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INTERNATIONAL A LEVEL MATHEMATICS • PURE 4

Pure Mathematics 4

Exam Questions by Topic: Mark Scheme

Pearson Edexcel International Advanced Level (WMA14/01) • Calculator allowed

Every question in the booklet, marked line by line the way an examiner marks it

Chapters (tap a chapter to jump to it)	Qs	Marks	Page	My score
1 Proof Proof by contradiction: irrational numbers, primes, integers, inequalities	10	73	own PDF	
2 Partial Fractions Linear and repeated factors, improper fractions	10	78	own PDF	
3 Coordinate Geometry in the (x, y) Plane Parametric equations and conversion to Cartesian form	10	76	3	
4 Binomial Expansion $(a + bx)^n$ for rational n , validity, using partial fractions	10	81	own PDF	
5 Differentiation Parametric and implicit differentiation, rates of change	10	81	own PDF	
Integration Volumes of revolution, substitution, parts, partial fractions, differential equations	10	86	own PDF	
7 Vectors 3D vectors, vector equations of lines, intersections, scalar product	10	82	own PDF	

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How to use this mark scheme

The same marking rules Pearson examiners use for WMA14.

Mark every line, not just the answer. Method marks are given for a correct method even when the final answer is wrong. If you lost marks, find the *first* line where your working differs from the scheme. That is the thing to fix.

Code	Meaning
M1	Method mark: for knowing a correct method and attempting to apply it. Small slips in the working do not lose this mark.
dM1	Dependent method mark: can only be given if the method mark before it has been given.
A1	Accuracy mark: for a correct answer or line of working. It can only be given if the method mark it depends on has been given.
B1	Independent mark: for a correct result or statement, whatever method was used.
*	The answer is given in the question. Every step must be shown, and the answer must follow from correct working (cso).
cao	Correct answer only.
cso	Correct solution only: no errors anywhere in the working.
ft	Follow through: the mark can be given for correct work using your own earlier (wrong) answer.
oe	Or equivalent: any correct equivalent form is accepted.

“Solutions relying on calculator technology are not acceptable.” When a question says this, a correct final answer with no algebraic working scores **no marks**. The notes under each scheme line say when this applies.

Marking a proof. A proof by contradiction earns its final mark only when it has all three parts: a clear assumption at the start (“Assume there is ...”), correct deductions in the right order, and a statement of the contradiction with a conclusion. A sketch or calculator values are not a proof.



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Chapter 3: Coordinate Geometry in the (x, y) Plane

Mark scheme. Marks are shown on the right of each line.

Question 1

5 marks

Part	Scheme	Marks
(a)	$x(t+1) = 2t \Rightarrow t(x-2) = -x \Rightarrow t = \frac{x}{2-x}$	M1
	Substitutes into y : $t-3 = \frac{4x-6}{2-x}$ and $t+1 = \frac{2}{2-x}$	M1
	$y = \frac{4x-6}{2}$, so all points lie on the line $y = 2x - 3$	A1
	<i>The answer must contain no t (Oct 2022 report: y "in terms of both x and t" after a partial rearrangement).</i>	(3)
(b)	Solves with the line: $2t+t-3 = 13(t+1)$ gives $t = -\frac{8}{5}$; or $x = \frac{16}{3}$ on the line $y = 2x - 3$	M1
	$t = -\frac{8}{5}$ is not in $t > -1$ (equivalently $x = 2 - \frac{2}{t+1} < 2$ on C), so C does not meet the line	A1
	<i>Finding the point $(\frac{16}{3}, \frac{23}{3})$ on the full line and stopping there scores M1 A0.</i>	(2)
Total for Question 1: 5 marks		

Question 2

5 marks

Part	Scheme	Marks
(a)	Uses $\sec^2 t = 1 + \tan^2 t$ with $\tan t = x - 2$: $y = 4(1 + (x-2)^2) - 3$	M1
	$f(x) = 4(x-2)^2 + 1$	A1
		(2)
(b)	Domain: $\tan(-\frac{\pi}{4}) = -1$, $\tan \frac{\pi}{3} = \sqrt{3}$, so $1 \leq x \leq 2 + \sqrt{3}$	B1
	$t = 0$ (so $x = 2$) is inside the interval: minimum $y = 1$; ends give $y = 5$ and $y = 13$	M1
	$1 \leq f(x) \leq 13$	A1
	$5 \leq f(x) \leq 13$ (the two end values only) is the common error: the minimum is at $t = 0$, inside the interval.	(3)
Total for Question 2: 5 marks		



