadratic formula to solve each equation, giving exact answers. If there are no real roots, say so.

(a) $x^2 + 6x + 4 = 0$

(b) $x^2 + 6x + 9 = 0$

(c) $x^2 + 6x + 11 = 0$

(d) $2x^2 + 6x + 3 = 0$

[4 marks]**B3** Write each expression in the form $a(x+b)^2 + c$

(a) $x^2 + 10x + 3$

(b) $x^2 - 7x + 2$

(c) $5x^2 + 20x + 12$

(d) $10 - 4x - x^2$

[4 marks]**B4** Find the value of $b^2 - 4ac$, and state how many real roots each equation has.

(a) $x^2 - 5x + 7 = 0$

(b) $4x^2 - 12x + 9 = 0$

(c) $3x^2 + 2x - 5 = 0$

(d) $5x^2 = 2x - 1$

[4 marks]

CORE *continued: graphs and disguised quadratics*

B5 Write down the coordinates of the turning point of each curve, and state whether it is a maximum or a minimum.

(a) $y = (x - 3)^2 + 5$

(b) $y = 2(x + 1)^2 - 7$

(c) $y = 4 - (2x - 1)^2$

[3 marks]

B6 Solve

(a) $x^4 - 17x^2 + 16 = 0$

(b) $x - 6\sqrt{x} + 8 = 0$

(c) $x^6 + 7x^3 - 8 = 0$

(d) $(x + 1)^2 - 5(x + 1) + 6 = 0$

[4 marks]

STRETCH

B7 Write each expression in the form $p(x + q)^2 + r$, where p , q and r are constants. In part (c), give q and r in terms of k .

(a) $3x^2 - 9x + 2$

(b) $5x - 1 - 2x^2$

(c) $x^2 + 2kx + 5$

(d) $(2x - 1)(x + 3)$

[4 marks]

B8 Find the value, or values, of k for which each equation has equal roots.

(a) $x^2 + kx + 9 = 0$

(b) $kx^2 + 4x + k = 0$

(c) $x^2 + (k - 1)x + k + 2 = 0$

(d) $(k - 2)x^2 + 2kx + k + 3 = 0$

[4 marks]



STRETCH*continued: disguised quadratics and graphs***B9** Solve, giving exact answers.

(a) $4^x - 6(2^x) + 8 = 0$

(b) $x - 4\sqrt{x} - 1 = 0$

(c) $\frac{2}{x^2} + \frac{3}{x} - 2 = 0$

(d) $x^4 + 3x^2 - 10 = 0$

[4 marks]**B10** Find the equation of each quadratic curve, giving your answer in the form $y = ax^2 + bx + c$ **(a)** It crosses the x -axis at $x = 1$ and $x = 5$, and passes through $(0, 10)$ **(b)** Its turning point is $(2, -3)$, and it passes through $(0, 5)$ **(c)** It touches the x -axis at $x = -3$, and crosses the y -axis at $(0, -18)$ **(d)** It crosses the x -axis at $x = 1 \pm \sqrt{2}$, and the y -axis at $(0, -3)$ **[4 marks]**

C**Reasoning**

Show your working and explain your thinking. A correct answer without a reason does not score full marks.

CORE

Secure the method

- C1** Priya solves the equation $x^2 = 6x$ by dividing both sides by x , and writes $x = 6$
Explain the mistake Priya has made, and solve the equation correctly. [2 marks]
- C2** Ben writes $2x^2 + 8x + 3 = 2(x + 4)^2 - 29$
Show that Ben is wrong, and write $2x^2 + 8x + 3$ correctly in the form $a(x + b)^2 + c$ [2 marks]
- C3** By completing the square, show that $4x^2 - 12x + 10$ is positive for all real values of x , and state its minimum value. [3 marks]
- C4** A question says: *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.* Solve $3x^2 - 6x - 45 = 0$
Ali writes $(x - 5)(x + 3) = 0$, so $x = 5$ or $x = -3$
Ali's answers are correct. Explain why Ali would still lose marks, and write a correct solution. [3 marks]



STRETCH

- C5** Jo says: "The equation $(k - 2)x^2 + 4x + 1 = 0$ has exactly one real root only when $b^2 - 4ac = 0$ "
Find the value of k that Jo's condition gives. Explain why Jo's statement is not true, and find a second value of k for which the equation has exactly one real root. [3 marks]
- C6** Prove that, for every real value of the constant k , the equation $x^2 + kx + k - 2 = 0$ has two distinct real roots. [3 marks]
- C7** Sam solves $x - 2\sqrt{x} - 8 = 0$ and writes
 $(\sqrt{x} - 4)(\sqrt{x} + 2) = 0$, so $\sqrt{x} = 4$ or $\sqrt{x} = -2$, so $x = 16$ or $x = 4$
Explain why one of Sam's answers is wrong, and state the correct solution. [3 marks]
- C8** (a) By completing the square, prove that the solutions of the equation $ax^2 + bx + c = 0$, where $a \neq 0$, are
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(b) Hence explain why the equation has equal roots when $b^2 = 4ac$ [5 marks]



D**Problem solving**

You decide the method. Read the whole question first, and show every step.