Question 3

5 marks

Part	Scheme	Marks
(a)	$y = \sin 2t = 2 \sin t \cos t = x \cos t$	M1
	$y^2 = x^2 \cos^2 t = x^2 \left(1 - \frac{x^2}{4}\right)$	M1
	$4y^2 = x^2 (4 - x^2) \quad *$	A1*
		(3)
(b)	$\sin 2t = 0: t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ (or $y = 0$ in the Cartesian equation)	M1
	$(-2, 0), (0, 0), (2, 0)$	A1
		(2)
Total for Question 3: 5 marks		

Question 4

5 marks

Part	Scheme	Marks
	$y = 2 \left(\cos t \cos \frac{\pi}{6} - \sin t \sin \frac{\pi}{6} \right) = \sqrt{3} \cos t - \sin t$	B1
	$\sin t = \frac{x}{2}$, so $y + \frac{x}{2} = \sqrt{3} \cos t$	M1
	Squares and uses $\cos^2 t = 1 - \sin^2 t = 1 - \frac{x^2}{4}$	M1
	$y^2 + xy + \frac{x^2}{4} = 3 - \frac{3x^2}{4}$	A1
	$x^2 + xy + y^2 = 3$, so $k = 3$	A1
	<i>The value of k must be stated (Oct 2024 report: "very common for the value of k to be omitted").</i>	
		(5)
Total for Question 4: 5 marks		



Question 5

5 marks

Part	Scheme	Marks
(a)	$t = e^x - 1: y = (e^x - 1)^2 - 2(e^x - 1)$	M1
	$y = e^{2x} - 4e^x + 3$, so $p = -4, q = 3$	A1
		(2)
(b)	$y = 0: t(t - 2) = 0$ or $(e^x - 1)(e^x - 3) = 0$	M1
	$(0, 0)$ and $(\ln 3, 0)$	A1
		(2)
(c)	$y = (t - 1)^2 - 1$ and $t = 1$ is allowed: $y \geq -1$	B1
		(1)
Total for Question 5: 5 marks		

Question 6

11 marks

Part	Scheme	Marks
(a)	$y = 0: t(t^2 - 3) = 0$	M1
	$t = \pm\sqrt{3}$ (and $t = 0$ at O)	A1
	Both give $A(3, 0)$, with $t = \sqrt{3}$ and $t = -\sqrt{3}$	A1
		(3)
(b)	$y^2 = t^2(t^2 - 3)^2$	M1
	Substitutes $t^2 = x$	M1
	$y^2 = x(x - 3)^2$ *	A1*
		(3)
(c)	$t^3 - 3t = 2t^2 \Rightarrow t(t^2 - 2t - 3) = 0$	M1
	$t = 3$ or $t = -1$	A1
	$(9, 18)$ and $(1, 2)$	A1
		(3)
(d)	$t = 0$ or $t^2 - mt - 3 = 0$, which always has two different non-zero roots ($m^2 + 12 > 0$); two values of t give the same point only at A , when the roots are $\pm\sqrt{3}$	M1
	$m = 0$: the line $y = 0$ meets C only at O and A	A1
		(2)
Total for Question 6: 11 marks		



Question 7

9 marks

Part	Scheme	Marks
(a)	$\cos t = \frac{x+1}{4}, \sin t = \frac{y-3}{4}$ and uses $\cos^2 t + \sin^2 t = 1$ $(x+1)^2 + (y-3)^2 = 16$ centre $(-1, 3)$, radius 4	M1 A1 A1 (3)
(b)	Upper semicircle ($\sin t \geq 0$ so $y \geq 3$) Ends $(3, 3)$ and $(-5, 3)$; highest point $(-1, 7)$	B1 B1 (2)
(c)	$4 \sin t + 3 = 4 \cos t - 1 + k \Rightarrow 4 \sin t - 4 \cos t = k - 4$ $4\sqrt{2} \sin\left(t - \frac{\pi}{4}\right) = k - 4$ $-\frac{\pi}{4} \leq t - \frac{\pi}{4} \leq \frac{3\pi}{4}$: two solutions when $\frac{\sqrt{2}}{2} \leq \sin\left(t - \frac{\pi}{4}\right) < 1$ $8 \leq k < 4 + 4\sqrt{2}$ <i>Alternative: $k = 8$ is the line through the end $(-5, 3)$ (it also meets $(-1, 7)$), and $k = 4 + 4\sqrt{2}$ is the tangent, where the distance from the centre equals 4. Using the whole circle gives $4 - 4\sqrt{2} < k < 4 + 4\sqrt{2}$: M1 M1 M0 A0.</i>	M1 M1 M1 A1 (4)
Total for Question 7: 9 marks		



Question 8

10 marks

Part	Scheme	Marks
(a)	$\cos t = \frac{x}{50}, \sin t = \frac{y}{30}$ and $\cos^2 t + \sin^2 t = 1$ $\frac{x^2}{2500} + \frac{y^2}{900} = 1$	M1 A1 (2)
(b)	$d^2 = 2500 \cos^2 t + 900 \sin^2 t = 2500 \cos^2 t + 900 (1 - \cos^2 t)$ $= 900 + 1600 \cos^2 t$ *	M1 A1* (2)
(c)	$1600 = 900 + 1600 \cos^2 t \Rightarrow \cos t = \pm \frac{\sqrt{7}}{4}$ $t = 0.848$ and the other three by symmetry $t = 0.848, 2.29, 3.99, 5.44$	M1 M1 A1 (3)
(d)	$d > 40$ when $\cos^2 t > \frac{7}{16}$: for $0 \leq t < 0.848, 2.29 < t < 3.99, 5.44 < t < 2\pi$, total $4 \times 0.848 = 3.392$ minutes 204 seconds (3 minutes 24 seconds)	M1 A1 (2)
(e)	e.g. the runner will not keep exactly to one line of the track, or the runner's speed is not likely to follow the model; a real track has straight sections	B1 (1)
Total for Question 8: 10 marks		



Question 9

10 marks

Part	Scheme	Marks
(a)	$\cos 2t = \cos^2 t - \sin^2 t = \cos^2 t (1 - \tan^2 t)$ $\cos^2 t = \frac{1}{\sec^2 t} = \frac{1}{1 + \tan^2 t}$ $y = \frac{1 - x^2}{1 + x^2}^*$	M1 M1 A1* (3)
(b)	$1 - x^2 = 2 - (1 + x^2)$ $y = -1 + \frac{2}{1 + x^2}$	M1 A1 (2)
(c)	Assume there is a point on C with $y \leq -1$ Then $-1 + \frac{2}{1 + x^2} \leq -1$, so $\frac{2}{1 + x^2} \leq 0$ But $1 + x^2 > 0$, so $\frac{2}{1 + x^2} > 0$: a contradiction. So there is no point with $y \leq -1$ *	B1 M1 A1* (3)
(d)	$\frac{1 - x^2}{1 + x^2} = \frac{1}{2} \Rightarrow x^2 = \frac{1}{3}$ (or $\cos 2t = \frac{1}{2}$, $t = \pm \frac{\pi}{6}$) $\left(\frac{\sqrt{3}}{3}, \frac{1}{2}\right)$ and $\left(-\frac{\sqrt{3}}{3}, \frac{1}{2}\right)$	M1 A1 (2)
Total for Question 9: 10 marks		



Question 10

11 marks

Part	Scheme	Marks
(a)	$\left(\frac{x}{2}\right)^{\frac{2}{3}} = \cos^2 t$ and $\left(\frac{y}{2}\right)^{\frac{2}{3}} = \sin^2 t$ Adds: $\frac{x^{\frac{2}{3}} + y^{\frac{2}{3}}}{2^{\frac{2}{3}}} = 1$ $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2^{\frac{2}{3}}$, so $k = \sqrt[3]{4}$	M1 A1 A1 (3)
(b)	$x^2 + y^2 = 4(\cos^6 t + \sin^6 t)$ $\cos^6 t + \sin^6 t = (\cos^2 t + \sin^2 t)^3 - 3\cos^2 t \sin^2 t (\cos^2 t + \sin^2 t)$ $= 1 - 3\sin^2 t \cos^2 t = 1 - \frac{3}{4} \sin^2 2t$, using $\sin t \cos t = \frac{1}{2} \sin 2t$ $x^2 + y^2 = 4 - 3\sin^2 2t$ *	M1 M1 M1 A1* (4)
(c)	Least when $\sin^2 2t = 1$: $x^2 + y^2 = 1$ Least distance $\boxed{1}$ $t = \frac{\pi}{4} \cdot \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$	M1 A1 A1 (3)
(d)	$\boxed{2}$, at $(2, 0)$, $(0, 2)$, $(-2, 0)$ and $(0, -2)$	B1 (1)
Total for Question 10: 11 marks		