CORE*Secure the method*

- D1** Figure 1 shows a sketch of the curve C with equation $y = f(x)$, where $f(x)$ is a quadratic expression.

The curve C cuts the x -axis at the points $(1, 0)$ and $(9, 0)$, and has a minimum point at $(5, -8)$

- (a) Find $f(x)$, giving your answer in the form $ax^2 + bx + c$, where a , b and c are constants.

(2)

- (b) Write down the coordinates of the point where C cuts the y -axis.

(1)

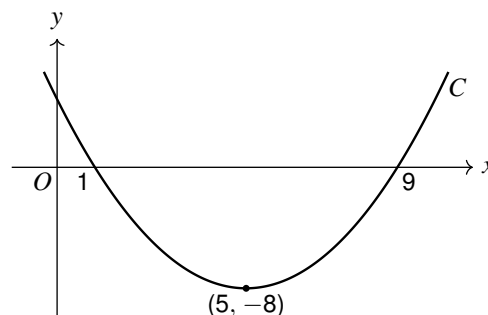


Figure 1

[3 marks]

- D2** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 2 shows the line l with equation $y = x + 11$ and the curve C with equation $y = 2x^2 - 5x - 9$

The line l meets the curve C at the points P and Q , as shown in Figure 2.

Use algebra to find the coordinates of P and the coordinates of Q .

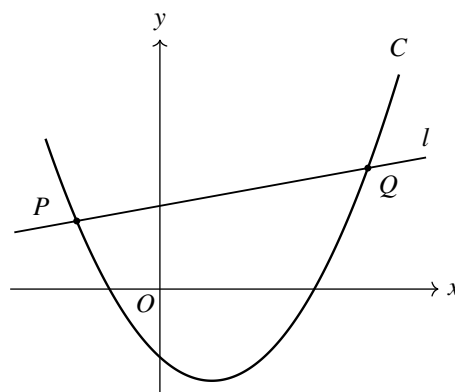


Figure 2

[4 marks]

- D3** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Given that $x \neq 0$,

- (a) show that the equation $3x^2 + \frac{4}{x^2} = 13$ can be written as $3u^2 - 13u + 4 = 0$, where $u = x^2$

(1)

- (b) Hence solve $3x^2 + \frac{4}{x^2} = 13$, giving each answer in its simplest exact form.

(2)

[3 marks]



- D4** A café owner models the daily profit, £ P , made from selling sandwiches when each sandwich is sold for £ x , by the equation

$$P = 360x - 40x^2 - 500 \quad 1 \leq x \leq 8$$

- (a) Write $360x - 40x^2 - 500$ in the form $A - B(x - C)^2$, where A , B and C are positive constants to be found. (2)

- (b) Hence state, according to the model, the maximum daily profit and the selling price that gives this maximum. (1)

[3 marks]

STRETCH

- D5** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

$$(k - 1)x^2 + 6x + k + 7 = 0$$

where k is a constant.

- (a) Solve the equation when $k = -1$, giving your answers in the form $\frac{a \pm \sqrt{b}}{c}$, where a , b and c are integers. (2)

- (b) Find the set of values of k for which the equation has no real roots. (2)

[4 marks]

- D6** Figure 3 shows a sketch of the curve C with equation $y = x^2 - 2x + 4$

The straight lines l_1 and l_2 both pass through the origin O . Each line touches C at exactly one point: l_1 touches C at A and l_2 touches C at B , as shown in Figure 3.

Given that the line with equation $y = mx$, where m is a constant, touches C ,

- (a) show that $m^2 + 4m - 12 = 0$

(2)

- (b) Hence find the coordinates of A and the coordinates of B .

(2)

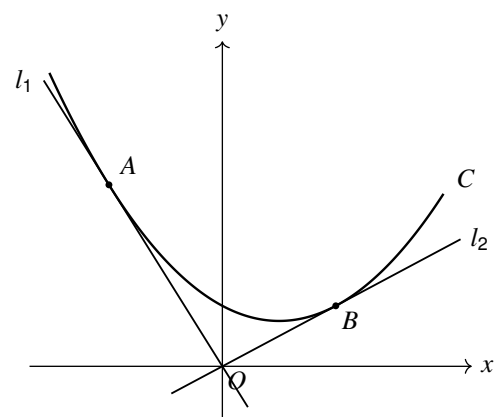


Figure 3

[4 marks]



- D7** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Figure 4 shows a sketch of the curve C with equation $y = 2x^2 - 12x + 13$

The curve C cuts the x -axis at the points A and B , and M is the minimum point of C . The triangle AMB is shaded in Figure 4.

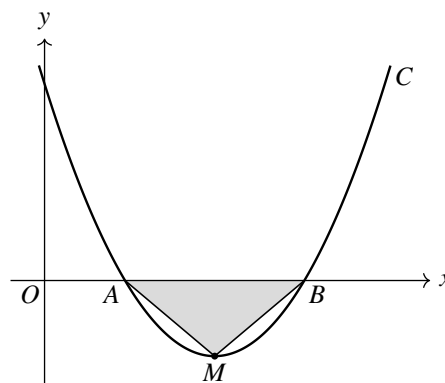


Figure 4

- (a) Write $2x^2 - 12x + 13$ in the form $a(x+b)^2 + c$, where a , b and c are integers.

(2)

- (b) Hence find the exact x coordinates of A and B , giving each in the form $p + q\sqrt{10}$, where p and q are rational.

(1)

- (c) Find the exact area of triangle AMB .

(1)

[4 marks]

- D8** In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

$$x - 6\sqrt{x} + k = 0 \quad x \geq 0$$

where k is a constant.

- (a) Solve the equation when $k = 5$

(2)

- (b) Find the set of values of k for which the equation has two distinct real solutions.

(3)

[5 marks]



E**Exam practice**

Exam-style questions in the style of WMA11. Write your answers in the spaces, and show every step.

CORE

Exam questions for everyone

E1 *In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.*

The curve C has equation $y = x^2 + 2x + 6$

The point $A(1, 0)$ does not lie on C . The straight line l passes through A and has gradient k , where k is a constant.

(a) Write down an equation for l

.....
(1)

Given that l meets C at least once,

(b) show that $k^2 - 8k - 20 \geq 0$

(2)

(c) Hence find the set of possible values of k .

.....
(2)

(Total for Question E1 is 5 marks)



E2 Figure 5 shows a sketch of part of the curve C with equation $y = f(x)$, where

$$f(x) = 8x - 2x^2 - 5$$

(a) Express $f(x)$ in the form $a + b(x + c)^2$, where a , b and c are constants to be found.

(3)

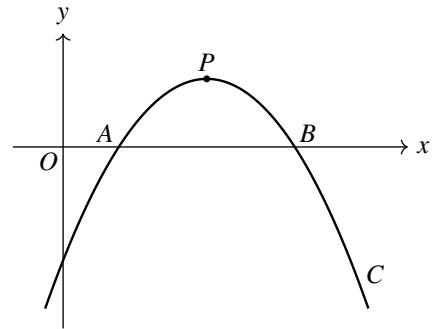


Figure 5

The point P is the maximum point of C .

(b) State the coordinates of P .

.....

(1)

The curve C cuts the x -axis at the points A and B , as shown in Figure 5.

(c) Find the exact length of AB , giving your answer as a simplified surd.

.....

(2)

(Total for Question E2 is 6 marks)



STRETCH

E3 In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

Given that $x \neq 0$,

(a) use the substitution $u = x + \frac{1}{x}$ to show that the equation

$$2x^2 - 9x + 14 - \frac{9}{x} + \frac{2}{x^2} = 0$$

can be written as $2u^2 - 9u + 10 = 0$

(2)

(b) Hence solve $2x^2 - 9x + 14 - \frac{9}{x} + \frac{2}{x^2} = 0$, giving each solution as an exact value and showing each stage of your working.

(3)

(c) Using algebra, find the set of values of the constant k for which the equation $x + \frac{1}{x} = k$ has at least one real solution.

(3)

(Total for Question E3 is 8 marks)

Want more exam practice? The **Exam Questions by Topic** booklets have 10 more full exam questions on Pure 1 Chapter 2, Quadratics, with a full mark scheme.

[Open the booklet](#)

Finished? Mark your work with the **full worked solutions**, fill in your scores on the front page, then keep the skill sharp with unlimited questions on the **Algebra revision engine**.



Join students already acing their exams.

Book your FREE lesson at www.eclassroom.org.uk